Robustness of rotor position observer for permanent magnet synchronous motors with unknown magnet flux

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Abstract: We introduce a new sensorless rotor position observer for permanent magnet synchronous motors which does not require the knowledge of the magnet's flux : only electrical measurements and (approximate) knowledge of the resistance and inductance are needed. This observer extends the gradient observer from Lee et al. [2010] with the estimation of the magnet's flux and makes it globally convergent provided the rotation speed remains away from zero. We study its sensitivity to uncertainties on the resistance and inductance and to the presence of saliency. Its performances in open-loop are illustrated via an implementation using real data and compared to other existing magnet flux independent observers in terms of computational cost and robustness.

1. INTRODUCTION

1.1 Context

To minimize the cost and increase the reliability of Permanent Magnet Synchronous Motors (PMSM), it is still important to make progress on estimating their state variables, in particular the rotor position and speed, with a minimum of sensors and fast algorithms. To this end, studies have been made for a long time on the so-called "sensorless" control which uses no mechanical variables measurement, only electrical ones. A review of the first used methods was given in Acarnley and Watson [2006]), then a Luenberger observer was proposed in Poulain et al. [2008]. More recently, a very simple gradient observer, proposed in Lee et al. [2010] and analyzed in Ortega et al. [2011], has been shown to be extremely effective in practice as rotor position estimator. From the theoretical view point it is only conditionally convergent but it was shown in Malaizé et al. [2012] how, via a very minor modification, it can be made globally convergent thanks to convexity arguments.

These observers require typically the knowledge of the resistance, magnet flux and inductance. Unfortunately while the latter may be considered as known and constant (as long as there is no magnetic saturation), the other two do vary significantly with the temperature and these variations should be taken into account in the observer. For example, for a given injected current, when the magnet's temperature increases, its magnetic flux decreases, and the produced torque becomes smaller. Therefore, an online estimation of the magnet's flux enables to :

- adapt the control law in real time and thus ensure a torque control which is robust to the machine's temperature ;
- have an estimation of the rotor's temperature
- have an estimation of the magnet's magnetization degradation with time.

That is why efforts have been made to look for observers which do not rely on the knowledge of those parameters. For instance, in Romero et al. [2016], the authors propose and study via simulations an adaptive observer to make the gradient observer previously mentioned independent from the resistance.

In this paper, we focus on observers which require the knowledge of the resistance and the inductance, but not of the magnet flux. First steps in this direction are reported in Henwood et al. [2012] with the design of a Luenberger observer (see Henwood [2014] for a much more detailed analysis), and in Bobtsov et al. [2015a,b, 2016], with the design of an observer based on tools from parameter linear identification. In fact, we will show that those two observers rely on the same regression equation but the former solves it at each time whereas the latter solves it with time with a gradient-like scheme. Convergence comes with an assumption of invertibility of the regressor matrix for the former, and on a persistent excitation condition for the latter.

Here, we start by proposing, for the same goal, another observer which is a direct extension, with estimation of the magnet flux, of the gradient observer obtained in Lee et al. [2010]. We claim its global convergence.

Then, for these various observers, we study the sensitivity of the estimates to errors in the resistance and inductance, and also to the action of (ignored) saliency and we illustrate and compare their performances in open-loop through simulations on real data.

1.2 System model and problem statement

Using Joule's and Faraday's laws, a simpler PMSM model expressed in a fixed $\alpha\beta$-frame reads

$$\dot{\Psi} = u - R\,i \tag{1}$$

where Ψ is the total flux generated by the windings and the permanent magnet, (u, i) are the voltage and intensity of the current in the fixed frame and R the stator winding resistance. The quantities u, i and Ψ are two dimensional vectors, and, for the case of a non-salient PMSM, the total flux may be expressed as

$$\Psi = Li + \Phi \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{2}$$

where L is the inductance, Φ the magnet's flux, and θ the electrical phase. This relation implies

$$|\Psi - Li|^2 - \Phi^2 = 0 \tag{3}$$

and the electrical phase θ is nothing but the argument of $\Psi - Li$. Therefore, in the case where L and i are known, θ can be recovered simply through an estimate of the total flux Ψ.

Our interest in this work is about observers of Ψ using measurements of u and i, (approximate) knowledge of R and L but not of Φ. In particular, we look at the computational cost and study how the estimate they give depend on uncertainties on R, L and saliency. To guarantee observability, we assume, all along, the electric rotation speed $\omega = \dot{\theta}$ remains away from 0.

Notations : The rotation matrix of angle θ is denoted $\mathcal{R}(\theta)$, i-e

$$\mathcal{R}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \ .$$

2. GRADIENT OBSERVER

In Lee et al. [2010], the authors proposed the gradient observer :

$$\dot{\hat{\Psi}} = u - \hat{R}i - 2q\,(\hat{\Psi} - \hat{L}i)\,\left(\left|\hat{\Psi} - \hat{L}i\right|^2 - \Phi^2\right). \qquad (4)$$

where $\hat{R}$ and $\hat{L}$ are estimates of R and L the model parameters. This observer turned out to be quite efficient in practice but it was proved in Ortega et al. [2011] that it was only conditionally convergent. In particular it may admit several equilibrium points depending on the rotation speed ω. In fact, later in Malaizé et al. [2012], it was shown that taking rather the following gradient observer, based on the "convexified" expression $\max\left(\left|\hat{\Psi} - \hat{L}i\right|^2 - \Phi^2\,,\,0\right)$,

$$\dot{\hat{\Psi}} = u - \hat{R}i - 2q\,(\hat{\Psi} - \hat{L}i)\,\max\left(\left|\hat{\Psi} - \hat{L}i\right|^2 - \Phi^2\,,\,0\right) \quad (5)$$

enables to achieve global asymptotic stability when $\hat{R} = R$ and $\hat{L} = L$.

In this paper, we propose the following observer for Ψ and Φ

$$\begin{cases} \dot{\hat{\Psi}} \ = \ u - \hat{R}i - 2\gamma\,(\hat{\Psi} - \hat{L}i)\,\left(\left|\hat{\Psi} - \hat{L}i\right|^2 - \hat{\Phi}^2\right) \\ \dot{\hat{\Phi}} \ = \ \gamma\,\hat{\Phi}\left(\left|\hat{\Psi} - \hat{L}i\right|^2 - \hat{\Phi}^2\right) \end{cases} \qquad (6)$$

where γ is an arbitrary strictly positive real number. It is a straightforward extension of the gradient observer (4).

Let $\Psi(\psi, t)$ be the solution at time t of model (1) satisfying (3) initialized at ψ at time 0. Similarly, let $(\hat{\Psi}(\hat{\psi}, \hat{\phi}, t), \hat{\Phi}(\hat{\psi}, \hat{\phi}, t))$ be the solution at time t of observer (6) initialized at $(\hat{\psi}, \hat{\phi})$ at time 0.

Theorem 1. Assume that there exists a strictly positive number $\underline{w}$ such that ω is lower-bounded by $\underline{w}$, and that the state variables of the PMSM are bounded. Assume also $\hat{R} = R$ and $\hat{L} = L$. Then, for any strictly positive real number γ, for any $(\psi, \hat{\psi}, \phi)$ in $\mathbb{R}^4 \times (0, +\infty)$, we have

$$\lim_{t\to\infty} |\hat{\Psi}(\hat{\psi}, \hat{\phi}, t) - \Psi(\psi, t)| + |\hat{\Phi}(\psi, \phi, t) - \Phi| = 0 \ .$$

Taking $\hat{\theta}$ as the argument of $\hat{\Psi} - \hat{L}i$, we also obtain according to (2)

$$\lim_{t\to\infty} \hat{\theta} - \theta = 0 \ .$$

The proof of Theorem 1 goes with changing coordinates, building an appropriate weak Lyapunov function and studying the invariant sets. Unfortunately it is too long and too technical to be given here.

This theorem tells us that unlike for observer (4), no convexification is needed to achieve global convergence of the gradient observer (6). Hence, even when the parameter Φ is known, we may prefer to use observer (6) instead of observer (4). In this way, although the observer state is augmented with $\hat{\Phi}$, we get global convergence and independence with respect to Φ.

3. ALTERNATIVE PATH

The observer presented in the previous section is based on the system

$$\begin{cases} \dot{\Psi} \ = u - Ri \\ \dot{\Phi} \ = 0 \\ \ y \ = |\Psi - Li|^2 - \Phi^2 \end{cases} \qquad (7)$$

with inputs (u, i), state (Ψ, Φ) and measurement y which is constantly zero. This system is nonlinear because of its output function. Fortunately, this function is quadratic in (Ψ, Φ), and $(\dot{\Psi}, \dot{\Phi})$ does not depend on (Ψ, Φ). Hence linearity can be obtained by time derivation. Namely, we have

$$\dot{y} = 2\,(\Psi - Li)^T\,(u - Ri - \dot{\widehat{Li}})$$

which is linear in Ψ and independent from Φ. The new problem we face now is the presence of the time derivative $\dot{\widehat{Li}}$. A well known fix to this, is to use a strictly causal filter. Namely, let

$$\dot{\eta} \ = \ -\lambda(\eta + y) \quad , \quad y_f = \eta + y \qquad (8)$$

with λ any complex number with strictly positive real part. It is easy to check that the evaluation of $y_f + (c+2Li)^T\Psi - (z + L^2|i|^2)$, along any solution, decreases as $\exp(-\lambda t)$ when c and z are solutions of

$$\begin{cases} \dot{c} \ = \ -\lambda c - 2\lambda Li - 2(u - Ri) \\ \dot{z} \ = \ -\lambda z + c^T(u - Ri) - \lambda L^2|i|^2 \ . \end{cases} \qquad (9)$$

So, instead of the design model (7), we can use :

$$\begin{cases} \ \dot{\Psi} \ = \ u - Ri \\ y_f \ = \ -(c + 2Li)^T\Psi + (z + L^2|i|^2) \end{cases} \qquad (10)$$

with inputs (u, i, c, z), state Ψ and measurement y_f. Also because of (8), we pick y_f constantly zero as we did above with y. The system (10) can be seen as a linear time varying system and therefore any observer design for such systems apply. It can be a Kalman filter or more simply the following gradient observer :

$$\begin{cases} \dot{c} = -\lambda c - 2\lambda\hat{L}i - 2(u - \hat{R}i) \\ \dot{z} = -\lambda z + c^T(u - \hat{R}i) - \lambda\hat{L}^2|i|^2 \\ \dot{\hat{\Psi}} = u - \hat{R}i + \gamma\left(c + 2\hat{L}i\right)\left(-(c + 2\hat{L}i)^T\hat{\Psi} + z + \hat{L}^2|i|^2\right). \end{cases} \qquad (11)$$

where γ is an arbitrary strictly positive real number. In Bobtsov et al. [2015a], the authors propose the following non minimal version of this observer :

$$\begin{cases} \ \hat{\Psi} \ = \ \xi_{14} + \xi_{89} \\ \dot{\xi}_{14} \ = \ u - \hat{R}i \\ \ \dot{\xi}_5 \ = \ -\lambda(\xi_5 - |\xi_{14} - \hat{L}i|^2) \\ \dot{\xi}_{89} \ = \ \gamma\Omega\left(y - \Omega^T\xi_{89}\right) \\ \ \Omega \ = \ -\lambda(c + 2\hat{L}i) \\ \ y \ = \ -\lambda|\xi_{14} - \hat{L}i|^2 - \lambda\xi_5 \end{cases} \qquad (12)$$

where c verifies the dynamics (11) and we have the relation

$$z = \xi_{14}^T(c + \xi_{14}) + \xi_5 \ .$$

with z satisfying (11).

Convergence of these observers (11) or (12) is guaranteed as long as Ω satisfies a persistent excitation condition which, as proved in Bobtsov et al. [2015a], holds when the rotation speed is sufficiently rich.

Inspired from nonlinear Luenberger observers, another observer is proposed in Henwood et al. [2012]. It consists in using m filters of the type (9), with poles λ_k, with k in $\{1,\dots,m\}$, to obtain m equations in $\hat{\Psi}$

$$(c_k + 2\hat{L}i)^T\hat{\Psi} - (z_k + \hat{L}^2|i|^2) = 0 \tag{13}$$

which are solved in a least square sense. It is proved in Henwood [2014] that the matrix of the $c_k + \hat{L}i$ is full column rank when ω stay away from 0, $m \geq 3$ and the λ_k are chosen in a generic way.

Actually, observer (11), observer (12) of Bobtsov et al. [2015a], or the one in Henwood et al. [2012], are identical except in their way of solving in $\hat{\Psi}$ equations (13). The former two solve (13) with only one λ ($m = 1$) but dynamically along time. The later solve them at each time, with at least two λ ($m \geq 2$).

In the remainder of the paper, we intend to compare the performances of observer (6) introduced in the previous section with those of this other family of observers, in particular observer (11).

4. PERFORMANCES

4.1 Computational cost

We already see that the small dimension of observer (6) and its great simplicity of implementation provides a significant advantage. Indeed, in our matlab simulations, CPU time was found to be twice smaller than for the other observers presented in Section 3. This numerical efficiency constitutes an important feature since those observers are intended to run online where processing power is often limited.

4.2 Sensitivity to the presence of saliency when i_d is constant

According to Bodson and Chiasson [1998], the simplest way to take saliency into account in the model of a PMSM is to keep (1) but to replace the expression (2) of the total flux by

$$\Psi = L_0 i + L_1 \begin{pmatrix}\cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta\end{pmatrix} i + \Phi \begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} \tag{14}$$

where L_1 is a second order inductance. Thanks to the identity

$$\begin{pmatrix}\cos 2\theta + 1 & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta + 1\end{pmatrix}\begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix} = 2\begin{pmatrix}\cos\theta & 0 \\ \sin\theta & 0\end{pmatrix}$$

the above expression of Ψ can be rewritten as

$$\Psi - (L_0 - L_1)i = (\Phi + 2L_1 i_d)\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} \tag{15}$$

with the notation

$$i_{dq} = \begin{pmatrix} i_d \\ i_q \end{pmatrix} = \mathcal{R}(-\theta)\, i\ . \tag{16}$$

This shows that, when i_d is constant, we recover exactly the design model (7) provided we replace L and Φ by

$$L_s = L_0 - L_1 \quad , \quad \Phi_s = |\Phi + 2L_1 i_d|\ .$$

Hence Theorem 1 holds in the case with saliency at least when the signals obtained from the motor are such that i_d is constant. Specifically, by implementing observer (6) with $\hat{L} = L_s$, we directly obtain :

$$\lim_{t\to\infty} |\hat{\Psi}(\psi,\phi,t) - \Psi(t)| + |\hat{\Phi}(\psi,\phi,t) - \Phi_s| = 0$$

This means that $\hat{\Psi}$ converges to Ψ and $\hat{\Phi}$ to the "equivalent flux" Φ_s. But this time, it is not sufficient to compute the argument of $\hat{\Psi} - L_s i$ to obtain an estimate of θ, since according to (15), it converges either to θ or $\theta + \pi$ depending on the sign of $\Phi + 2L_1 i_d$. In fact, defining θ_0 as the argument of $\Psi - L_s i$ and $i_{dq,0}$ as

$$i_{dq,0} = \begin{pmatrix} i_{d,0} \\ i_{q,0} \end{pmatrix} = \mathcal{R}(-\theta_0)\, i$$

we have :

- if $\Phi + 2L_1 i_d > 0$, then $\Phi_s = \Phi + 2L_1 i_d$, $\theta_0 = \theta$, $i_{d,0} = i_d$ and $\Phi_s - 2L_1 i_{d,0} = \Phi > 0$
- if $\Phi + 2L_1 i_d < 0$, then $\Phi_s = -\Phi - 2L_1 i_d$, $\theta_0 = \theta + \pi$, $i_{d,0} = -i_d$ and $\Phi_s - 2L_1 i_{d,0} = -\Phi < 0$.

Therefore, computing the argument $\hat{\theta}_0$ of $\hat{\Psi} - L_s i$, and $\hat{i}_{dq,0}$ defined by

$$\hat{i}_{dq,0} = \begin{pmatrix} \hat{i}_{d,0} \\ \hat{i}_{q,0} \end{pmatrix} = \mathcal{R}(-\hat{\theta}_0)\, i\ ,$$

and taking

$$\begin{aligned}\hat{\theta} &= \hat{\theta}_0 && \text{if } \hat{\Phi} - 2L_1 \hat{i}_{d,0} \geq 0 \\ \hat{\theta} &= \hat{\theta}_0 + \pi && \text{otherwise ,}\end{aligned}$$

we obtain convergence of $\hat{\theta}$ to θ. This convergence is a clear argument in favor of observer (6) with respect to observer (5). Indeed, the flexibility provided by the estimation of Φ enables to apply the same observer to salient motors without losing convergence of θ. The same conclusions hold for the observers presented in Section 3. Not to be forgotten, all this holds when i_d is constant.

4.3 Sensitivity to errors on R and L when (i_d, i_q, ω) is constant

In Theorem 1, we claimed convergence for observer (6) assuming perfect knowledge of the resistance and the inductance and the absence of saliency. Then, in the latter subsection, we extended this result to salient models as long as the current in the dq frame i_d is constant. We study here the possible consequences of having $\hat{R}$ and $\hat{L}$ different from R and L. For this we restrict our attention to the case where $\mathcal{R}(-\theta)\, i = i_{dq}$ and ω are constant. This configuration is often considered in practice, since it corresponds to a constant rotation speed with a constant load torque. In this case the model with saliency made of (1) and (14) has an asymptotic behavior given by

$$u = \mathcal{R}(\theta) u_{dq}\ , \quad i = \mathcal{R}(\theta) i_{dq}\ , \quad \Psi = \mathcal{R}(\theta)\Psi_{dq}$$

where u_{dq}, i_{dq} and Ψ_{dq} are constant satisfying

$$\omega J \Psi_{dq} = u_{dq} - R i_{dq} \quad , \qquad \Psi_{dq} - L_s i_{dq} = \begin{pmatrix}\Phi_s \\ 0\end{pmatrix}$$

where

$$J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\ .$$

Let Ψ_{eq} be defined as

$$\Psi_{eq} = \frac{1}{\omega} J^{-1}\mathcal{R}(\theta)(u_{dq} - \hat{R} i_{dq})$$

It satisfies

a) $$\dot{\Psi}_{eq} = u - \hat{R} i$$

i-e the same dynamics as Ψ but with $\hat{R}$ instead of R.

b) $$\Psi_{eq} - \hat{L} i = \mathcal{R}(\theta)\underbrace{\left(\frac{1}{\omega}J^{-1}\left(u_{dq} - \hat{R} i_{dq}\right) - \hat{L} i_{dq}\right)}_{\text{constant}}\ . \tag{17}$$

		$R+1\%R$		$L+1\%L$	
ω	Obs	$\tilde{\theta}$ (rad)	$\tilde{\Phi}/\Phi$	$\tilde{\theta}$ (rad)	$\tilde{\Phi}/\Phi$
500 rpm	(6)	0.015	2.6 %	$5.4\,10^{-3}$	0.3 %
	(11)	0.015	2.6 %	$5.2\,10^{-3}$	0.3 %
2000 rpm	(6)	$3.8\,10^{-3}$	0.7 %	$5.4\,10^{-3}$	0.3 %
	(11)	$3.3\,10^{-3}$	0.6 %	$4.9\,10^{-3}$	0.3 %

Table 1. Sensitivity of observers (6) and (11) with respect to R and L at two different electrical rotation speeds with the notation $\tilde{\theta} = |\hat{\theta} - \theta|$ and $\tilde{\Phi}/\Phi = \frac{|\hat{\Phi}-\Phi|}{\Phi}$.

Thus, with Φ_{eq} the constant real number defined as

$$\begin{aligned}\Phi_{eq} &= \left|\frac{1}{\omega}J^{-1}\left(u_{dq} - \hat{R}i_{dq}\right) - \hat{L}i_{dq}\right| \\ &= \left|\begin{pmatrix}\Phi_s\\0\end{pmatrix} + \left([R-\hat{R}]\frac{J^{-1}}{\omega} + L_s - \hat{L}\right) i_{dq}\right|\end{aligned}$$

we have

$$|\Psi_{eq} - \hat{L}i|^2 - \Phi_{eq}^2 = 0 \ .$$

It follows that Ψ_{eq} is solution of the model (1)-(3) if we replace (R, L, Φ) by $(\hat{R}, \hat{L}, \Phi_{eq})$. So, according to Theorem 1, the observer (6), implemented with $\hat{R}$ and $\hat{L}$, gives

$$\lim_{t\to\infty} |\hat{\Psi}(\psi,\phi,t) - \Psi_{eq}(t)| + |\hat{\Phi}(\psi,\phi,t) - \Phi_{eq}| = 0 \ .$$

Hence $\hat{\Phi}$ converges to $\left|\begin{pmatrix}\Phi_s\\0\end{pmatrix} + \left([R-\hat{R}]\frac{J^{-1}}{\omega} + L_s - \hat{L}\right) i_{dq}\right|$.

And with $\hat{\theta}$ computed as the argument of $\hat{\Psi} - \hat{L}i$, we have asymptotically

$$\begin{aligned}\left|\hat{\Psi} - \hat{L}i\right| &\begin{pmatrix}\cos(\hat{\theta}-\theta)\\ \sin(\hat{\theta}-\theta)\end{pmatrix} \\ &= \mathcal{R}(-\theta)\left(\Psi_{eq} - \hat{L}i\right) \\ &= \frac{1}{\omega}J^{-1}\left(u_{dq} - \hat{R}i_{dq}\right) - \hat{L}i_{dq} \\ &= \begin{pmatrix}\Phi_s\\0\end{pmatrix} + \left([R-\hat{R}]\frac{J^{-1}}{\omega} + L_s - \hat{L}\right) i_{dq} \ , \qquad (18)\end{aligned}$$

where we have used (17). In other words the error $\hat{\theta}-\theta$ converges to the argument of $\begin{pmatrix}\Phi_s\\0\end{pmatrix} + \left([R-\hat{R}]\frac{J^{-1}}{\omega} + L_s - \hat{L}\right) i_{dq}$. Up to the first order, this is exactly the same result as the one obtained in Henwood [2014] for the Luenberger observer presented in Henwood et al. [2012]. Of course we recover the fact that without any errors on R and L, the asymptotic value of $\hat{\Phi}$ is Φ_s and $\hat{\theta}$ converges to θ.

We illustrate formula (18) via simulations with ideal data obtained for $L = 0.65$ mH, $R = 0.167\,\Omega$, $\Phi = 7.3$ mWb, $i_d = -3.46$ A, $i_q = 6$ A, for two different regimes. The results are given in Table 1 for observers (6) and (11). Both observers were implemented with an Euler scheme with $dt = 1.2\,10^{-4}$ s and give similar results. The reader may check that the absolute error on θ and the relative error on Φ correspond exactly to the expected theoretical errors.

5. TESTS WITH REAL DATA

To illustrate the results above about the sensitivity with respect to the parameters, to saliency, but also to noise, we applied in open-loop (and offline) the observers (6) and (11) to real data obtained from two PMSM used in test beds at IFPEN : Motor 1 and Motor 2. The available data are the measurements of voltages u_m and currents i_m in the $\alpha\beta$ fixed frame, the measurement of the rotor position θ_m, the physical parameters given in Table 2.

Parameter	Motor 1	Motor 2
Regime	variable : Figure 1	constant : 2000 rpm
L_d	0.72 mH	0.142 mH
L_q	0.78 mH	0.62 mH
Φ	8.94 mWb	18.5 mWb
R	$0.151\,\Omega$	$0.023\,\Omega$
Pairs of poles (n_p)	10	2

Table 2. Parameters for Motor 1 and 2.

The norms of u_m and i_m for each motor are given in Figures 2 and 5. Note that unlike Motor 2, Motor 1 is submitted consecutively to four regimes : around 150 rpm, 450 rpm, 1000 rpm and finally 1500 rpm (see Figure 1).

The motors differ in terms of saliency. According to Bodson and Chiasson [1998], L_0 and L_1 in (14) are given by

$$L_0 = \frac{L_d + L_q}{2} \quad , \quad L_1 = \frac{L_d - L_q}{2} \ .$$

and therefore

$$L_s = L_0 - L_1 = L_q \ .$$

We conclude that saliency is weak for Motor 1 ($\frac{L_1}{L_0} \approx 4\%$), but significant for Motor 2 ($\frac{L_1}{L_0} \approx 80\%$).

We have implemented the observers using the measured values u_m and i_m as u and i, and an explicit Euler scheme with the sample time ($dt_1 = 10^{-4}$ s, $dt_2 = 2\,10^{-5}$ s). We chose the parameters of the observers to ensure the responses have all approximately the same time constant ($\gamma_{(6)} = 20000$, $\gamma_{(11)} = 50000$, $\lambda = 50$) and so that convergence is obtained in less than two rotations of the motor. The results are presented in Figures 3-4 for Motor 1 and in Figures 6-7 for Motor 2. The performances are globally better for Motor 1 than Motor 2, but it is mainly due to the fact that the data were noisier for the latter.

For θ (Figures 3 and 6), both observers provide similar results, with a final oscillatory error of amplitude smaller than 0.05 rad for Motor 1 (0.09 rad for the last regime) and 0.12 rad for Motor 2. But (the mean value of) the estimation $\hat{\theta}$ does not converge to the measurement θ_m. There are static errors. They are likely due, in part at least, to an offset in the sensor for θ_m. But there is more since, according to Figure 3, these biases depend on the regime. One explanation comes from (18) where the regime ω appears explicitly. Another possible explanation has been proposed and studied in Henwood [2014]. It is the effects of the dynamics of the sensors providing the measurements u_m and i_m. When they are modelled simply by

$$\dot{i}_m = -\tau_i(i_m - i) \quad , \qquad \dot{u}_m = -\tau_u(u_m - u)$$

the phase shift of these first order systems (depending on the regime) is directly translated in a static error on $\hat{\theta}$ and consequently on $\hat{\Phi}$. We refer the reader to Henwood [2014] for more details.

Concerning Φ (Figures 4 and 7), although both observers provide again the same mean for the final errors, the transient of observer (11) seems to be more oscillatory. This difference could be explained by the fact that $\hat{\Phi}$ is directly estimated by observer (6) while it is reconstructed from the norm of $\hat{\Psi} - L_q i$ for observer (11). Here again (the mean value of) $\hat{\Phi}$ does not tend to Φ. Let us concentrate on the data from Motor 2 and from the first regime of Motor 1, where the norm of the current is constant. Assuming that the offset $\hat{\theta} - \theta_m$ mentioned above is only due to the position of the sensor and therefore that $\hat{\theta}$ is actually the correct rotor position, we compute i_d as the first component of $\mathcal{R}(-\hat{\theta})i$ and find

$$\begin{array}{ll} \text{Motor 1:} & i_{d,1} = -4.2\,\text{A} \\ \text{Motor 2:} & i_{d,2} = -201\,\text{A}\ . \end{array}$$

If the values of R, L_d, L_q and Φ in Table 2 are correct, we can expect $\hat{\Phi}$ to tend to $\Phi_s = \Phi + 2L_1 i_d$, i-e

$$\begin{array}{ll} \text{Motor 1:} & \Phi_{s,1} = 9.2\,\text{mWb} \\ \text{Motor 2:} & \Phi_{s,2} = 115\,\text{mWb}\ . \end{array}$$

This is verified for both motors on Figures 4 (first regime) and 7. We could conclude that the values of R and L used in the observers are correct. Unfortunately we cannot go further in the analysis since, for the other regimes in Figure 4, the steady state is not reached.

6. CONCLUSION

We have introduced a new rotor position observer for sensorless permanent magnet synchronous motors (PMSM). It is designed from a non salient model and uses measurements of voltages and current, and estimations of resistance and inductance. But it does not need the knowledge of the magnet flux. We have claimed its convergence in an ideal context and for a rotating motor.

We have compared it with the equivalent observers proposed in Henwood et al. [2012], Henwood [2014] and Bobtsov et al. [2015b]. The main difference is that this new observer is less demanding in terms of computations. On the other hand it gives qualitatively the same kind of performance, in terms of speed of convergence, sensitivity to errors in the resistance or the inductance and also in presence of saliency.

At least three important issues remain to be addressed:

a) Sensitivity to measurement noise or more interestingly the definition of a tuning policy in presence of such disturbances. This kind of study has been made in Henwood [2014] for the Luenberger observer proposed in Henwood et al. [2012] . The same kind of tools should be useful in our context.
b) Use of the observer in closed loop. Tests via simulations or test beds for the observers in Henwood et al. [2012] and Bobtsov et al. [2015b] are reported in those papers. But as far as we know no theoretical results are yet available.
c) Extension to non salient models. We are unaware of any observer for this case.

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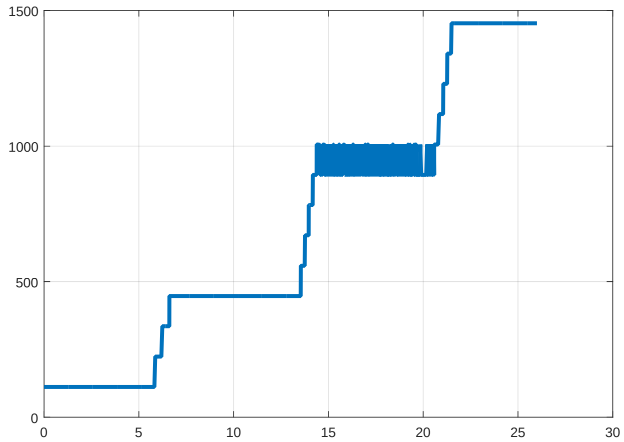

Fig. 1. Motor 1 : Regime (rpm).

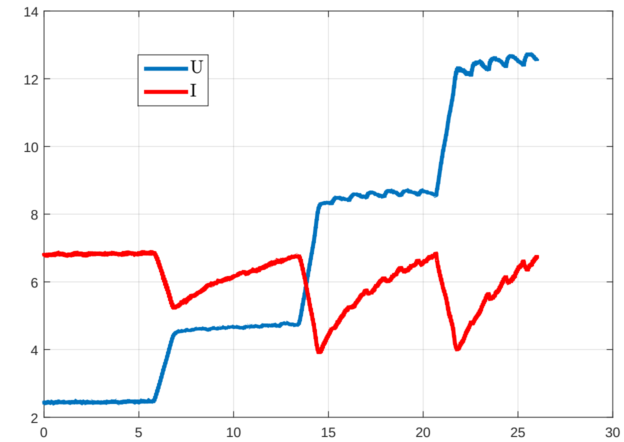

Fig. 2. Motor 1 : Norm of the voltage u_m (V) and current i_m (A).

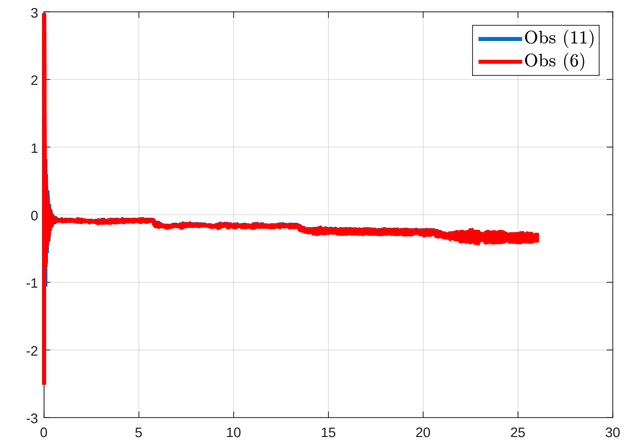

Fig. 3. Motor 1: Error $\hat{\theta} - \theta_m$ (rad) given by observers (6) and (11) , where θ_m is a measurement of θ

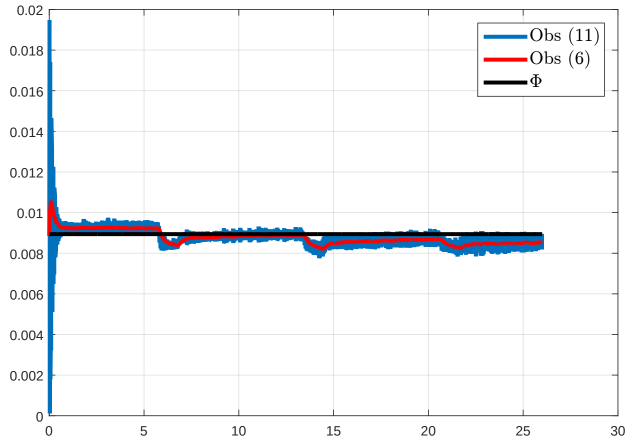

Fig. 4. Motor 1: $\hat{\Phi}$ given by observers (6) and (11) compared to Φ

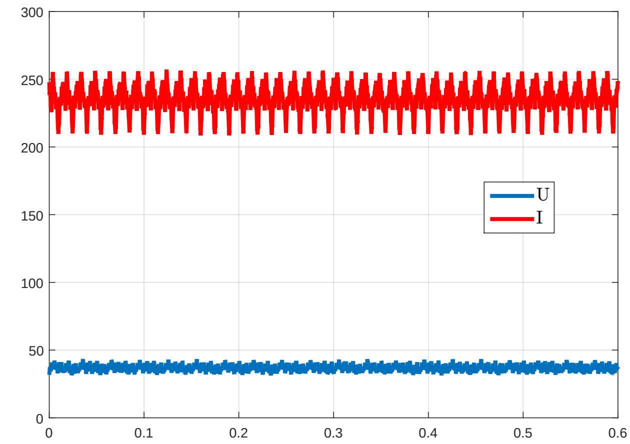

Fig. 5. Motor 2 : Norm of the voltage u_m (V) and current i_m (A).

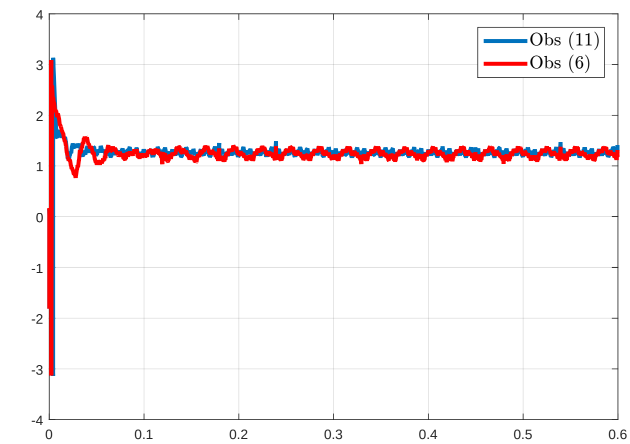

Fig. 6. Motor 2: Error $\hat{\theta} - \theta_m$ (rad) given by observers (6) and (11) , where θ_m is a measurement of θ

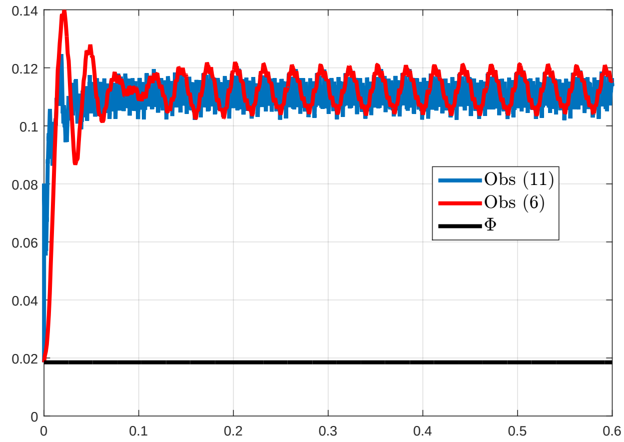

Fig. 7. Motor 2: $\hat{\Phi}$ given by observers (6) and (11) compared to Φ

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