Symmetry and field-oriented control of induction motors

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1 Introduction

In 1974 Blaschke and Hasse introduced the so called *field-oriented* method for the control of induction motors [1, 5]. This method is based on a nonlinear coordinate change, admitting a clear physical interpretation since corresponding to a rotation by the rotor flux angle. In these coordinates the equations of the induction motor are very similar to the equations of a DC motor. Since the control of DC motors is much simpler and better understood, field-oriented methods have become very popular. This paper is devoted to a symmetry interpretation of field-oriented control methods (see, e.g., [4] for related works on symmetry and control of linear systems). We show that:

1. the direct field-oriented method admits a nice interpretation in terms of symmetries; for a control problem invariant with respect to a given Lie group of finite dimension this method just consists in a reduction to the base dynamics.

2. the indirect field-oriented method consists in fact in using *symmetries for adding control variables.*

In section 2 we recall the classical model of the induction machine, and its invariance under an arbitrary rotation of the electrical variables. In section 3 we present the direct and indirect field-oriented control methods. In section 4 we propose in a abstract setting the generalization of the direct and indirect field-oriented methods to a control problem invariant under a transformation group of finite dimension s: the direct method consists then in eliminating s state variables corresponding to the orbits; the indirect method consists in adding s new control variables.

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2 Model of the induction motor

2.1 The differential-algebraic model

We recall here the standard two-phase equivalent machine representation of a symmetrical three-phase induction motor, see for instance [6] for a complete derivation of the equations.

The electrical equations describing the stator and rotor circuits are

$$R_s \imath_s + \frac{d\varphi_s}{dt} = u_s \tag{1}$$

$$R_r \imath_r + \frac{d\varphi_r}{dt} = j n_p \omega \varphi_r, \tag{2}$$

where R_s is the stator resistance, $\imath_s$ the stator current, φ_s the stator flux, u_s the voltage applied to the stator, R_r the rotor resistance, $\imath_r$ the rotor current, φ_r the rotor flux, ω the rotor angular velocity, and $j := \sqrt{-1}$; the rotor flux and current are expressed in the stator frame, hence the term $j n_p \omega \varphi_r$ with ω the mechanical speed. We use the complex representations for currents, fluxes and voltages: for instance $\imath_s = \imath_{sa} + j \imath_{sb}$, where $\imath_{sa}$ and $\imath_{sb}$ denote the currents in each of the two stator phases. Since the motor can be actuated through u_s, there are two independent (real) controls u_{sa} and u_{sb}.

Under the assumptions of linearity of the magnetic circuits (valid as long as the currents are not too large) and neglecting iron losses, fluxes and currents are related by

$$\varphi_s = L_s \imath_s + L_m \imath_r \tag{3}$$

$$\varphi_r = L_m \imath_s + L_r \imath_r, \tag{4}$$

where, L_s the stator inductance, L_r the rotor inductance, L_m the mutual inductance between the stator and the rotor.

As a consequence of the Lorenz force law, the torque produced by the motor is

$$n_p \Im(\imath_r^\star \varphi_r),$$

where $\Im(z)$ stands for the imaginary part of the complex number z and $z^\star$ for its conjugate. The motion of the rotor is thus given by

$$J \frac{d\omega}{dt} = n_p \Im(\imath_r^\star \varphi_r) - \tau_L, \tag{5}$$

with J the moment of inertia of the rotor and τ_L the load torque. Equations (1–5) form the differential-algebraic model of the induction motor.

2.2 Rotational invariance

Consider the transformation, g_δ, $\delta \in \mathbb{R}$, defined by

$$\begin{pmatrix} u_s \\ \imath_s \\ \varphi_s \\ \imath_r \\ \varphi_r \end{pmatrix} \mapsto \begin{pmatrix} U_s \\ I_s \\ \Phi_s \\ I_r \\ \Phi_r \end{pmatrix} = \exp(j\delta) \begin{pmatrix} u_s \\ \imath_s \\ \varphi_s \\ \imath_r \\ \varphi_r \end{pmatrix},$$

while ω remains unchanged, which rotates the electrical variables by the angle δ. Clearly, (1)–(5) remains unchanged by this transformation:

$$\begin{aligned} \frac{d\Phi_s}{dt} + R_s I_s &= U_s \\ \frac{d\Phi_r}{dt} - jn_p\omega\Phi_r + R_r I_r &= 0 \\ \Phi_s &= L_s I_s + L_m I_r \\ \Phi_r &= L_m I_s + L_r I_r \\ J\frac{d\omega}{dt} &= \Im(\Phi_r I_r^\star) - \tau_L. \end{aligned}$$

In other words, the transformation g_δ is a *symmetry* of the system. The set $(g_\delta)_{\delta\in\mathbb{R}}$ forms a transformation group G isomorphic to the abstract group $\mathbb{S}^1$.

Notice also that the control objectives (in general tracking for ω and rotor flux module) are also invariant. Hence, the complete control problem is invariant under the transformation group G.

2.3 The current-fed explicit model

For the sake of simplicity, we will use in the sequel a reduced model of the motor. Indeed, for most motors the leakage factor $1 - L_s L_r / L_m^2$ is small, which allows to neglect the stator dynamics. This yields the classical reduced model

$$J\frac{d\omega}{dt} = L_m/L_r \Im(\imath_s^\star \varphi_r) - \tau_L \tag{6}$$

$$T_r\frac{d\varphi_r}{dt} = (-1 + jn_p T_r\omega)\varphi_r + L_m \imath_s, \tag{7}$$

where $T_r = L_r/R_r$ and $\imath_s$ can be considered as the control.

This reduced model is of course invariant under the transformation group G with g_δ now reduced to

$$\begin{pmatrix} \imath_s \\ \varphi_r \end{pmatrix} \mapsto \begin{pmatrix} I_s \\ \Phi_r \end{pmatrix} = \exp(j\delta) \begin{pmatrix} \imath_s \\ \varphi_r \end{pmatrix}.$$

3 Field-oriented methods

3.1 Direct field-oriented control

The field-oriented control method [1, 5] consists in using the so-called $(d-q)$ frame defined by the transformation $g_{-\alpha}$ where α is the rotor flux angle:

$$\varphi_r = \rho \exp(\jmath\alpha), \quad \rho > 0.$$

Setting $\imath_{sd} + \jmath\, \imath_{sq} := exp(-\jmath\alpha)\, \imath_s$ (6,7) becomes

$$\begin{aligned} J\frac{d\omega}{dt} &= L_m/L_r\ \rho\ \imath_{sq} - \tau_L \\ T_r\frac{d\rho}{dt} &= (L_m\imath_{sd} - \rho) \\ T_r\rho\frac{d\alpha}{dt} &= n_p T_r \omega\rho + L_m \imath_{sq}. \end{aligned}$$

Thanks to the rotational invariance, the equations for ω and ρ do not depend on α. Moreover, since the control objectives are invariant, the dynamic of α can be forgotten and we simply have to design a controller for the two first equations:

$$J\frac{d\omega}{dt} = L_m/L_r\ \rho\ \imath_{sq} - \tau_L \tag{8}$$

$$T_r\frac{d\rho}{dt} = (L_m\imath_{sd} - \rho) \tag{9}$$

with $\imath_{sd}$ and $\imath_{sq}$ as controls. If the state variables were measured, the controller would be obvious since there are two states and two controls (this explains why the direct current $\imath_{sd}$ is often called the magnetizing current and the quadrature current $\imath_{sq}$ the "torque" current). In general, the flux is not measured and some flux observer must be used.

3.2 Indirect field-oriented control

In the indirect field-oriented method we use instead of rotor flux angle α a time-varying angle $\delta(t)$

$$\begin{aligned} J\frac{d\omega}{dt} &= L_m/L_r \Im(I_s^\star \Phi_r) - \tau_L \\ T_r\frac{d\Phi_r}{dt} &= (-1 + jT_r(n_p\omega - \frac{d\delta}{dt}))\Phi_r + L_m I_s, \end{aligned}$$

where $\varphi_r = \Phi_r \exp(j\delta(t))$, $\imath_s = I_s \exp(j\delta(t))$. The arbitrary time function $\omega_s(t)$ called the stator velocity,

$$\frac{d\delta}{dt} = \omega_s,$$

can be seen as an extra control. This provides a system with with 3 control inputs and 3 states, which is obviously very easy to control when all the states are measured.

Notice that contrarily to the direct field oriented method, the transformation is always known (since δ is computed). This is very interesting because the control law can be designed without a flux observer: it suffices to use the flux reference as an estimate of the actual flux [10, 3, 2].

4 Generalization

We now generalize the field-oriented methods to the Lie group framework (see, e.g.,[9] for a summary of standard results on Lie groups).

Consider the control system

$$\frac{dx}{dt} = f(x, u), \quad x \in M \tag{10}$$

where M is the state manifold and a Lie transformation group G of dimension s acting on the manifold M.

Definition 1. System (10) admits G as a symmetry group if and only if for each $g \in G$ there exists a regular static feedback $U = k_g(x, u)$ is the new control) such that for all x and u

$$Dg(x).f(x, u) = f(g(x), k_g(x, u)).$$

One also says that (10) is invariant under G.

This definition just says that the vector field f is invariant up to a regular static feedback, i.e., the equation $\dot{x} = f(x, u)$ remains unchanged after the change of variables $(x, u) \mapsto (X, U) = (g(x), k(x, u))$, that is $\dot{X} = f(X, U)$.

4.1 Reduction to the base dynamics

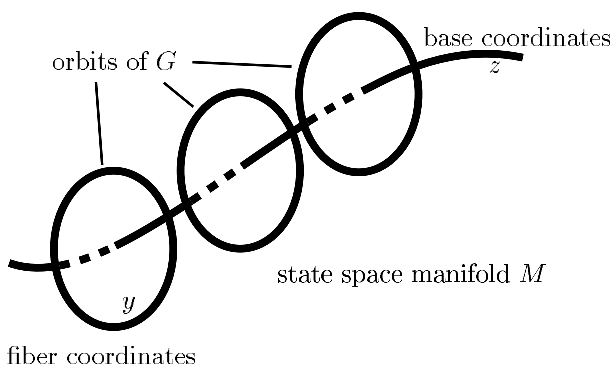

Figure 1: foliation of the state manifold.

We first generalize the direct field-oriented method: it consists in reducing the dynamics to the base dynamics which is decoupled from the fiber dynamics (see figure 1). We assume that the action of G is regular with s the dimension of the orbits.

Locally around any point of M, it is always possible to choose rectifying coordinates $(y, z) \in \mathbb{R}^s \times \mathbb{R}^{n-s}$ ((y, z) close to 0, such that the action of any $g \in G$ near the identity reads

$$g \cdot (y, z) = (\gamma(y, z), z)$$

with γ a smooth map. The z-coordinates, which are left unchanged by G, are called the base-coordinates. The y-coordinates are called fiber or orbit coordinates. For the induction motor z (resp. y) corresponds to (ω, ρ) (resp. α). Denoting by (f^y, f^z) the components of f according to the (y, z)-coordinates,

$$f = f^y \frac{\partial}{\partial y} + f^z \frac{\partial}{\partial z},$$

invariance implies that, for each $g \in G$ close to identity, there exists a feedback $v = k_g(x, u)$ such that

$$f^z(y, z, u) = f^z(\gamma(y, z), z, k_g(y, z, u)).$$

Since (y, z) and $(0, z)$ belong to the same orbit, there exists $g \in G$ close to identity sending (y, z) to $(0, z)$. For the induction motor g consists in a rotation by minus the rotor flux angle α. Thanks to the feedback $v = k_g(y, z, u)$, we must now control only

$$\dot{z} = f^z(0, z, v),$$

which is independent of the orbit variables y. The feedback $v = k(y, z, u)$ can be obtained by solving the overdetermined system:

$$f^z(y, z, u) = f^z(0, z, v).$$

Invariance ensures this system admits a solution.

4.2 Addition of control variables

We assume here the action of G on M locally effective, hence we do not distinguish between elements of $\mathcal{G}$, the Lie algebra of G, and the infinitesimal generators of the action of G on M. The Lie algebra $\mathcal{G}$ is spanned by s independent vector fields $\{S_1, \dots, S_s\}$ on M. For each $x \in M$, the tangent space to the orbit passing through x is nothing but the vector space spanned by $\{S_1(x), \dots, S_s(x)\}$.

Take $t \mapsto g_t$ a curve on G, then its velocity $\dot{g}_t$ can be expressed as a linear combination of the S_i's:

$$\dot{g}_t(x) = \sum_{i=1}^{s} \omega_i(t) S_i(g_t(x)),$$

where the s real functions $t \mapsto \omega_i(t)$ are arbitrary when the curve $t \mapsto g_t$ is arbitrary.

Under the time-varying change of coordinates $X = g_t(x)$, $\dot{x} = f(x,u)$ reads

$$\begin{aligned} \frac{dX}{dt} &= Dg_t(x) \cdot \frac{dx}{dt} + \dot{g}_t(x) \\ &= Dg_t(x) \cdot f(x,u) + \sum_{i=1}^{s} \omega_i(t) S_i(g_t(x)). \end{aligned}$$

Invariance implies the existence of a feedback $U = k_{g_t}(x,u)$ such that $Dg_t(x) \cdot f(x,u) = f(X,U)$. Hence

$$\frac{dX}{dt} = f(X,U) + \sum_{1}^{s} \omega_i S_i(X), \tag{11}$$

where the ω_i can be considered as extra control variables.

The design of a controller for $\dot{x} = f(x,u)$, then splits into two steps. The first step consists in designing a control law for (11), e.g., a feedback $U = K(X)$ and $\omega_i = r_i(X)$, $i = 1, \ldots, s$: this design is easier since it involves s additional independent inputs.

The second step is just the computation of the actual control u from the knowledge of the feedback functions K and r_i and the actual state x. We have $X = g_t(x)$ with g_t the flow of the time-varying vector field

$$\sum_{1}^{r} w_i(t) S_i(z).$$

At this stage the computation of g_t is highly simplified by the use of a matrix representation of G: g_t reduces then to the integration of a matrix equation of the form

$$A^{-1}(t) \frac{dA}{dt}(t) = \sum_{1}^{r} w_i(t) N_i$$

where $A(t)$ is the time-varying matrix associated to g_t and the constant matrix N_i corresponds to the infinitesimal generator S_i. For the induction motor (6,7), the computation of g_t boils down to a direct integration of ω_s, which plays the role of ω_1 with $s = 1$.

Once g_t is known, X is known and the actual control u results form $Dg_t(x) f(x,u) = f(X,U)$. Notice the static feedback $U = K(X)$ and $w_i = r_i(X)$ designed for (11) leads to a dynamic feedback for the original system, the dynamic part being associated to g_t.

5 Conclusion

The dynamics of the induction motor admits a rather rich structure. In this paper we have shown that field-oriented control strategies strongly rely on the rotational invariance of the systems and thus its physical structure. Another important physical property of such system is dissipation and passivity and can be exploited for control design (see, e.g., [8, 11] for recent developments).

Dissipation implies that the electrical part of the dynamics is contracting. More precisely, we have,with the stator and rotor flux variables

$$\frac{d}{dt}\begin{pmatrix}\varphi_s\\ \varphi_r\end{pmatrix} = A(t)\begin{pmatrix}\varphi_s\\ \varphi_r\end{pmatrix} + \begin{pmatrix}u_s\\ 0\end{pmatrix}$$

where

$$A(t) = \begin{pmatrix}R_s & 0\\ 0 & R_r\end{pmatrix}\begin{pmatrix}L_r & L_m\\ L_m & L_s\end{pmatrix}^{-1} + \begin{pmatrix}0 & 0\\ 0 & -jn_p\omega(t)\end{pmatrix}.$$

The matrix $A(t)$ admits a symmetric part (for the Hermitian product defined by $\Delta = \mathrm{diag}(R_r^{-1}, R_s^{-1})$), that is $\Delta A + A^\star \Delta$, negative definite (see [7] for generalizations) It is clear that this property is invariant under rotation, i.e., multiplication by a complex of modulus one. This implies that the derived Lyapounov functions are invariant by rotation. Such combinations of Lyapounov and symmetries will be investigated in the future.

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