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Real-Time Combustion Torque Estimation on a Diesel Engine Test Bench Using Time-Varying Kalman Filtering

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Abstract— **We propose an estimator of the combustion torque on a Diesel Engine using as only sensor the easily available instantaneous crankshaft angle speed. The observer consists in a Kalman filter designed on a physics-based time-varying model for the engine dynamics. Convergence is proven, using results from the literature by establishing the uniform controllability and observability properties of this periodic system. A test bench and development environment is presented. Performance is studied through simulations and real test bench experiments.**

I. INTRODUCTION

Performance and environmental requirements impose advance control strategies for automotive applications. In this context, controlling the combustion represents a key challenge. A first step is the control of the combustion torque which characterizes the performance of the engine and is the result of various inputs such as injection quantity and timing, EGR (exhaust gas recirculation) rate . . . .

Ideally this torque could be measured using fast pressure sensors in each cylinder. Unfortunately their cost and reliability prevent them from reaching commercial products lines. As a consequence an interesting problem is the design of a real-time observer for the combustion torque using the reliable and available instantaneous crankshaft angle speed as only measurement.

Combustion torque determination by the measurement of the crankshaft angle speed has been addressed previously in the literature. Most of the proposed solutions have their foundations on a Direct or Indirect Fourier Transform of a black box model (see [10], [11], [7]). Other focus on a stochastic approach (see [8]) but the problem of real-time estimation is not addressed. Other approach such as mean indicated torque are also proposed (see [13] and [14] for example). Solving this first problem opens the door to more exciting applications such as misfiring detection ([1] and [15]) and combustion analysis.

For the design of a combustion torque observer, we use a physics-based model underlying the role of time-varying inertia. A Kalman filter observer is designed and validated both experimentally (on the presented test bench) and theoretically (proof of the convergence). It is computationally tractable on a typical XPC Target (or DSpace system) embedded system capable of handling a 500 μs sampling time.

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The contribution is as follows. In the Section II, we explain the engine dynamics. We describe the combustion torque observer design in Section III. In Section IV, we describe the experimental setup. Simulation and experimental results are presented in Section V. Future directions are given in Section VI.

II. CRANKSHAFT DYNAMICS

A. Continuous time dynamics

In this part, we briefly describe the dynamics of the system stressing out the role of the combustion torque, T_{comb}, also referred as the indicated torque. Following [12], the torque balance on the crankshaft can be written

$$T_{comb} - T_{mass} - T^*_{load} = 0 \quad (1)$$

where $T^*_{load} = T_{load} + T_{fric}$ is referred as "the extended load torque" and T_{load} and T_{fric} are known. The mass torque T_{mass} is the derivative of the kinetic energy E_{mass} of the moving masses in the engine as described in Figure 1.

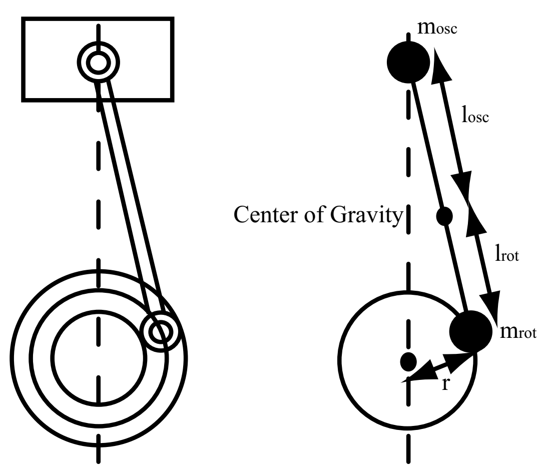

Fig. 1. Mass Model.

$$E_{mass} = \int_0^{2\pi} T_{mass} d\alpha = \frac{1}{2} J(\alpha)\dot{\alpha}^2$$

The mass torque T_{mass} can be expressed as

$$\frac{dE_{mass}}{dt} = T_{mass}\dot{\alpha}$$
$$= (J\ddot{\alpha} + \frac{1}{2}\frac{dJ}{d\alpha}\dot{\alpha}^2)\dot{\alpha}$$

with

$$\left\{\begin{array}{lcl} J(\alpha) & = & m_{rot}r^2 + m_{osc}\sum_{j=1}^{4}(\frac{ds_j}{d\alpha})^2 \\ f = \frac{1}{2}\frac{dJ}{d\alpha} & = & m_{osc}\sum_{j=1}^{4}\frac{ds_j}{d\alpha}\frac{d^2 s_j}{d\alpha^2} \end{array}\right.$$

the computation of the various elements of J are described in [6] and are usually perfectly known for a particular engine. $J(\alpha)$ and $\frac{dJ}{d\alpha}(\alpha)$ are periodic functions in α over an engine cycle.

B. Discrete Time-varying Linear Approximation

The torque balance writes

$$J(\alpha)\ddot{\alpha} = T_{comb}(\alpha) - T^*_{load}(\alpha) - f(\alpha)\dot{\alpha}^2$$

We can reformulate this equation as

$$\dot{\alpha}\frac{d\dot{\alpha}}{d\alpha} = \frac{1}{J(\alpha)}(T_{comb}(\alpha) - T^*_{load}(\alpha) - f(\alpha)\dot{\alpha}^2) \quad (2)$$

Using a first order approximation on the right hand-side of the previous equation, we can break the dependence on time and on the crankshaft angle and only save a dependence on the square of the crankshaft angle speed.

$$\dot{\alpha}^2(n+1) - \dot{\alpha}^2(n) \approx \frac{2\Delta\alpha}{J(n)}(T_{comb}(n) - T^*_{load}(n) - f(n)\dot{\alpha}^2(n))$$

In practice an angular path $\Delta\alpha = 6^o$ is used. Using the square of the crankshaft angle speed $\dot{\alpha}^2$ as the first state variable x_1, we get the linear equation

$$x_1(n+1) = \left(1 - \frac{2\Delta\alpha}{J(n)}f(n)\right)x_1(n) + \frac{2\Delta\alpha}{J(n)}x_2(n) \quad (3)$$

where

$$\left\{\begin{array}{l} x_1(n) = \dot{\alpha}^2(n) \\ x_2(n) = T_{comb}(n) - T^*_{load}(n) \end{array}\right.$$

This formulation of the problem as a two dimensional linear time-varying system suggests that classical methods for combustion torque estimation (x_2) can be used.

C. Mass torque as a filter

The combustion torque generates the movement of the crankshaft. The oscillations of the combustion torque and of the load torque decrease when the engine is accelerating. This oscillation can be described by a low-pass $h(z)$ filter excited by a white noise u(z) as in [9].

$$x_2(z) = h(z)u(z) \quad (4)$$

In the following, $x_2(n)$, is a colored noise.

III. Combustion Torque estimation using Kalman filtering

As stated in Equation (3), the crankshaft is subject to torque excitations created by the combustion process in each cylinder (T_{comb}) which is a highly varying signal (due to combustion cycles and their imperfections). The resulting angular speed has a slowly varying component and a fast varying one resulting from the combustion process.

A colored white noise can be a good representation for the combustion torque. x_2 can be modelled in the z-transform domain as the product of a filter $h(z)$ and a white noise $u(z)$

$$x_2(z) = h(z)u(z)$$

where $h(z)$ is :

$$h(z) = \frac{b_0 + b_1 z^{-1} + \cdots + b_p z^{-p}}{1 + a_1 z^{-1} + \cdots + a_q z^{-q}} \quad (5)$$

This filter is chosen stable, the roots are $\{\lambda_i\}_{i\in\{1,\ldots,q\}}$.

A. Reference model

Gathering past values of x_2 over $[k-q+1,\ k]$, we obtain a time-varying linear system.

$$\left\{\begin{array}{lcl} x_{k+1} & = & A_k x_k + B_k u_k \\ y_k & = & C_k x_k + w_k \end{array}\right. \quad (6)$$

with the state

$$x_k = \begin{pmatrix} \dot{\alpha}^2(k) \\ T_{comb}(k) - T^*_{load}(k) \\ \ldots \\ T_{comb}(k-q+1) - T^*_{load}(k-q+1) \end{pmatrix} \in \mathbb{R}^{q+1}$$

The matrices A_k, B_k and C_k are

$$A_k = \begin{bmatrix} 1 - \frac{2\Delta\alpha}{J(k)}f(k) & v_k \\ 0 & M \end{bmatrix} \in \mathcal{M}_{q+1,q+1}(\mathbb{R}) \quad (7)$$

$$B_k = \begin{bmatrix} 0 & 0 & 0 \\ b_0 & \ldots & b_p \\ 0 & 0 & 0 \\ \ldots & \ldots & \ldots \\ 0 & 0 & 0 \end{bmatrix} \in \mathcal{M}_{q+1,p+1}(\mathbb{R}) \quad (8)$$

$$C_k = \begin{bmatrix} 1 & 0 & \ldots & 0 \end{bmatrix} \in \mathcal{M}_{1,q+1}(\mathbb{R}) \quad (9)$$

with

$$v_k = \begin{bmatrix} \frac{2\Delta\alpha}{J(k)} & 0 & \ldots & 0 \end{bmatrix} \in \mathcal{M}_{1,q}(\mathbb{R}) \quad (10)$$

and

$$M = \begin{bmatrix} -a_1 & -a_2 & -a_3 & \ldots & -a_q \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \in \mathcal{M}_{q,q}(\mathbb{R})$$

Finally, u_k is a white noise. This system is $N = 120$-periodic (since the angle dynamics (2) is 4π periodic and the angle sample is $\pi/30$).

B. Time-varying prediction algorithm

We use a time-varying Kalman predictor for the combustion torque. For purpose we introduce the system

$$\hat{x}_{k+1/k} = A_k \hat{x}_{k/k-1} + L_k(y_k - C_k \hat{x}_{k/k-1}) \tag{11}$$

with the initial condition

$$x_{0/-1} = m_0$$

where L_k is the Kalman gain matrix

$$L_k = A_k P_k C_k^T (C_k P_k C_k^T + R_k)^{-1} \tag{12}$$

In this last expression, the covariance error $P_k = cov(x_k - \hat{x}_{k/k-1})$ is recursively computed through

$$\begin{array}{rcl} P_{k+1} & = & A_k P_k A_k^T + B_k Q_k B_k^T \\ & & -A_k P_k C_k^T (C_k P_k C_k^T + R_k)^{-1} C_k P_k A_k^T \end{array} \tag{13}$$

with $P_0 = cov(x_0)$. At last Q_k and R_k are matrices to be chosen.

C. Convergence

In the general time-varying case, there is no proof of the convergence of the Kalman observer algorithm. Nevertheless linear periodic systems have received a lot of attention for the last twenty years. The Discrete Periodic Riccati Equation (DPRE) properties are used to extend the Kalman filter to periodic systems. A key challenge is to prove the existence and uniqueness of a Symmetric Periodic Positive Solution (SPPS). In short, Bittanti et *al.* exposes sufficient conditions to prove convergence of the estimator. Theses properties are the detectability and stabilizability of the system. To check theses, the Gramian is a handy tool. Since B_* (i.e. the set of all B_k for $k \in \mathbb{N}$) and C_* are constant matrices, the criteria of detectability (resp. stabilizability) is equivalent to the observability (resp. controllability) criteria. The next subsection focuses on checking these last properties, through controllability and observability Gramians. We prove the positiveness of both Gramians and conclude using theorem 1.

Reference model properties

1) A_k's eigenvalues:
To check stability, we investigate A_k's eigenvalues. All the A_k matrices are block upper-triangular, so

$$eig(A_k) = \{1 - \frac{2\Delta\alpha}{J(k)} f(k)\ ,\ \lambda_1\ ,\ \dots\ ,\ \lambda_p\}$$

Both J and $f(k) = \frac{1}{2}\frac{dJ}{d\alpha}(k)$ are periodic while $\frac{2}{J(k)} f(k) = \frac{d\log(J)}{d\alpha}(k)$ is periodic with a 0 mean value. The system is thus unstable when $f(k) > 0$ which occurs half of the time along the engine cycle.

2) Stability of A_*:
The properties of each A_k do not allow us to conclude stability of $A_* = \{A_k\}_{k\in\mathbb{N}}$ as a set. It is a common result that A_* is asymptotically stable if and only if the characteristic multipliers are included in the unitary circle (see [2]). To compute these multipliers we compute by induction the transition matrices
$\forall (k_1, k_2) \in \mathbb{N}^2 \quad k_2 \geq k_1$

$$\Phi(k_2, k_1) = \left[\begin{array}{cc} \pi_{k_2,k_1} & \phi_{k_2,k_1} \\ 0 & M^{k_2-k_1} \end{array} \right]$$

with

$$\phi_{k_2,k_1} = \left\{ \begin{array}{cl} 0 & \text{if} \quad k_2 = k_1 \\ \sum_{j=k_1}^{k_2-1} (\pi_{k_2,j+1} v_j M^{j-k_1}) & \text{if} \quad k_2 > k_1 \end{array} \right. \tag{14}$$

and

$$\pi_{k_2,k_1} = \left\{ \begin{array}{cl} 1 & \text{if} \quad k_2 = k_1 \\ \prod_{i=k_1}^{k_2-1} (1 - \frac{2\Delta\alpha}{J(i)} f(i)) & \text{if} \quad k_2 > k_1 \end{array} \right.$$

We finally have

$$eig(\Phi(N+1, 1)) = \{\pi_{N+1,1}\ ,\ \lambda_1^N\ ,\ \dots\ ,\ \lambda_p^N\}$$

The analytical expression of $J(n)$ allows us to state the $\frac{N}{2}$-periodicity of $J(n)$ and $\frac{d}{d\alpha}(\frac{1}{J})(n)$. Note that this last expression is also symmetric with respect of the $n \mapsto -n$ mapping. Thus

$$\forall k \in \{1, \frac{N}{2}\} \quad \frac{2\Delta\alpha}{J(k)} f(k) + \frac{2\Delta\alpha}{J(N-k)} f(N-k) = 0$$

thus

$$\prod_{i=1}^{N} (1 - \frac{2\Delta\alpha}{J(i)} f(i)) = \prod_{i=1}^{\frac{N}{2}} (1 - (\frac{2\Delta\alpha}{J(i)} f(i))^2) < 1$$

finally $eig(\Phi(N+1, 1)) \subset \mathcal{D}_{0,1}$. Stability of the system is proven. The following result holds

Lemma 1: The system $x_k = A_k x_k + B_k u_k$, $y_k = C_k x_k + w_k$ where A_k, B_k and C_k are given by Equations (7), (8) and (9) is asymptotically stable.

3) Controllability:
To show the controllability of the system, we compute the controllability Gramian W_c over an interval $[k_0, k_0 + k]$ and check its uniform positiveness over k. Since the system is N-periodic, we just have to check positiveness over $k \in [1,\ N]$.

$$W_c(k_f, k_0) = \sum_{i=k_0}^{k_f-1} \Phi_{k_f,i+1} B_i B_i^T \Phi_{k_f,i+1}^T$$

Let us look wether $W_c(k_0 + nN, k_0)$ is positive definite with $n = q+1$ the size of A. Let $V_c(k, i)$ denote the second column of $\Phi(k, i)$ as given in Equation (14). On the other hand

$$B_i^T \Phi(k_f, i+1)^T = \left[\begin{array}{c} b_0 V_c(k_f, i+1)^T \\ b_1 V_c(k_f, i+1)^T \\ \dots \\ b_p V_c(k_f, i+1)^T \end{array} \right]$$

So

$$W_c(k_0+nN,k_0)>0 \Leftrightarrow \bigcap_{i=k_0+1}^{k_0+nN} Ker(V_c(k_0+nN,i)^T)=\{0\}$$

Let

$$\mathcal{V}_c(k_2)=\left[\begin{array}{ccc} V_c(k_2,k_2) & \dots & V_c(k_2,k_2-(q-1)) \end{array}\right]$$

Using $b=\left[\begin{array}{cccc} 1 & 0 & \dots & 0 \end{array}\right]^T \in \mathcal{M}_{q,1}(\mathbb{R})$ we note

$$\begin{aligned}\mathbf{M} &= \left[\begin{array}{ccccc} b & Mb & M^2b & \dots & M^{q-1}b \end{array}\right] \\ &= \left[\begin{array}{cccc} 1 & * & * & * \\ 0 & 1 & * & * \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{array}\right]\end{aligned}$$

And we realize that

$$\mathcal{V}_c(k_2)=\left[\begin{array}{c} * \quad * \\ \mathbf{M} \end{array}\right]$$

The preceding matrix has rank q as $\mathbf{M}$ has.

$$\bigcap_{i=k_0}^{k_0+nN-1} Ker(V_c(k_0+nN-1,i)) \subset Ker(\mathcal{V}_c(k_0+nN))=\{0\}$$

Controllability is thus proven and the following result holds

Lemma 2: The system $x_k = A_k x_k + B_k u_k,\ y_k = C_k x_k + w_k$ where A_k, B_k and C_k are given by Equations (7), (8) and (9) is controllable.

4) Observability:
We now compute the observability Gramian W_o over an interval $[k_0, k_0+k]$ and check its uniform positiveness over k. Again, since the system is periodic, we just have to check positiveness of W_o over $k \in [1,\ N]$. The observability Gramian over $[k_0, k_f]$ is defined by

$$W_o(k_f,k_0)=\sum_{i=k_0}^{k_f} \Phi_{k_f,i}^T C_i^T C_i \Phi_{k_f,i}$$

To check wether $W_o(k_0+nN,k_0)$ is positive definite, we pose

$$\begin{aligned}V_o(k,i) =& C_i\Phi_{k,i} \\ =& \left[\begin{array}{cc} \pi_{k_f,i} & \phi_{k_f,i} \end{array}\right]\end{aligned}$$

We have

$$W_o(k_0+nN,k_0)>0 \Leftrightarrow \bigcap_{i=k_0}^{k_0+nN} Ker(V_o(k_0+nN,i))=\{0\}$$

As before, we pose

$$\mathcal{V}_o(k_2)=\left[\begin{array}{c} V_o(k_2,k_2) \\ V_o(k_2,k_2-1) \\ \dots \\ V_o(k_2,k_2-(q-1)) \end{array}\right] \tag{15}$$

We note $L_1^{(j)}$ the first line of M^j. Due to the analytic expression of v_j as defined in (10) we notice that ϕ_{k_2,k_1} is a linear combination of the elements of $\{L_1^{(j)}\}_{j=0,\dots,k_2-k_1}$. This yields

$$rank(\left[\begin{array}{c} \phi_{k_2,k_2} \\ \phi_{k_2,k_2-1} \\ \dots \\ \phi_{k_2,k_2-(q-1)} \end{array}\right]) = rank(\mathbf{L}) \tag{16}$$

with

$$\mathbf{L}=\left[\begin{array}{c} L_1^{(0)} \\ L_1^{(1)} \\ \dots \\ L_1^{(q-1)} \end{array}\right]$$

Yet $|det(\mathbf{L})| = |det(M)|^{q-1}$. So $\mathbf{L}$ is a full rank matrix and so is $\mathcal{V}_o(k_2)$.

$$\bigcap_{i=k_0}^{k_0+nN} Ker(V_o(k_0+nN,i)) \subset Ker(\mathcal{V}_o(k_0+nN))=\{0\}$$

Observability is proven and the following result holds

Lemma 3: The system $x_k = A_k x_k + B_k u_k,\ y_k = C_k x_k + w_k$ where A_k, B_k and C_k are given by Equations (7), (8) and (9) is observable.

5) Riccati equation for discrete-time periodic systems:
We now focus on the properties of the DPRE described by (13) adapting the results of Theorem 1. The weight matrices R_k and Q_k previously defined are supposed to be constant symmetric definite positive matrices. We have

$$R_k=\tilde{R}\tilde{R}^T \text{ and } Q_k=\tilde{Q}\tilde{Q}^T$$

where $\tilde{R}$ and $\tilde{Q}$ are symmetric definite positive matrices. Let

$$\hat{B}_k=B_k\tilde{Q}_k \text{ and } \hat{C}_k=\tilde{R}_k^{-1}C_k$$

Equation (13) becomes

$$\begin{aligned}P_{k+1} \;=\; & A_kP_kA_k^T+\hat{B}_k\hat{B}_k^T \\ & -A_kP_k\hat{C}_k^T(\hat{C}_kP_k\hat{C}_k^T+I)^{-1}\hat{C}_kP_kA_k^T\end{aligned} \tag{17}$$

As a result, the previous simple change of coordinates yields Theorem 1 formulation.

6) Conclusion on time-varying Kalman filter convergence:
Whatever the choice of the filter h in Equation (5) which defines the combustion model, we proved that the system is stable while each matrix A_k is not. Moreover, we proved the controllability and the observability of the reference system. We finally get on Bittanti et *al*'s conditions (the

observability (resp. controllability) condition is invariant by multiplication of C_* (resp. B_*) by a definite positive matrix) with more general weighting matrices R_k and Q_k as used in Equation (13). All these steps lead to the convergence of the observer.

Proposition 1: With R_k and Q_k constant symmetric definite positive matrices, the Kalman filter state defined in Equations (11,12,13) converges towards the reference model state (6) whatever the choice of the combustion model (5).

IV. Experimental Setup for Control Design

In this paper we deal with a 4-cylinder Diesel engine. For this work we have at hand a Diesel test bench. For simulation purposes this reference system is approximated using a Chmela combustion model [5] (nondimensional combustion model that relies on the concept of mixing controlled combustion avoiding the detailed description of the individual mixture formation and fuel oxidation process) coded in Simulink.

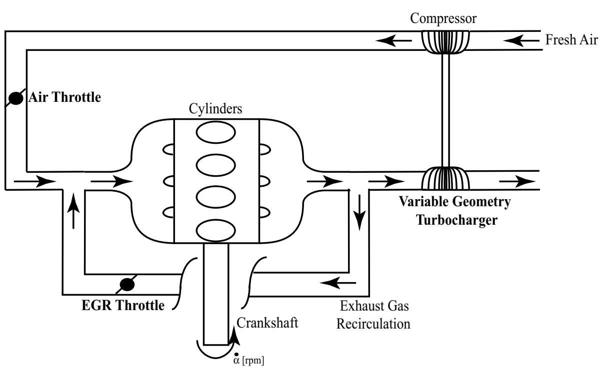

Fig. 2. Engine Scheme.

In our work, we try to restrict most of the design and tuning work to the simulation environment. This reduces the costly work on the engine test bench.

The same code is kept and implemented from the simulation environment to the embedded control system. This HiL (Hardware in the Loop) platform is easily transferred to a fast prototyping system. Typically 1 second of engine simulation is computed in 30 seconds on a 1 GHz Pentium based computer.

V. Simulation and Experimental Results

A. Filter choice

In the following, $x_2(n)$, is a colored noise.

$$h(z) = \frac{(1 - e^{-\delta\Delta\alpha})^2}{(z - e^{-\delta\Delta\alpha})^2} \tag{18}$$

In the discrete-time domain, the state variable $x_2(n)$ can be expressed as

$$x_2(n+2) - 2x_2(n+1)e^{-\delta\Delta\alpha} + x_2(n)e^{-2\delta\Delta\alpha} = (1 - e^{-\delta\Delta\alpha})^2 u(n)$$

B. Results and Comments

In the next figures, we have the comparison of the performance of the observer presented in [4], and the observer presented here. This last observer relies on a pole-placement for a extended state space model of the engine that assumes $\dot{x}_2 = 0$. Though giving qualitatively interesting results it suffers from a lag and a lack of accuracy. T_{mass} is estimated through our observer, then T_{comb} is computed by adding T_{mass} and T^*_{load} according to (1).

1) Simulation results:

We present a simulation corresponding to the following set point:

- Engine Speed : 1000 rpm
- BMEP (Brake Mean Effective Pressure) : 5 bar

To simulate the unbalance, we introduce offsets in the mass injected in each cylinder.

- Cylinder 1: 10% of the reference mass
- Cylinder 2: 0% of the reference mass
- Cylinder 3: 0% of the reference mass
- Cylinder 4: -20% of the reference mass

In Figure 4 the set point is a low engine speed and a low load. This point is very interesting because it represents where the driver feels internal loads and vibrations most. Correcting the unbalance at this points increases the driver's comfort.

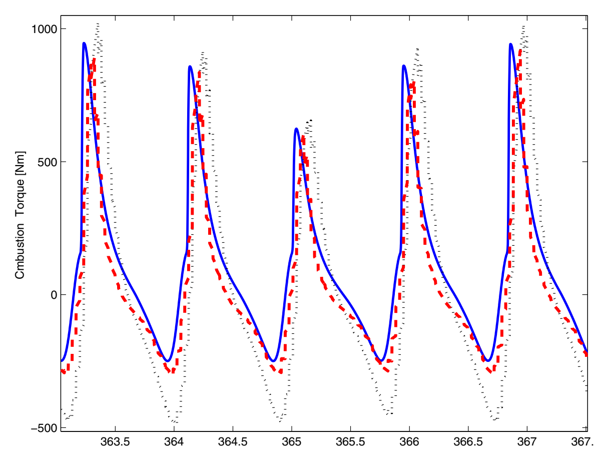

Fig. 4. Combustion torque (1000 rpm, 5 bar). bold (blue) : reference combustion torque, dashed (red) : combustion torque estimated by the time-varying filter, dotted (black) : combustion torque estimated by pole placement as in [4]. Notice the good match between the bold and the dashed signals

2) Experimental Results:

Figures 5 and 6 display the result of the estimator on experimental data. We reconstruct the combustion torque from the bench with the in-cylinder pressure and we test the observer on the flywheel velocity measurement. The set point is different from the simulation one to check robustness.

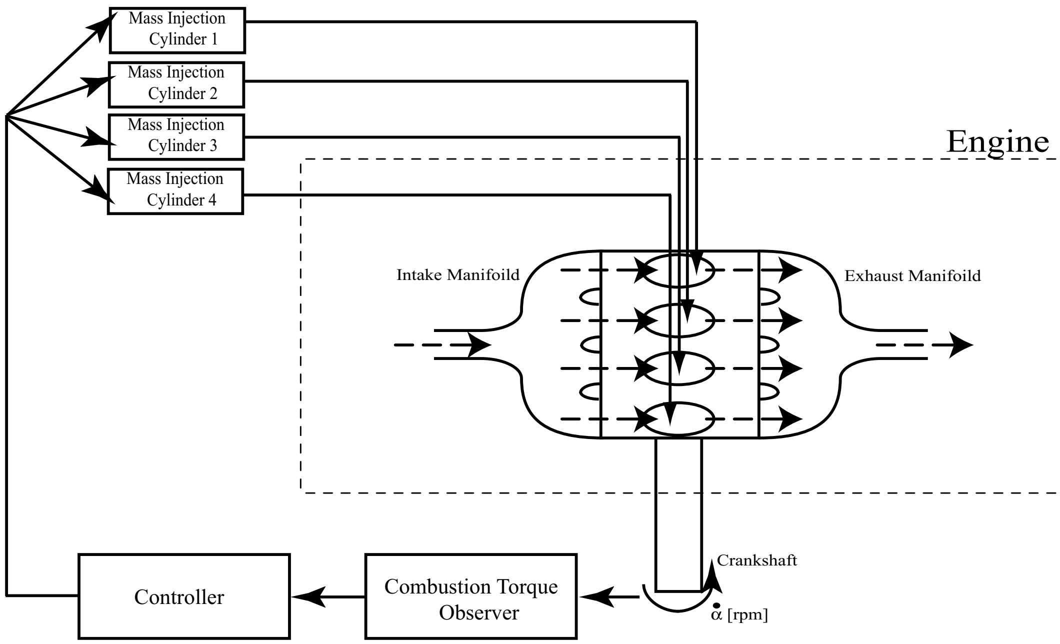

Fig. 3. Global Scheme.

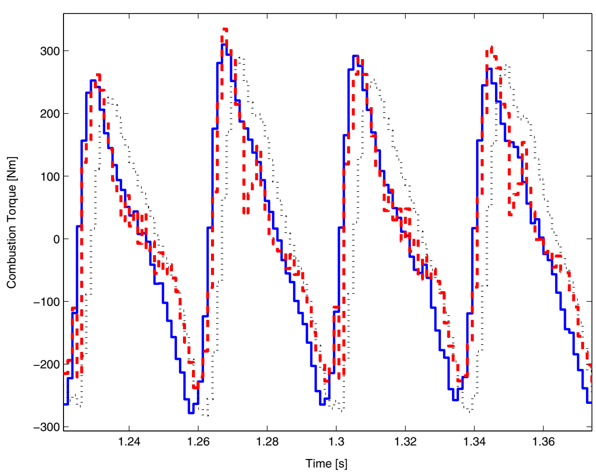

Fig. 5. Combustion torque on the test bench (800 rpm, 2 bar). bold (blue) : reference combustion torque, dashed (red) : combustion torque estimated by the time-varying filter, dotted (black) : combustion torque estimated by pole placement as in [4]. Notice the good match between the bold and the dashed signals

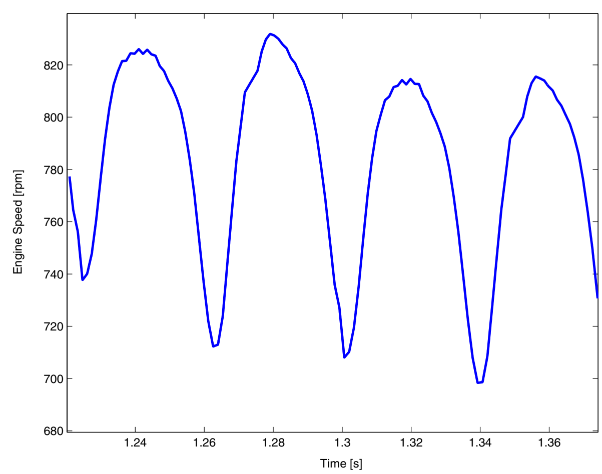

Fig. 6. Engine Speed $[rpm]$ on the test bench used as input of our Kalman filter

3) Comments: Today these results are very satisfactory. An exhaustive testing campaign is underway to evaluate the Kalman filter design under various set points (engine speed and load). The predictor gives better results than the one presented in [4]. In both simulation and test bench cases, we are able to predict the combustion torque dynamics well. Further, we can easily detect the torque unbalance and have a good estimation of the peaks of the combustion torque.

VI. Conclusion and Future Directions

The results of the presented time-varying observer are good. As is, a drawback of our approach is that extensive computations have to be done inline (namely matrix Equations (11), (12) and (13)). Nevertheless, we know that the covariance matrix P_k converges to a periodic solution (see Theorem 3). Moreover the $\{P_k\}_{k\in\mathbb{N}}$ matrices converge towards a periodic solution $\{\bar{P}_k\}_{k=1..N}$. These asymptotic solutions can be computed off-line and used as a gain-scheduling observer.

A numerical study (see results in Table I performed on a Matlab environment with a 1.7 GHz Pentium M (compiled code)) shows the computational effort required

order	2	4	6	8
CPU-time (TV)	0.286	1.03	2.70	5.33
CPU-time (APS)	0.0252	0.0250	0.0336	0.0280

TABLE I

CPU-TIMES ARE GIVEN IN MS FOR A SINGLE FILTER UPDATE. TV: TIME VARYING EXACT RICCATI SOLUTION. APS : ASYMPTOTIC PERIODIC SOLUTION

for various order filters modelling combustion (h filter in Equation (5)). It appears that the preceding substitution of the actual solutions with their asymptotic periodic values has a significant impact on the CPU load, while providing similarly good results.

Finally, we believe that a Kalman filter is a good tool to solve the combustion torque estimation problem for Diesel engines. Its computational demand and efficiency are well balanced. We plan to report further test bench results when an exhaustive test campaign is performed, including EGR 4-cylinders and HCCI combustion mode engines. Note also that tests on a 6-cylinders are scheduled. In this last problem, we have to focus on the overlapping phenomenon of the cylinders torques, that is not present in the 4-cylinders setup.

APPENDIX

Three main theorems are exposed in [3]. They allow to conclude on the convergence of the Kalman predictor in the linear periodic case.

Theorem 1 (Bittanti et al. *[3].):* [Predictor Convergence]
With the above notations, consider the optimal Kalman gain

$$L_k = A_k P_k \hat{C}_k^T (\hat{C}_k P_k \hat{C}_k^T + I)^{-1}$$

associated with *any* semi-definite solution P of (17). If $(A_*, \hat{B}_*)$ is stabilizable and $(A_*, \hat{C}_*)$ detectable, then the corresponding closed-loop matrix $\hat{A}_* = A_* - L_* \hat{C}_*$ is exponentially stable

Theorem 2 (Bittanti et al. *[3].):* [Existence and Uniqueness of a SPPS]
There exists a unique SPPS solution $\bar{P}_*$ of the DPRE and the corresponding closed-loop matrix $\hat{A}_* = A_* - L_* \hat{C}_*$ is asymptotically stable iff $(A_*, \hat{B}_*)$ is detectable and $(A_*, \hat{C}_*)$ reachable.

Theorem 3 (Bittanti et al. *[3].):* [Convergence toward SPPS]
Suppose that $(A_*, \hat{B}_*)$ is stabilizable and $(A_*, \hat{C}_*)$ detectable. Then every symmetric and positive semi-definite solution of the DPRE converges to the unique SPPS solution.

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