Abstract

We study an electromagnetic actuator controlling the valve position on a camless engine. This actuator is made of a pair of electromagnets and a pair of springs. It forces the displacements up and down of a plate, itself pushing the armature related to the valve’s shutter. The landing must be silent which implies that the landing velocity must be small enough to avoid hitting the cylinder head. This terminal phase is particularly delicate since the force delivered by the electromagnets is large and poorly known. A reference trajectory and a feedback law including an observer are designed to realize the required displacements with soft landing. Simulations are presented.

Figure 1: The actuator during the valve closing phase (right) and the opening one (left).

1 INTRODUCTION

We study an electromagnetic actuator controlling the valve position on a camless engine. This actuator is made of a pair of electromagnets and a pair of springs. It forces the displacements up and down of a plate, itself pushing the armature related to the valve’s shutter (see Fig. 1). This setup has been proposed to save energy, to be simpler to produce than usual mechanical cams, and to potentially improve the motor control. Prototypes of such an equipment have been designed and tested by several firms among which are Ford and PSA Peugeot Citroën.

However, three important drawbacks have been noted: first, the actuator is efficient only within a small domain, the springs producing a much bigger force than the electromagnets up to a small distance of the final position. Second, unwanted noise is produced by the valve’s shutter hitting the cylinder head. It results from the fact that, in terminal phase, the air gap of the electromagnets being close to zero, the electromagnetic force is suddenly very large and particularly badly known. Thus, soft (and silent) landing means slowing down the shutter fast enough with precise positioning, which is not easy with such an inaccurate force. Finally, the frequency of the PWM unit, supposed to produce the desired current in the coils, is limited and current rate saturations may result: when the electromagnetic force becomes large enough, near the end point, to effectively control the plate’s motion, the current has to grow at a rate which is not allowed by the PWM unit. Therefore, a feedforward trajectory of the current is needed to obtain the required force at the required place and time. This is why we propose a control approach based on a reference trajectory design on the one hand and a feedback design, including an observer, on the other hand, the feedback itself reacting too slowly according to the above PWM limitation.

This problem has been studied in various ways, in particular by iterative learning control [4][5]. We propose here a different approach using nonlinear control methods. More precisely, based on flatness [3]), we construct a feasible reference trajectory in the phase plane (thus independent of the motor’s rotation speed), rather than with respect to time, ensuring fast opening and closing and soft landing, and then a robust feedback law by potential methods ([2][1]), that makes the reference trajectory an attractive invariant manifold. Since we only measure the plate’s position and the current and voltage, this feedback can be implemented thanks to an observer of the velocity and the flux.

This paper is organized as follows: a model of the ac-
2 MODEL OF THE ACTUATOR

Let us denote by \( x, \dot{x}, m \) the vertical position, velocity and mass of the plate respectively, and \( i \) the coil’s current. The absolute value of the electromagnetic force produced by the coils, if we neglect the magnetic losses, is given by:

\[
F = M \frac{\dot{x}^2}{(N - sx)^2}
\]

the constants \( M \) and particularly \( N \) being inaccurately known. Thus, the plate’s dynamics are given by:

\[
\begin{align*}
\dot{x} &= v \\
\dot{v} &= -\frac{k}{m}x - \frac{c_f}{m}v - g + s \frac{M}{n} \frac{i^2}{(N - sx)^2}
\end{align*}
\]

(1)

where \( s = 1 \) in the shutting phase and \( s = -1 \) in the opening one.

Recall that the magnetic permeance \( \Lambda \), still neglecting the magnetic losses, is given by:

\[
\frac{M}{(N - sx)} = n^2 \Lambda,
\]

(2)

\( n \) being the number of loops of the coil, and that the magnetic flux \( \Phi \) across the plate is given by:

\[
\Phi = 2n \Lambda i = \mu_0 S \frac{ni}{N - sx}.
\]

(3)

The flux dynamics are:

\[
\dot{\Phi} = \frac{1}{n} u - \frac{r}{2n^2 \Lambda} \Phi
\]

(4)

where \( r \) is the resistance of the coil and \( u \) its input voltage.

Let us denote by \( \Phi \) the square root of the electromagnetic acceleration:

\[
\Phi^2 = \frac{M}{m} \frac{\dot{x}^2}{(N - sx)^2}.
\]

(5)

Thus: \( \Phi = \frac{1}{\sqrt{2m \mu_0 S}} \Phi = \frac{n}{2\sqrt{mM}} \Phi \). This proves that \( \Phi \) is proportional to the flux and, with an understandable abuse of language, we call it the flux from now on. Its dynamics are:

\[
\dot{\Phi} = \frac{1}{n \sqrt{2m \mu_0 S}} u - \frac{r(N - sx)}{n^2 \mu_0 S} \Phi
\]

(6)

The complete model, expressed with respect to the variables \((x, v, \Phi)\), is thus given by:

\[
\begin{align*}
\dot{x} &= v \\
\dot{v} &= -\frac{k}{m}x - \frac{c_f}{m}v + s\Phi^2 \\
\Phi &= \frac{1}{2\sqrt{mM}} u - \frac{r}{2M} (N - sx) \Phi
\end{align*}
\]

(7)

According to (3) or (5), the current is subject to the state constraint:

\[
|i| = \sqrt{\frac{m}{M} \Phi(N - sx)} \leq i_{\text{max}}
\]

(8)

and the voltage must also satisfy the constraint:

\[
|u| \leq u_{\text{max}}.
\]

(9)

On the real set-up, the electrical dynamics of \( \Phi \) or \( \Phi \) cannot be modified by the user. The only external control variable is the current, the internal control loop being implemented in hardware by a PWM unit.

3 TRAJECTORY PLANNING

We are looking for an open-loop trajectory \((x(t), v(t), \Phi(t), u(t))\) satisfying (7), (8) and (9) and softly landing. In fact, expressing the trajectory with respect to time is nowhere needed. On the contrary, expressing the desired trajectory with respect to \( x \) is useful since its expression is independent of the motor’s regime and since, the velocity being corrupted by errors, the positionning precision with respect to time may be degraded. Finally, this is possible since the system is flat with \( x \) as flat output: eliminating the time, the new system reads:

\[
\begin{align*}
\dot{v} &= -\frac{k}{m}x - \frac{c_f}{m}v - g + s\Phi^2 \\
v\Phi' &= \frac{1}{2\sqrt{mM}} u - \frac{r}{2M} (N - sx) \Phi
\end{align*}
\]

(10)

with \( v' = \frac{dv}{dx} \) and \( \Phi' = \frac{d\Phi}{dx} \). Thus,

\[
\Phi'^2 = s(vv' + \frac{k}{m}x + \frac{c_f}{m}v + g)
\]

(11)

and

\[
u = 2\sqrt{mM}\Phi' + r\sqrt{\frac{m}{M}}(N - sx) \Phi.
\]

(12)

Since \( \Phi' \) is also a function of \( v, v', v'' \), with \( v'' = \frac{dv'}{dx} = \frac{dv}{dx^2} \), we have proved that \( \Phi \) and \( u \) can be expressed as functions of \( x, v, v', v'' \), and thus that the system (10) is flat. It results that any trajectory of (10) may be obtained, without integration of the system’s equations, as a function of the corresponding trajectory \( x \mapsto v(x) \) of the flat output.
From now on, we denote by \( \vartheta, \phi \) and \( v \) the functions \( x \mapsto v, x \mapsto \Phi \) and \( x \mapsto u \) respectively, to avoid confusing with time functions.

It remains thus to determine the curve \( x \mapsto \vartheta(x) \).

At the initial position \( x_0 \), the initial velocity \( v_0 \) is deduced by the fact that we start with zero current, i.e. \( i_0 = 0 \), at \( x_0 \).

The initial voltage \( u_0 \) is also given. Thus:
\[
\vartheta(x_0) = v_0, \quad \phi(x_0) = 0
\]
and using the fact that \( v' = \vartheta \), we get:
\[
\vartheta'(x_0) = -\frac{k}{m} \frac{x_0}{v_0} - \frac{c_f}{m} \frac{\Delta}{v_0}.
\]

In addition, using (10),
\[
\phi'(x_0) = \frac{1}{2\sqrt{mM}} \left( \frac{u_0}{v_0} \right).
\]

Differentiating twice the expression of \( v' \) in (10) we get:
\[
\vartheta''(x_0) = -\frac{1}{v_0} \left( v_0 \left( v_0' + \frac{c_f}{m} \right) + \frac{k}{m} \right) \Rightarrow v_0'' = \vartheta''(x_0)
\]
\[
\vartheta'''(x_0) = -\frac{1}{v_0} \left[ v_0' \left( 3v_0' + \frac{c_f}{m} \right) - 2\vartheta'^2(x_0) \right] \Rightarrow \vartheta'''(x_0)
\]

At the final position \( x_1 = x \), the velocity is \( \vartheta(x_1) = v_1 \) and, to control the landing, we impose the final slope \( v_1' = \vartheta'(x_1) \) and curvature \( v_1'' = \vartheta''(x_1) \). We compute \( x_1 \) by imposing the relation:
\[
x_1 = x = x_0 + \frac{v_s}{v_1'},
\]
where \( x_0 \) is the position of the support and \( v_s \) the admissible impact velocity on the support, which means that the final velocity is enough to guarantee that the valve closes.

We thus have 4 initial conditions and 3 final conditions and the desired trajectory \( x \mapsto \vartheta(x) \) may be obtained, by interpolation, as a 6th degree polynomial:
\[
\vartheta(x) = v_0 + v_0'x\xi(x) + \frac{v_0''}{2}x^2\xi^2(x) + \frac{v_0'''}{6}x^3\xi^3(x) + a_4\xi^4(x) + a_5\xi^5(x)
\]
with \( X = x_1 - x_0 = x - x_0 \), \( \xi(x) = \frac{x-x_0}{X} \), the coefficients \( a_4, a_5, a_6 \) being:
\[
\begin{align*}
a_4 &= 15(v_1 - v_0) - 5(v_1' + 2v_0')X + \frac{1}{2}(v_1'' - 6v_0'')X^2 - \frac{1}{2}v_0'''X^3 \\
a_5 &= -24(v_1 - v_0) + 3(3v_1' + 5v_0')X - (v_1'' - 4v_0'')X^2 + \frac{1}{2}v_0'''X^3 \\
a_6 &= 10(v_1 - v_0) - 2(2v_1' + 3v_0')X + \frac{1}{2}(v_1'' - 3v_0'')X^2 - \frac{1}{6}v_0'''X^3.
\end{align*}
\]

Since in these formulae \( M \) and \( N \) are not precisely known, we replace them by estimates noted \( M_\text{et} \) and \( N_\text{et} \). Note that these estimations are only needed to compute the reference voltage. An explicit construction of an estimator of \( N_\text{et} \) is presented in section 6.

Indeed, from this trajectory, a time-parameterized trajectory is easily deduced: if \( t_0 \) is the duration to travel from the opposite support to \( x_0 \), the total travelling time is:
\[
T = t_0 + \int_{x_0}^{x_1} \frac{1}{\vartheta(\xi)} d\xi
\]

The 4 free parameters \( x_0, u_0, v_1' \) et \( v_1'' \) may be chosen such that the constraints on \( i \) and \( \Phi \) are satisfied with a travelling time \( T \) as short as possible. It is interesting to remark that with \( u_0 = 0 \) we start close to the initial open-loop situation, which indeed doesn’t hit the saturations. At the other end of the trajectory, increasing \( |v_1'| \) accelerates the landing and decreasing \( |x_0| \) decreases the duration \( T \).

The graphs of \( \vartheta \) and the associated current and voltage are displayed in figure 2 in the opening case with the following parameters:
\[
x_0 = -2 \text{ mm}, \quad v_0 = -2.91 \text{ m/s}, \quad u_0 = 0 \text{ V} \\
x_1 = -4.0018 \text{ mm}, \quad v_1' = -2800 \text{ 1/s}.
\]

Since, in this example, we have not imposed the curvature \( v_1'' \), \( \vartheta \) is a 5th degree polynomial.

\section{Control Synthesis}

The functions \((\vartheta, \phi)\) previously designed define an invariant set of system (7). Indeed, for any initial condition
In fact, since 
\[ (x, v, \Phi) \text{ satisfying:} \]
\[ (v, \Phi) = (\bar{\Phi}(x), \phi(x)) \]  
(22)
the control:
\[ u = v(x) \]  
(23)
satisfies (9) and is such that the corresponding solution
\[ (x(t), v(t), \Phi(t)) \text{ satisfies for all time t:} \]
\[ (v(t), \Phi(t)) = (\bar{\Phi}(x(t)), \phi(x(t))). \]  
(24)
Moreover, this trajectory coincides with the planned refer-
tion trajectory.
On this set, the dynamics reduce to:
\[ \dot{x} = \bar{\Phi}(x) \]  
(25)
Outside this set, we have:
\[ \dot{x} = v = \bar{\Phi}(x) + (v - \bar{\Phi}(x)). \]  
(26)
Thus, it suffices to asymptotically stabilize the invariant set
\[ v = \bar{\Phi}(x). \] According to (10),
\[ v - \bar{\Phi}(x) = \left[ -\frac{k}{m} x - \frac{c_f}{m} v + \phi(\Phi)^2 - \Phi'(x) v \right] \]
\[ = -s \phi(\Phi)^2 + \left[ \Phi'(x) \dot{\bar{\Phi}}(x) + \frac{k}{m} x + \frac{c_f}{m} \dot{\bar{\Phi}}(x) \right] \]
\[ + \left[ -\frac{k}{m} x - \frac{c_f}{m} v + s \phi(\Phi)^2 - \Phi'(x) v \right] \]
\[ = - s \left[ \Theta'(x) + \frac{c_f}{m} \right] [v - \bar{\Phi}(x)] + s \left[ \phi(\Phi)^2 - \phi(\Phi)^2 \right]. \]  
(27)
Assuming that \( \Phi^2 \) is a control variable, the invariant set
may be stabilized by choosing:
\[ \phi(\Phi)^2 = \phi(x)^2 - s g(x) [v - \bar{\Phi}(x)] \]  
(28)
where \( g \) is an arbitrary function satisfying
\[ g(x) + \left[ \Phi'(x) + \frac{c_f}{m} \right] > 0. \]  
(29)
In fact, since \( \Phi \) is not a control variable, but, according to
(5), the corresponding current is:
\[ i^2 = \frac{m}{M} (\bar{N} - sx)^2 (\phi(x)^2 - s g(x) [v - \bar{\Phi}(x)]), \]  
(30)
where \( \bar{M} \) and \( \bar{N} \) are estimates of \( M \) and \( N \). Since these
estimates produce various consequences on the closed-loop
behavior, we decompose the expression of \( i^2 \) as the sum of
a constant, a linear term in \( x - x_1 \) and a linear term in \( v \).
From (11) we have:
\[ i^2 = s \frac{m}{M} (\bar{N} - sx)^2 (\Phi'(x) \dot{\Phi}(x) + \frac{k}{m} x \]
\[ + \frac{c}{m} \dot{\Phi}(x) - g(x) [v - \bar{\Phi}(x)]). \]  
(31)
Since, by construction, the function \( \Phi \) vanishes at \( x_1 \), we
deduce that:
\[ i^2 = s \frac{k}{m} (\bar{N} - sx_1)^2 x_1 \]
\[ + \frac{s k (x - x_1)^2}{M} \left[ (\bar{N} - s x)^2 - sx_1 (2 \bar{N} - s (x + x_1)) \right] \]
\[ + \frac{s m (N - x)^2}{M} \left[ (\Theta'(x) + \frac{c_f}{m}) \Phi(x) - g(x) [v - \Phi(x)] \right]. \]  
(32)
In the first term of the right-hand side, the parameters \( \hat{M} \)
and \( \hat{N} \) may be used to move the equilibrium point. In the
other terms, their contribution is to change the position and
velocity gains, and therefore the convergence rates. There-
fore, we introduce two values \( \hat{M}_1 \) and \( \hat{N}_1 \) a priori distinct
from \( \bar{M} \) and \( \bar{N} \) respectively. We therefore get:
\[ \bar{P}^2 = s \frac{k}{M} (\hat{N}_1 - sx_1)^2 x_1 \]
\[ + \frac{s k (x - x_1)^2}{M} \left[ (\bar{N} - s x)^2 - sx_1 (2 \bar{N} - s (x + x_1)) \right] \]
\[ + \frac{s m (\bar{N} - x_1)^2}{M} \left[ (\Theta'(x) + \frac{c_f}{m}) \Phi(x) - g(x) [v - \Phi(x)] \right]. \]  
(33)
Taking into account the constraint (8), we obtain:
\[ i_{\text{cons}} = \sqrt{\max \{0, \min \left\{ i_{\text{max}}^2, \bar{P}^2 \right\} \}. \]  
(34)

5 PHASE PORTRAIT ANALYSIS AND TUNING PROCEDURE

Let us assume for a moment that the current loop is perfect,
i.e. \( i = i_{\text{cons}} \), that the constraint on \( i^2 \) is satisfied, i.e. \( i_{\text{cons}} = \bar{i} \),
and that the mass \( m \) and the coefficient \( M \) are perfectly
known: \( \bar{M}_1 = \bar{M} = M \).

To study the closed-loop behavior in presence of estimation
errors on \( N \), we rewrite \( \bar{v} \) as follows:
\[ \bar{v} = -U'(x) - k(x) v, \]  
(35)
where \( U' \) is the derivative of a function \( U \), interpreted as a
potential, with the total energy defined by:
\[ E(x, v) = \frac{1}{2} v^2 + U(x) \]  
(36)
or
\[ \dot{E} = -k(x) v . \]  
(37)
Thus, the energy is dissipated by the system if the function
\( k \) is non negative. According to the Legendre-Dirichlet
Theorem, if the critical points of \( U \), i.e. the points that are
solutions of \( U'(x) = 0 \), are equilibrium points and they are
stable and attractive if they correspond to a local minimum
of \( U \). If such a point is unique in the domain of interest, i.e.
on \( (-\infty, N) \) for \( s = 1 \) and \( (-N, +\infty) \), for \( s = -1 \), and if the
potential is proper on this domain, this point is a globally
asymptotically stable equilibrium.

For \( k \), we directly have:
\[ k(x) = \frac{c_f}{m} + g(x) \frac{(\bar{N} - sx)^2}{(N - sx)^2}. \]  
(38)
The system is thus always dissipative if \( g \) is non negative,
which implies (29).

The potential \( U(x) \) may be directly determined by computing \( \bar{v} \). These computations are omitted here. We just
Figure 3: The potential $U(x)$.

Figure 4: Equilibrium points and associated eigenvalues locus.

To conclude, we may choose $\hat{N}_1$ as close as possible to $N$ and $\hat{N} > N$ large, such that $N^2 + 4\hat{N} - \hat{N}^2 < 0$.

Also, $g$ can be chosen as $g(x) = -g_0 \theta'(x)$, with $g_0$ not too large to avoid saturations.

6 SPEED AND FLUX OBSERVER DESIGN AND ESTIMATOR OF $N$

To estimate the velocity $v$ from position and current and voltage measurements without using the constant $N$, we set:

$$z = v - k_v x$$

where $k_v$ is an arbitrary positive real. Differentiating, we get

$$\dot{z} = -\left(\frac{k}{m} + k_v \left(\frac{c_i}{m} + k_v\right)\right)x - \left(\frac{c_i}{m} + k_v\right)z + s \Phi^2.$$  

Thus, if we set

$$\hat{z} = \hat{v} - k_v x$$  

we obtain, using the fact that $\hat{z} - z = \hat{v} - v$ and $\hat{z} - z = \hat{v} - v$, the estimation error

$$\hat{v} - v = -\left(\frac{c_i}{m} + k_v\right)(\hat{v} - v) + s \left(\frac{\Phi^2 - \Phi^2}{\Phi^2}\right).$$

In addition, this observer may be completed by using the dynamics of $\Phi$, $u$ and $i$ being measured. We set

$$\zeta = \Phi - k_\Phi x.$$  

thus

$$\dot{\zeta} = \frac{1}{2\sqrt{mM}} (u - ri) - k_\Phi \hat{v}$$

and, noting $\hat{\zeta}$ the estimate of $\zeta$, the following system is an observer

$$\dot{\hat{\zeta}} = \frac{1}{2\sqrt{mM}} (u - ri) - k_\Phi \hat{v}$$

with

$$\hat{\Phi} = \hat{\zeta} + k_\Phi x.$$  

Thus, the error dynamics satisfies:

$$\dot{\hat{\zeta}} - \hat{\zeta} = \hat{\Phi} - \Phi = -k_\Phi (\hat{v} - v).$$

To summarize, the following reduced observer has been constructed:

$$\begin{align*}
\dot{\hat{z}} &= -\left(\frac{k}{m} + k_v \left(\frac{c_i}{m} + k_v\right)\right)x - \left(\frac{c_i}{m} + k_v\right)\hat{z} + s \Phi^2 \\
\dot{\hat{\zeta}} &= \frac{1}{2\sqrt{mM}} (u - ri) - k_\Phi \hat{v} \\
\hat{v} &= \hat{z} + k_v x \\
\hat{\Phi} &= \hat{\zeta} + k_\Phi x
\end{align*}$$
and the estimation error on \( v \) and \( \Phi \) is given by

\[
\begin{aligned}
\dot{\hat{v}} - \hat{v} &= \left( \frac{c_f}{m} + k_v \right) (\hat{v} - v) + s (\hat{\Phi} + \Phi) \left( \Phi - \Phi \right) \\
\hat{\Phi} - \Phi &= k_k (\hat{v} - v).
\end{aligned}
\]

It can be made stable by choosing \( k_v > 0 \) and \( k_k > 0 \). More precisely, for every solution of the closed-loop system such that \( \hat{\Phi} + \Phi \leq \Phi_{\text{max}} \), it suffices that \( k_v \) and \( k_k \) satisfy:

\[
\left( \frac{c_f}{m} + k_v \right)^2 > 2 (s k_k) \Phi_{\text{max}}.
\]

In addition, an estimate of \( N \), given \( \hat{M} \) and \( \hat{m} \), is given by:

\[
\hat{N} = -x + \sqrt{\frac{M}{\hat{m}}} \frac{i}{\hat{\Phi}}.
\]

7 SIMULATION IN THE OPENING PHASE

In Figure 5, we give a simulation displaying a typical opening behavior. The red curves are the references obtained by \( \vartheta \) and the associated current and voltage. The blue ones are the real ones. The magenta ones describe the flux and velocity estimates.

The function \( \vartheta \) is the one of Figure 2 obtained with the parameters: \( x_0 = -2 \, \text{mm}, \, v_0 = -2.91 \, \text{m/s}, \, u_0 = 0 \, \text{V}, \, x_1 = -4.0018 \, \text{mm}, \, v_1 = -2800 \, \text{1/s}. \)

The other parameters used to stabilize the set \( v = \vartheta(x) \) are: \( g_\vartheta = 7 \) and \( g(x) = -g_\vartheta \vartheta'(x) \), \( k_v = 10000 \), \( k_k = 10000 \times k_v/120 \), \( \hat{N} = 4.25 \, \text{mm}, \hat{N} = 4.084 \, \text{mm}, \hat{M} = 2.47267 \times 10^{-7} \).

The phase portrait in the right below graph of Figure 5 shows, on the right of the point marked by a star, the function \( \vartheta \) and the real trajectory. When the coil is activated, the error \( v - \vartheta(x) \) is significant but has the right sign and the controller can react by increasing the current (second graph on the left). Such a current cannot be delivered at once since the voltage is saturated (3rd graph on the left). However, the flux increases and catches up with its reference. The velocity and flux are given in the two first graphs on the right, showing a satisfactory convergence of the observer.

8 CONCLUSION

We have presented a nonlinear controller able to track a reference trajectory corresponding to soft landing and playing the role of an invariant manifold. This control law is completed by a reduced observer of the velocity and flux. However, the final precision might be improved by taking into account the magnetic losses, as in [4].

REFERENCES


