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Stabilization of an Electrostatic MEMS Including Uncontrollable Linearization

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***Abstract*— Electrostatic micro-actuators are not linearly controllable in a set containing the origin due to a quadratic term of electrical variable appearing as the input to the mechanical subsystem. Consequently, such systems are not feedback linearizable and thus not differentially flat on this set and the application of techniques based on feedback linearization leads usually to an unbounded control. This work aims at developing control schemes which should be bounded everywhere in the whole operational range. As there are no existing general frameworks for tackling the control design for the system under consideration, the approach of Lyapunov design combined with backstepping is used. The obtained control scheme is proved to stabilize the system at the above mentioned uncontrollable set. Furthermore, we address the output feedback control using a reduced order observer and certainty-equivalence implementation. The closed-loop stability is demonstrated by both stability analysis and numerical simulation.**

I. INTRODUCTION

This paper addresses the problem of the control of a one degree of freedom (1DOF) parallel-plate electrostatic actuator driven by a voltage source. The schematic representation of such a device is given in Fig. 1, where m is the mass of the movable upper electrode, b is the damping coefficient, k is the elastic constant, A is the area of electrodes, G is the air gap, G_0 is the zero-voltage gap, x is the normalized deflection, and R is the loop resistance. This is one of the most popular devices in micro-electromechanical systems (MEMS), such as micro-mirrors, optical gratings, variable capacitors, and accelerometers. This simple MEMS is often modeled as a rigid body, though it may be considered as a reduced-order model of an infinite dimensional micro-device, e.g. micro-beam or micro-plate [16].

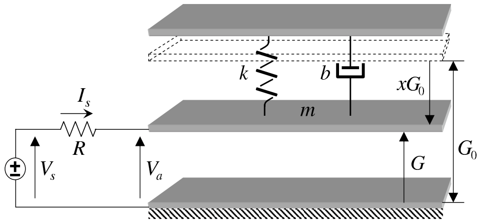

Fig. 1. 1DOF parallel-plate electrostatic actuator.

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Micro-actuator is a key component in such applications as adaptive optics in which a deformable mirror is actuated by an underlying two-dimensional array of such devices, forming the desired configuration of reflecting surface for correcting optical aberrations [19]. Potentially, a deformable mirror in adaptive optics systems can contain several hundred thousands of micro-actuators. Therefore the control system is of great importance in order to obtain the enhanced performance required in this application.

It is straightforward to show that due to the quadratic term q^2 appearing in the mechanical subsystem, where q represents the charge on the device (cf. (1) in Section II), the Jacobian linearization of such a system is not controllable at points where $q = 0$. Consequently, controls derived from feedback linearization will usually explode as the trajectory of the system is approaching these points. We remark that the only uncontrollable equilibrium point for System (1) is the origin, corresponding to the zero-voltage position. As the zero-voltage position is often taken as the initial position of the device, one needs to operate frequently around this singular point. It happens also in the application of adaptive optics that a big amount of devices need only to deflect slightly from their initial position for producing desired configurations for certain patterns. Hence, a precise manipulation for small deflections while assuring the overall performance in the full operational range is a realistic requirement. Note that System (1) is globally asymptotically stable at the origin if the actuation voltage is set to zero. Therefore a simple solution for avoiding the singularity is to remove the control signal if one wants to bring the device to this position. However, with this control the accuracy and the resolution of the system around the origin will be compromised. In addition, the amplitude of control signal can still be very high over a region near the origin, which constitutes a serious obstacle to experimental implementations.

Control of linearly uncontrollable systems has attracted many attentions in recent years and one can find in the literature existing frameworks for tackling this problem for systems with special structures, in particular systems including *odd order* power integrators [2], [20], [9], [3]. However, for systems with *even order* power integrators, as the one considered in this work, no simple solution is known. In this work, we seek control schemes that should be bounded in the whole operational range while being capable of enhancing the performance of the system described in (1).

Note that to avoid the singularity due to the uncontrollable linearization, one can use such techniques as input-output linearization [12], which is an implementation of the charge feedback introduced in [15]. However, the performance of the system using this type of control is mostly dominated by the mechanical subsystem and might be poor if, e.g., the natural damping of the device is too low or too high. A remarkable work on avoiding this singularity while providing an enhanced performance is the passivity-based control [10], [11]. In this work, we present an alternative solution based on backstepping and Lyapunov design. We will show that our design will result in a continuous or smooth control and the system considered can be stabilized by an output feedback control.

It is interesting to note that the dynamics of some magnetic levitation systems have similar properties as the one studied in this paper [7], [8], [13]. A more popular example is the magnetic levitation of a steel ball (see, e.g., §8.3 of [5]). In these systems the electrical variable (the current or the flux) acts as the input to the mechanical subsystem in quadratic form. The approach proposed in this paper can then be applied to these systems. Since electrostatic and electromagnetic forces are among the most popular actuation mechanisms in electromechanical systems, we might expect that our work is of practical interest.

II. System Description

According to [14], [21], the dynamical model of 1DOF parallel-plate electrostatic actuator in a normalized coordinate is given by :

$$\dot{x} = v, \tag{1a}$$

$$\dot{v} = -2\zeta v - x + \frac{1}{3}q^2, \tag{1b}$$

$$\dot{q} = -\frac{1}{r}q(1-x) + \frac{2}{3r}u_s, \tag{1c}$$

where x is the deflection, v is the deflection speed, q is the charge, u_s is the actuation voltage (the control signal), $\zeta > 0$ is the damping ratio, and r is the resistance in the actuation circuit loop. All variables appearing in (1) are defined in normalized coordinates and are dimensionless. Physically, the movement of the actuator is limited by the fixed electrode. As in practice an insulating layer is often added on the fixed electrode in order to prevent the device from shorting, the maximum displacement of the movable plate is $x = 1 - \delta$, where δ is the normalized thickness of the insulating layer. System (1) is thus defined on the state space $\mathcal{X} = \{(x, v, q) \in \mathbb{R}^3 \mid x \leq 1 - \delta\}$.

To address the problem of stabilizing the system at the set-points, we consider an equilibrium $(\bar{x}, \bar{v}, \bar{q}, \bar{u}_s)$. To determine all equilibria of System (1) we note firstly that $\bar{v} \equiv 0$. Secondly, since the electrostatic force is always attractive regardless of the sign of the charge, there are no equilibria for $x < 0$. Therefore, $\bar{x} \in [0, 1 - \delta]$ and at the equilibrium the charge is given by $\bar{q} = \pm\sqrt{3\bar{x}}$. Letting $x_1 = x - \bar{x}$, $x_2 = v - \bar{v}$, $x_3 = q - \bar{q}$, and $u = u_s - \bar{u}_s$, the system (1) becomes

$$\dot{x}_1 = x_2, \tag{2a}$$

$$\dot{x}_2 = -x_1 - 2\zeta x_2 + \frac{2\bar{q}}{3}x_3 + \frac{1}{3}x_3^2, \tag{2b}$$

$$\dot{x}_3 = \frac{1}{r}\left(\bar{q}x_1 + x_1 x_3 + (\bar{x} - 1)x_3\right) + \frac{2}{3r}u, \tag{2c}$$

which is defined on the state space

$$\bar{\mathcal{X}} = \left\{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 \leq 1 - \delta - \bar{x}\right\}.$$

The set-point control of System (1) is then transformed to the stabilization of System (2) at the origin.

Note that (2c) can be written as

$$\dot{x}_3 = \bar{u} \tag{3}$$

where $\bar{u}$ is a new control defined by

$$\bar{u} = \frac{1}{r}\left(\bar{q}x_1 + x_1 x_3 + (\bar{x} - 1)x_3\right) + \frac{2}{3r}u. \tag{4}$$

The contact of the two plates happens when $x_1 = 1 - \delta - \bar{x}$, at the boundary of $\bar{\mathcal{X}}$ defined as

$$\partial\bar{\mathcal{X}} = \left\{(x_1, x_2, x_3) \in \bar{\mathcal{X}} \mid x_1 = 1 - \delta - \bar{x}\right\}.$$

After contact, if one continues to charge the device, the dynamics of the mechanical subsystem might collapse. When it happens, the mechanical subsystem will no long be governed by (2a)-(2b) but by

$$\dot{x}_1 = 0, \tag{5a}$$

$$\dot{x}_2 = 0. \tag{5b}$$

Hence, the system exhibits switching behavior on $\partial\bar{\mathcal{X}}$. Note that at any equilibrium $q^2 = \bar{q}^2 = 3\bar{x} \leq 3(1 - \delta)$, therefore q can be used as a switching signal, as proposed in [10]. To completely characterize the contact dynamics, we need the following assumptions:

Assumption 1: [10] The velocity of the moveable electrode before and after contact satisfy the relation $x_2(t_c^+) = -\mu x_2(t_c^-)$ where $0 \leq \mu \leq 1$, and $x_2(t_c^-)$ and $x_2(t_c^+)$ are the velocities of the moveable electrode just before and after contact respectively.

Assumption 2: [10] When the system is restricted on $\partial\bar{\mathcal{X}}$, the mechanical dynamics are governed by (5a)-(5b) if $q^2 \geq 3(1 - \delta)$ and switch back to (2a)-(2b) if $q^2 < 3(1 - \delta)$.

Assumption 3: The charge on the device just before and after contact remains unchanged.

Note that Assumption 1 is an intuitive consequence of Newton's law of motion. When the movable plate hits the fixed one, it will change the moving direction. After contact, the velocity would be reduced or become null since the kinetic energy might be partially or entirely absorbed at the contact. Assumption 3 is also in accordance with the physical property of electrostatic actuators, because, as the current across the device is always finite, one can not add or remove charges to or from the device instantaneously.

III. Stabilization by State Feedback

A. Stabilization Including the Uncontrollable Equilibrium

Our objective is to find a controller being able to stabilize System (2) at any equilibrium, including $(\bar{x}, \bar{v}, \bar{q}) = (0, 0, 0)$ at which the system is not linearly controllable. Consider now an energy-like Lyapunov function candidate:

$$V_1 = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2. \tag{6}$$

The time derivative of V_1 along the solutions of the corresponding subsystem of (2) is

$$\dot{V}_1 = -2\zeta x_2^2 + \frac{2\bar{q}}{3}x_2x_3 + \frac{1}{3}x_2x_3^2. \tag{7}$$

Therefore a virtual control of the form

$$x_{3d} = k_1 \mathrm{sgn}(\bar{q}) \min(x_2, 0)^{2n}(1 - \delta - x_1 - \bar{x}), \tag{8}$$

where k_1 is a positive constant, n is any positive integer, and $\mathrm{sgn} : \mathbb{R} \to \{-1, 1\}$ with $\mathrm{sgn}(x) = 1$ iff $x \geq 0$, would render $\dot{V}_1$ negative semidefinite for any $\bar{q}$. The term $\mathrm{sgn}(\bar{q})$ is for dealing with $\bar{q} < 0$. Note that x_{3d} is identically zero on $\partial\bar{\mathcal{X}}$. Obviously, this control will have the effect of adding damping to the system when $x_2 < 0$.

As the virtual control x_{3d} is differentiable, we can proceed with the backstepping design by augmenting V_1 as:

$$V = V_1 + \frac{1}{2}\left(x_3 - x_{3d}\right)^2. \tag{9}$$

The time derivative of V along the solutions of (2) yields

$$\begin{aligned}
\dot{V} =& -2\zeta x_2^2 + \frac{2\bar{q}}{3}x_2x_3 + \frac{1}{3}x_2x_3^2 \\
&+ \left(x_3 - x_{3d}\right)\left(\dot{x}_3 - \dot{x}_{3d}\right) \\
=& -2\zeta x_2^2 + \frac{2\bar{q}}{3}x_2x_{3d} + \frac{2\bar{q}}{3}x_2\left(x_3 - x_{3d}\right) \\
&+ \frac{1}{3}x_2x_{3d}^2 + \frac{1}{3}x_2\left(x_3^2 - x_{3d}^2\right) \\
&+ \left(x_3 - x_{3d}\right)\left(\dot{x}_3 - \dot{x}_{3d}\right) \\
=& -2\zeta x_2^2 + \frac{2\bar{q}}{3}x_2x_{3d} + \frac{1}{3}x_2x_{3d}^2 + \left(x_3 - x_{3d}\right) \\
&\times \left(\bar{u} - \dot{x}_{3d} + x_2\left(\frac{2\bar{q}}{3} + \frac{1}{3}\left(x_3 + x_{3d}\right)\right)\right).
\end{aligned}$$

Therefore a control of the form

$$\bar{u} = -k_2\left(x_3 - x_{3d}\right) + \dot{x}_{3d} - x_2\gamma(x_3, x_{3d}) \tag{10}$$

where

$$\gamma(x_3, x_{3d}) = \frac{2\bar{q}}{3} + \frac{1}{3}\left(x_3 + x_{3d}\right) \tag{11}$$

and k_2 is a positive constant, would render $\dot{V}$ negative semidefinite. It is straightforward to verify that the time derivative of V defined in (9) along the solutions of the constrained system defined on $\partial\bar{\mathcal{X}}$ is also negative semidefinite. Furthermore, the virtual control $x_{3d} \equiv 0$ on $\partial\bar{\mathcal{X}}$. Therefore we have by Assumption 1 and Assumption 3 that

$$\begin{gathered}
x_2^2(t_c^+) \leq x_2^2(t_c^-), \\
\left(x_3(t_c^+) - x_{3d}(t_c^+)\right)^2 = \left(x_3(t_c^-) - x_{3d}(t_c^-)\right)^2.
\end{gathered}$$

This implies $V(t_c^+) \leq V(t_c^-)$. Hence, V is a common Lyapunov function for the switched system. Finally, as the largest invariant set in

$$E = \left\{(x_1, x_2, x_3) \in \bar{\mathcal{X}} | \dot{V} = 0\right\}$$

is the origin, we can conclude from LaSalle's invariance principle that System (2) is globally asymptotically stable (GAS) at the origin with the proposed control.

The actual control can be obtained by reversing (4), which is given by

$$\begin{aligned}
u =& -k_2\left(x_3 - x_{3d}\right) - x_2\left(r\bar{q}x_2 + \frac{r}{2}\left(x_3 + x_{3d}\right)\right) \\
&+ \frac{3r}{2}\dot{x}_{3d} - \frac{3}{2}\left(\bar{q}x_1 + x_1x_3 + (\bar{x} - 1)x_3\right).
\end{aligned} \tag{12}$$

Note that as

$$\begin{aligned}
\dot{x}_{3d} =& k_1 \mathrm{sgn}(\bar{q}) \min(x_2, 0)^{2n-1}(2n(1 - \delta - x_1 - \bar{x})\dot{x}_2 \\
&- \min(x_2, 0)x_2)
\end{aligned} \tag{13}$$

is differentiable for all $n \geq 2$, u given in (12) is smooth for all $n \geq 2$. When $n = 1$, u is only continuous.

B. Stabilization Excluding the Uncontrollable Equilibrium

As the virtual control given in (8) does not add sufficient damping in the closing phase where x_2 is mostly positive, we present another one which will add damping for both the opening and the closing phases. However, as we will see later on, this control cannot guarantee the closed-loop stability at the uncontrollable equilibrium $(\bar{x}, \bar{v}, \bar{q}) = (0, 0, 0)$.

Consider the Lyapunov function V_1 given in (6). We chose a virtual control of the following form

$$x_{3d} = -\bar{q}k_1 \tanh(x_2) \tanh(1 - \delta - x_1 - \bar{x}). \tag{14}$$

Obviously, x_{3d} is smooth and vanishes at the contact. Note that since the value of x might be smaller than $-1 - \delta$, we need to saturate the amplitude of $1 - \delta - x = 1 - \delta - x_1 - \bar{x}$. With this virtual control, the time derivative of V_1 becomes

$$\begin{aligned}
\dot{V}_1 =& -2\zeta x_2^2 - \frac{1}{3}\bar{q}^2k_1x_2 \tanh(x_2) \tanh(1 - \delta - x_1 - \bar{x}) \\
&\times \left(2 - k_1 \tanh(x_2) \tanh(1 - \delta - x_1 - \bar{x})\right),
\end{aligned} \tag{15}$$

which is negative semidefinite if $0 < k_1 < 2$.

We can proceed once again with backstepping design and prove by following the same analysis as in Section III-B that System (2) is globally asymptotically stable at any equilibrium points, except for the uncontrollable one, with the control given in (12), where

$$\begin{aligned}
\dot{x}_{3d} =& -\bar{q}k_1\left(\left(1 - \tanh^2(x_2)\right) \tanh(1 - \delta - x_1 - \bar{x})\dot{x}_2\right. \\
&\left.+ \tanh(x_2)\left(\tanh^2(1 - \delta - x_1 - \bar{x}) - 1\right)x_2\right).
\end{aligned} \tag{16}$$

We can now expect to get enhanced performance by combining the two virtual controls given in (8) and (14) as:

$$x_{3d} = \begin{cases} k_1 \mathrm{sgn}(\bar{q}) \min(x_2, 0)^{2n}(1 - \delta - x_1 - \bar{x}), & \bar{x} \leq \bar{X}, \\ -\bar{q}k_1 \tanh(x_2) \tanh(1 - \delta - x_1 - \bar{x}), & \bar{x} > \bar{X}, \end{cases} \tag{17}$$

where $0 < \bar{X} < 1$ is a constant, while using the control given in (12). Since the system is stabilizable at any equilibrium with the first virtual control in (17), the choice of $\bar{X}$ depends only on performance consideration.

IV. Output Feedback Control

Usually, the charge on the device and the gap between the electrodes can be deduced from the input current, the voltage across the device, and the capacitance (see, e.g., [1]). However, direct sensing of velocity during normal operations for micro-devices is extremely difficult, if not impossible. We need therefore to construct a speed observer in order to provide the estimate of $v = x_2$ required for implementing the closed-loop control described in the previous section. It can be shown that System (1) with the deflection and the charge as outputs admits the observer canonical form [4]. Therefore it is possible to find a full order observer with linear error dynamics. However, we need only to directly construct a reduced order speed observer. Furthermore, since $x_2 \equiv v$, we can estimate the speed in the original coordinates.

Consider the following dynamical system:

$$\dot{z} = -((2\zeta + k_v)k_v + 1)x - (2\zeta + k_v)z + \frac{1}{3}q^2, \qquad (18)$$

with $z(0) = -(2\zeta + k_v)x(0)$, where k_v is a positive real number. Thus, by setting

$$\hat{v} = z + k_v x, \qquad (19)$$

the dynamics of estimation error $\varepsilon = v - \hat{v}$ will be given by:

$$\dot{\varepsilon} = -(2\zeta + k_v)\varepsilon, \ \ \varepsilon(0) = -x_2(0), \qquad (20)$$

which is globally exponentially stable at the origin with a decay rate defined by k_v. This implies that (18) and (19) form an exponential observer.

We use a certainty-equivalence implementation of state-feedback design by replacing x_2 (= v) by its estimate $\hat{x}_2$ (= $\hat{v}$). The control law is now given by

$$\bar{u} = -k_2\left(x_3 - \hat{x}_{3d}\right) + \dot{\hat{x}}_{3d} - \hat{x}_2\gamma(x_3, \hat{x}_{3d}), \qquad (21)$$

where

$$\hat{x}_{3d} = \begin{cases} k_1 \mathrm{sgn}(\bar{q}) \min(\hat{x}_2, 0)^{2n}(1 - \delta - x_1 - \bar{x}), & \bar{x} \le \bar{X}, \\ -\bar{q}k_1 \tanh(\hat{x}_2)\tanh(1 - \delta - x_1 - \bar{x}), & \bar{x} > \bar{X}, \end{cases} \qquad (22)$$

and $\dot{\hat{x}}_{3d}$ is computed by

$$\dot{\hat{x}}_{3d} = k_1 \mathrm{sgn}(\bar{q}) \min(\hat{x}_2, 0)^{2n-1}(2n(1 - \delta - x_1 - \bar{x})(k_v\hat{x}_2 + \dot{z}) - \min(\hat{x}_2, 0)\hat{x}_2) \qquad (23)$$

if $\bar{x} \le \bar{X}$ or by

$$\dot{\hat{x}}_{3d} = -\bar{q}k_1\left(\left(1 - \tanh^2(\hat{x}_2)\right)\tanh(1 - \delta - x_1 - \bar{x})(k_v\hat{x}_2 + \dot{z}) + \tanh(\hat{x}_2)\left(\tanh^2(1 - \delta - x_1 - \bar{x}) - 1\right)\hat{x}_2\right) \qquad (24)$$

otherwise. In (22) and (23), $n \ge 2$ is a positive integer.

Denoting $\xi = (x_1, x_2, x_3)$, in coordinates (ξ, ε) the closed-loop system can be expressed as

$$\begin{aligned} \dot{\xi} &= f(\xi, \varepsilon), \\ \dot{\varepsilon} &= -\bar{k}_v\varepsilon, \end{aligned} \qquad (25)$$

with $\bar{k}_v = 2\zeta + k_v$. The vector-value function $f(\xi, \varepsilon)$ can be derived from (2) with the control given above and is smooth for any $n \ge 2$. Thus one can show, by using the standard approaches in the literature, e.g. [17], that the above closed-loop system is semi-global asymptotically stable at the origin.

In fact, since the origin of the system $\dot{\xi} = f(\xi, 0)$ is GAS, by the converse Lyapunov theorem [6], there exists a positive definite and proper Lyapunov function $V_0(\xi)$ such that

$$\frac{\partial V_0}{\partial \xi}(\xi) f(\xi, 0) \le -\alpha(|\xi|) \qquad (26)$$

where α is some positive definite strictly increasing function on $[0, \infty)$, also called class-$\mathcal{K}$ function.

Consider the following Lyapunov function candidate

$$V(\xi, \varepsilon) = V_0(\xi) + \frac{1}{2}\varepsilon^2. \qquad (27)$$

We need to prove that for any $0 < c_1 < c_2$ there exists a k_v^* such that for all $\bar{k}_v > k_v^*$ the time derivative of V along solutions of the closed loop system is negative-definite for all initial conditions starting from the set

$$S_{\xi\varepsilon} = \{(\xi, \varepsilon) \ : \ c_1 \le V(\xi, \varepsilon) \le c_2\}. \qquad (28)$$

Since $S_{\xi\varepsilon}$ is a compact set, then if the control is smooth, there exists a positive constant k_3 such that, for all $(\xi, \varepsilon) \in S_{\xi\varepsilon}$,

$$\left|\frac{\partial V_0}{\partial \xi}(\xi)(f(\xi, \varepsilon) - f(\xi, 0))\right| \le k_3|\varepsilon^|.$$

Hence, the time derivative of V along the solutions of the closed-loop system satisfies

$$\begin{aligned} \dot{V} &\le -\alpha(|\xi|) + \frac{\partial V_0}{\partial \xi}(\xi)(f(\xi, \varepsilon) - f(\xi, 0)) - \bar{k}_v\varepsilon^2 \\ &\le -\alpha(|\xi|) - \bar{k}_v\varepsilon^2 + k_3|\varepsilon|. \end{aligned} \qquad (29)$$

By applying Lemma 2.1 of [18], (29) implies that there should exist a positive constant k_v^* such that for all $\bar{k}_v > k_v^*$

$$\dot{V} < 0, \ \ \forall (\xi, \varepsilon) \in S_{\xi\varepsilon}. \qquad (30)$$

It can be seen from (20) that the stability of the observer error dynamics is independent of ξ. Therefore, it can be shown, by following the same lines as in [17], that for a sufficiently large $\bar{k}_v$ every trajectory starting from $S_{\xi\varepsilon}$ under the constraint $\varepsilon(0) = -x_2(0)$ is ultimately bounded and will eventually enter into an arbitrarily small neighborhood of the origin contained in the basin of attraction of the closed-loop system. This proves our assertion.

As System (2) is stable with the developed control at any equilibrium point, including the one at which the system is not linearly controllable, we have achieved the design objective.

V. Simulation Results

We have performed numerical simulations to verify the stability and the performance of the control scheme proposed. As the performance of the state feedback control is usually superior to the one of its certainty-equivalence implementation, we present only the simulation results for the latter. The simulated device is a under-damped system with $\zeta = 0.1$ and the resistance in the driving circuit loop is supposed to be 1. The actuator is supposed to be driven by a bipolar voltage source.

We have found that the performance of the system at different set-points is not significantly affected by the variation of the controller gain k_2 and the observer gain k_v. Therefore once tuned their value is fixed. However, when the observer-based implementation of the virtual control (8) is used, the controller gain k_1 should be carefully adjusted at each set-point, in order to obtain satisfactory performance. We have used a polynomial interpolation of well tuned values at several set-points to present k_1 as a function of set-points. For the observer-based implementation of the virtual control (17), k_1 is set to 1.9. In the simulation, we chose $\bar{X} = 0.2$ as the threshold for selecting the virtual control in (22), $\mu = 0.5$ for determining the plate velocity at the contact, and $\delta = 0.05$.

Figure 2 shows the simulation results for the stabilization of 0.95, 0.9, 0.85, and 0.8 gap position in the closing phase with $x_1(0) = 0$, corresponding to the zero-voltage position. Figure 3 shows the simulation results for the stabilization of 0.8, 0.6, 0.4, 0.2, and 0.1 gap position. We can see that the controller works well at all the tested operational positions and remains stabilizing even in the presence of contact. The control signals are also depicted. We remark that in the initial phase, a hight control amplitude might be generated. This is because $x_{3d} = 0$ at the equilibrium, which might introduce an important deficit in terms of charge regulation. However, in the rest of the operation no excessive control efforts are employed. We can also see that, as expected, the control signals are quite smooth.

Figure 4 shows the simulation results for the stabilization of 0.85, 0.9, 0.95, and the full gap position in the opening phase with $x_1(0) = 0.2$. Figure 5 shows the simulation results for the stabilization of 0.2, 0.4, 0.6, and 0.8 gap position whit $x_1(0) = 0.95$, corresponding to the position where the device is completely closed. It can be seen that the controller works well in all the considered set-points, including the zero-voltage position where the system is not linearly controllable.

VI. Conclusions

This paper addressed the control of a parallel-plate electrostatic micro-actuator, which is a basic element in many MEMS-based applications. We have developed a state feedback control which is bounded and can stabilize the system at any set-point in the operational range, including the uncontrollable sets. To meet the practical operation environments, we constructed an output feedback control law based on the certainty-equivalence implementation using a

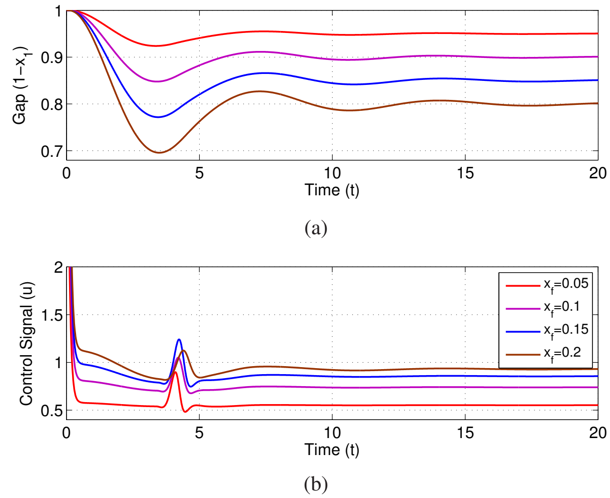

Fig. 2. Stabilization of 0.95, 0.9, 0.85, and 0.8 gap position in closing phase: (a) gap $(1 - x_1)$; (b) control signal u.

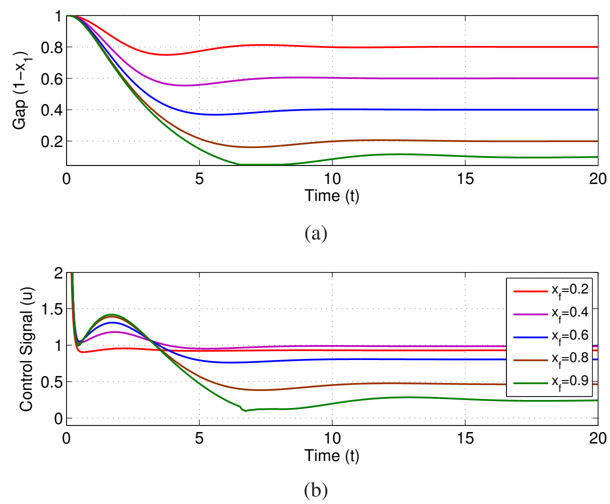

Fig. 3. Stabilization of 0.8, 0.6, 0.4, 0.2, and 0.1 gap position in closing phase: (a) gap $(1 - x_1)$; (b) control signal u.

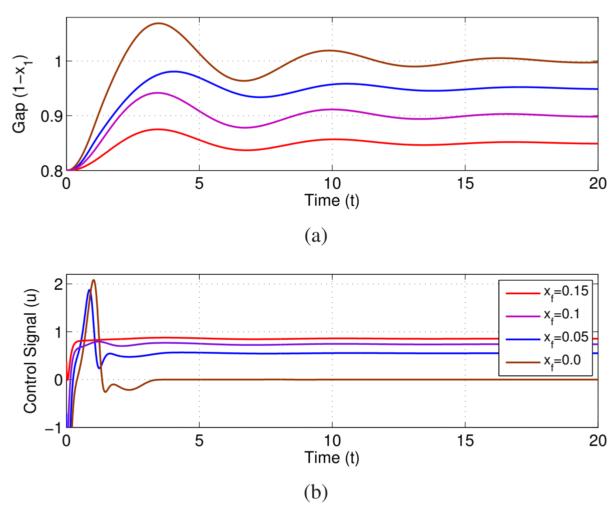

Fig. 4. Stabilization of 0.85, 0.9, 0.95, and the full gap position in opening phase: (a) gap $(1 - x_1)$; (b) control signal u.

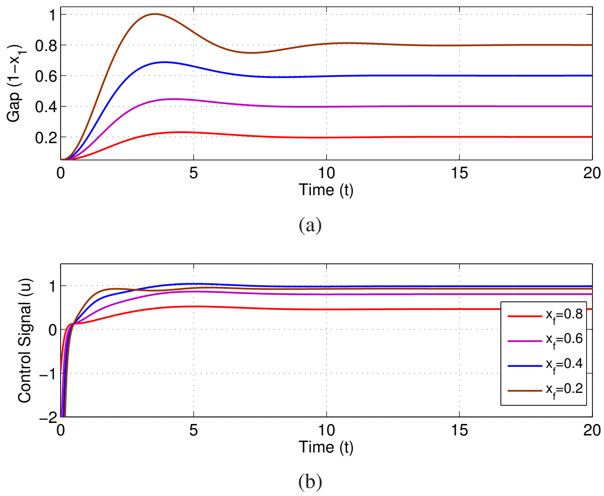

Fig. 5. Stabilization of 0.2, 0.4, 0.6, and 0.8 gap position in opening phase: (a) gap $(1 - x_1)$; (b) control signal u.

reduced-order speed observer and demonstrated the stability of the closed-loop system. The simulation results have shown that the proposed control scheme exhibited a satisfactory performance in different operational conditions. However, the presented control did not allow adding arbitrary damping. To obtain a higher performance, one may consider a hybrid solution. For example, one can use the present control in a neighborhood of the linearly uncontrollable equilibrium and a tracking control scheme (e.g., [21]) in other operational points. One may also take advantage of passivity-based design (see, e.g., [10]), which would allow adding arbitrary damping. The proposed approach can be applied to systems with similar properties, such as magnetic levitation systems.

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