# On the control of an electromagnetic actuator of valve positionning on a camless engine* 

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#### Abstract

We study an electromagnetic actuator controlling the valve position on a camless engine. This actuator is made of a pair of electromagnets and a pair of springs. It forces the displacements up and down of a plate, itself pushing the armature related to the valve's shutter. The landing must be silent which implies that the landing velocity must be small enough to avoid hitting the cylinder head. This terminal phase is particularly delicate since the force delivered by the electromagnets is large and poorly known. A reference trajectory and a feedback law including an observer are designed to realize the required displacements with soft landing. Simulations are presented.




Figure 1: The actuator during the valve closing phase (right) et and the opening one (left).

## 1 Introduction

We study an electromagnetic actuator controlling the valve position on a camless engine. This actuator is made of a pair of electromagnets and a pair of springs. It forces the displacements up and down of a plate, itself pushing the armature related to the valve's shutter (see Fig. (1). This setup has been proposed to save energy, to be simpler to produce than usual mechanical cams, and to potentially improve the

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motor control. Prototypes of such an equipment have been designed and tested by several firms among which are Ford and PSA Peugeot Citroën.

However, three important drawbacks have been noted: first, the actuator is efficient only within a small domain, the springs producing a much bigger force than the electromagnets up to a small distance of the final position. Second, unwanted noise is produced by the valve's shutter hitting the cylinder head. It results from the fact that, in terminal phase, the air gap of the electromagnets being close to zero, the electromagnetic force is suddenly very large and particularly badly known. Thus, soft (and silent) landing means slowing down the shutter fast enough with precise positionning, which is not easy with such an inaccurate force. Finally, the frequency of the PWM unit, supposed to produce the desired current in the coils, is limited and current rate saturations may result: when the electromagnetic force becomes large enough, near the end point, to effectively control the plate's motion, the current has to grow at a rate which is not allowed by the PWM unit. Therefore, a feedforward trajectory of the current is needed to obtain the required force at the required place and time. This is why we propose a control approach based on a reference trajectory design on the one hand and a feedback design, including an observer, on the other hand, the feedback itself reacting too slowly according to the above PWM limitation.

This problem has been studied in various ways, in particular by iterative learning control [4, 5]. We propose here a different approach using nonlinear control methods. More precisely, based on flatness ([3]), we construct a feasible reference trajectory in the phase plane (thus independent of the motor's rotation speed), rather than with respect to time, ensuring fast opening and closing and soft landing, and then a robust feedback law by potential methods ([2, 1]), that makes the reference trajectory an attractive invariant manifold. Since we only measure the plate's position and the current and voltage, this feedback can be implemented thanks to an observer of the velocity and the flux.

This paper is organized as follows: a model of the ac-
tuator is presented in section 2 , followed by the study of the trajectory planning in section 3 . In section 4 , the feedback control synthesis is given, the tuning aspects being analyzed in section 5 . Section 6 is devoted to the design of a velocity and flux observer and is followed by simulations in section 7 and concluding remarks.

## 2 MODEL OF THE ACTUATOR

Let us denote by $x, v, m$ the vertical position, velocity and mass of the plate respectively, and $i$ the coil's current. The absolute value of the electromagnetic force produced by the coils, if we neglect the magnetic losses, is given by

$$
F=M \frac{i^{2}}{(N-s x)^{2}}
$$

the constants $M$ and particularly $N$ being inaccurately known. Thus, the plate's dynamics are given by:

$$
\left\{\begin{array}{l}
\dot{x}=v  \tag{1}\\
\dot{v}=-\frac{k}{m} x-\frac{c_{f}}{m} v-g+s \frac{M}{m} \frac{i^{2}}{(N-s x)^{2}}
\end{array}\right.
$$

where $s=1$ in the shutting phase and $s=-1$ in the opening ond 1

Recall that the magnetic permeance $\Lambda$, still neglecting the magnetic losses, is given by

$$
\begin{equation*}
\frac{M}{(N-s x)}=n^{2} \Lambda \tag{2}
\end{equation*}
$$

$n$ being the number of loops of the coil, and that the magnetic flux $\varphi$ accross the plate is given by:

$$
\begin{equation*}
\varphi=2 n \Lambda i=\mu_{0} S \frac{n i}{N-s x} \tag{3}
\end{equation*}
$$

The flux dynamics are:

$$
\begin{equation*}
\dot{\varphi}=\frac{1}{n} u-\frac{r}{2 n^{2} \Lambda} \varphi \tag{4}
\end{equation*}
$$

where $r$ is the resistance of the coil and $u$ its input voltage.
Let us denote by $\Phi$ the square root of the elecromagnetic acceleration:

$$
\begin{equation*}
\Phi^{2}=\frac{M}{m} \frac{i^{2}}{(N-s x)^{2}} \tag{5}
\end{equation*}
$$

Thus: $\Phi=\frac{1}{\sqrt{2 m \mu_{0} S}} \varphi=\frac{n}{2 \sqrt{m M}} \varphi$. This proves that $\Phi$ is proportional to the flux and, with an understandable abuse of language, we call it the flux from now on. Its dynamics are:

$$
\begin{equation*}
\dot{\Phi}=\frac{1}{n \sqrt{2 m \mu_{0} S}} u-\frac{r(N-s x)}{n^{2} \mu_{0} S} \Phi \tag{6}
\end{equation*}
$$

[^1]The complete model, expressed with respect to the variables $(x, v, \Phi)$, is thus given by:

$$
\left\{\begin{array}{l}
\dot{x}=v  \tag{7}\\
\dot{v}=-\frac{k}{m} x-\frac{c_{f}}{m} v+s \Phi^{2} \\
\dot{\Phi}=\frac{1}{2 \sqrt{m M}} u-\frac{r}{2 M}(N-s x) \Phi
\end{array}\right.
$$

According to (3) or (5), the current is subject to the state constraint:

$$
\begin{equation*}
|i|=\sqrt{\frac{m}{M}}|\Phi(N-s x)| \leq i_{\max } \tag{8}
\end{equation*}
$$

and the voltage must also satisfy the constraint:

$$
\begin{equation*}
|u| \leq u_{\max } \tag{9}
\end{equation*}
$$

On the real set-up, the electrical dynamics of $\varphi$ or $\Phi$ cannot be modified by the user. The only external control variable is the current, the internal control loop being implemented in hardware by a PWM unit.

## 3 TRAJECTORY PLANNING

We are looking for an open-loop trajectory $\left(x_{r}(t), v_{r}(t), \Phi_{r}(t), u_{r}(t)\right)$ satisfying (7), 8) and 4) and softly landing. In fact, expressing the trajectory with respect to time is nowhere needed. On the contrary, expressing the desired trajectory with respect to $x$ is useful since its expression is independent of the motor's regime and since, the velocity being corrupted by errors, the positionning precision with respect to time may be degraded. Finally, this is possible since the system is flat with $x$ as flat output: eliminating the time, the new system is flat with $v$ (now function of $x$ ) as flat output since (7) reads:

$$
\begin{align*}
& v v^{\prime}=-\frac{k}{m} x-\frac{c_{f}}{m} v-g+s \Phi^{2} \\
& v \Phi^{\prime}=\frac{1}{2 \sqrt{m M}} u-\frac{r}{2 M}(N-s x) \Phi \tag{10}
\end{align*}
$$

with $v^{\prime}=\frac{d v}{d x}$ and $\Phi^{\prime}=\frac{d \Phi}{d x}$. Thus,

$$
\begin{equation*}
\Phi^{2}=s\left(v v^{\prime}+\frac{k}{m} x+\frac{c_{f}}{m} v+g\right) \tag{11}
\end{equation*}
$$

and

$$
\begin{equation*}
u=2 \sqrt{m M} v \Phi^{\prime}+r \sqrt{\frac{m}{M}}(N-s x) \Phi \tag{12}
\end{equation*}
$$

Since $\Phi^{\prime}$ is also a function of $v, v^{\prime}, v^{\prime \prime}$, with $v^{\prime \prime}=\frac{d v^{\prime}}{d x}=\frac{d^{2} v}{d x^{2}}$, we have proved that $\Phi$ and $u$ can be expressed as functions of $x, v, v^{\prime}, v^{\prime \prime}$, and thus that the system 10 is flat. It results that any trajectory of (10) may be obtained, without integration of the system's equations, as a function of the corresponding trajectory $x \mapsto v(x)$ of the flat output.

From now on, we denote by $\vartheta, \phi$ and $v$ the functions $x \mapsto v, x \mapsto \Phi$ and $x \mapsto u$ respectively, to avoid confusing with time functions.

It remains thus to determine the curve $x \mapsto \vartheta(x)$.
At the initial position $x_{0}$, the initial velocity $v_{0}$ is deduced by the fact that we start with zero current, i.e. $i_{0}=0$, at $x_{0}$. The initial voltage $u_{0}$ is also given. Thus:

$$
\begin{equation*}
\vartheta\left(x_{0}\right)=v_{0}, \quad \phi\left(x_{0}\right)=0 \tag{13}
\end{equation*}
$$

and using the fact that $v^{\prime}=\frac{\dot{v}}{v}$, we get:

$$
\begin{equation*}
\vartheta^{\prime}\left(x_{0}\right)=-\frac{k}{m} \frac{x_{0}}{v_{0}}-\frac{c_{f}}{m} \triangleq v_{0}^{\prime} \tag{14}
\end{equation*}
$$

In addition, using 10,

$$
\begin{equation*}
\phi^{\prime}\left(x_{0}\right)=\frac{1}{2 \sqrt{m M}}\left(\frac{u_{0}}{v_{0}}\right) \tag{15}
\end{equation*}
$$

Differentiating twice the expression of $v^{\prime}$ in we get:

$$
\begin{gather*}
\vartheta^{\prime \prime}\left(x_{0}\right)=-\frac{1}{v_{0}}\left(v_{0}^{\prime}\left(v_{0}^{\prime}+\frac{c_{f}}{m}\right)+\frac{k}{m}\right) \triangleq v_{0}^{\prime \prime}  \tag{16}\\
\vartheta^{\prime \prime \prime}\left(x_{0}\right)=-\frac{1}{v_{0}}\left[v_{0}^{\prime \prime}\left(3 v_{0}^{\prime}+\frac{c_{f}}{m}\right)-2 s \phi^{\prime 2}\left(x_{0}\right)\right]  \tag{17}\\
\triangleq v_{0}^{\prime \prime \prime}
\end{gather*}
$$

At the final position $x_{1}=\bar{x}$, the velocity is $\vartheta\left(x_{1}\right)=v_{1}$ and, to control the landing, we impose the final slope $v_{1}^{\prime}=\vartheta^{\prime}\left(x_{1}\right)$ and curvature $v_{1}^{\prime \prime}=\vartheta^{\prime \prime}\left(x_{1}\right)$. We compute $x_{1}$ by imposing the relation:

$$
\begin{equation*}
x_{1}=\bar{x}=x_{s}+\frac{v_{s}}{v_{1}^{\prime}}, \tag{18}
\end{equation*}
$$

where $x_{s}$ is the position of the support and $v_{s}$ the admissible impact velocity on the support, which means that the final velocity is enough to guarantee that the valve closes.

We thus have 4 initial conditions and 3 final conditions and the desired trajectory $x \mapsto \vartheta(x)$ may be obtained, by interpolation, as a 6 th degree polynomial:

$$
\begin{align*}
\vartheta(x)= & v_{0}+v_{0}^{\prime} X \xi(x)+\frac{v_{0}^{\prime \prime}}{2} X^{2} \xi^{2}(x) \\
& +\frac{v_{0}^{\prime \prime \prime}}{6} X^{3} \xi^{3}(x)+a_{4} \xi^{4}(x)  \tag{19}\\
& +a_{5} \xi^{5}(x)+a_{6} \xi^{6}(x)
\end{align*}
$$

with $X=x_{1}-x_{0}=\bar{x}-x_{0}, \xi(x)=\frac{x-x_{0}}{X}$, the coefficients $a_{4}, a_{5}, a_{6}$ being:

$$
\left\{\begin{align*}
a_{4}= & 15\left(v_{1}-v_{0}\right)-5\left(v_{1}^{\prime}+2 v_{0}^{\prime}\right) X  \tag{20}\\
& +\frac{1}{2}\left(v_{1}^{\prime \prime}-6 v_{0}^{\prime \prime}\right) X^{2}-\frac{1}{2} v_{0}^{\prime \prime \prime} X^{3} \\
a_{5}= & -24\left(v_{1}-v_{0}\right)+3\left(3 v_{1}^{\prime}+5 v_{0}^{\prime}\right) X \\
& -\left(v_{1}^{\prime \prime}-4 v_{0}^{\prime \prime}\right) X^{2}+\frac{1}{2} v_{0}^{\prime \prime \prime} X^{3} \\
a_{6}= & 10\left(v_{1}-v_{0}\right)-2\left(2 v_{1}^{\prime}+3 v_{0}^{\prime}\right) X \\
& +\frac{1}{2}\left(v_{1}^{\prime \prime}-3 v_{0}^{\prime \prime}\right) X^{2}-\frac{1}{6} v_{0}^{\prime \prime \prime} X^{3}
\end{align*}\right.
$$



Figure 2: Graph of $\vartheta$ and associated current and voltage

Since in these formulae $M$ and $N$ are not precisely known, we replace them by estimates noted $\widehat{M}$ and $\widehat{N}$. Note that these estimations are only needed to compute the reference voltage. An explicit construction of an estimator of $\widehat{N}$ is presented in section 6 .

Indeed, from this trajectory, a time-parameterized trajectory is easily deduced: if $t_{0}$ is the duration to travel from the opposite support to $x_{0}$, the total travelling time is:

$$
\begin{equation*}
T=t_{0}+\int_{x_{0}}^{x_{s}} \frac{1}{\vartheta(\xi)} d \xi \tag{21}
\end{equation*}
$$

since $\frac{d t}{d x}=\frac{1}{\vartheta(x)}$.
The 4 free parameters $x_{0}, u_{0}, v_{1}^{\prime}$ et $v_{1}^{\prime \prime}$ may be chosen such that the constraints on $i$ and $\Phi$ are satisfied with a travelling time $T$ as short as possible. It is interesting to remark that with $u_{0}=0$ we start close to the initial open-loop situation, which indeed doesn't hit the saturations. At the other end of the trajectory, increasing $\left|v_{1}^{\prime}\right|$ accelerates the landing and decreasing $\left|x_{0}\right|$ decreases the duration $T$.

The graphs of $\vartheta$ and the associated current and voltage are displayed in figure 2, in the opening case with the following parameters:

$$
\begin{gathered}
x_{0}=-2 \mathrm{~mm}, \quad v_{0}=-2.91 \mathrm{~m} / \mathrm{s}, \quad u_{0}=0 \mathrm{~V} \\
x_{1}=-4.0018 \mathrm{~mm}, \quad v_{1}^{\prime}=-2800 \mathrm{l} / \mathrm{s}
\end{gathered}
$$

Since, in this example, we have'nt imposed the curvature $v_{1}^{\prime \prime}, \vartheta$ is a 5 th degree polynomial.

## 4 CONTROL SYNTHESIS

The functions $(\vartheta, \phi)$ previously designed define an invariant set of system (7). Indeed, for any initial condition
$(x, v, \Phi)$ satisfying:

$$
\begin{equation*}
(v, \Phi)=(\vartheta(x), \phi(x)) \tag{22}
\end{equation*}
$$

the control:

$$
\begin{equation*}
u=v(x) \tag{23}
\end{equation*}
$$

satisfies (9) and is such that the corresponding solution $(x(t), v(t), \Phi(t))$ satisfies for all time $t$ :

$$
\begin{equation*}
(v(t), \Phi(t))=(\vartheta(x(t)), \phi(x(t))) \tag{24}
\end{equation*}
$$

Moreover, this trajectory coincides with the planned reference trajectory.

On this set, the dynamics reduce to:

$$
\begin{equation*}
\dot{x}=\vartheta(x) \tag{25}
\end{equation*}
$$

Outside this set, we have:

$$
\begin{equation*}
\dot{x}=v=\vartheta(x)+(v-\vartheta(x)) \tag{26}
\end{equation*}
$$

Thus, it suffices to asymptotically stabilize the invariant set $v=\vartheta(x)$. According to 10,

$$
\begin{align*}
& \overparen{v-\vartheta(x)}=\left[-\frac{k}{m} x-\frac{c_{f}}{m} v+s \Phi^{2}\right]-\vartheta^{\prime}(x) v \\
& =-s \phi(x)^{2}+\left[\vartheta^{\prime}(x) \vartheta(x)+\frac{k}{m} x+\frac{c_{f}}{m} \vartheta(x)\right]  \tag{27}\\
& \quad+\left[-\frac{k}{m} x-\frac{c_{f}}{m} v+s \Phi^{2}\right]-\vartheta^{\prime}(x) v \\
& =-\left[\vartheta^{\prime}(x)+\frac{c_{f}^{m}}{m}\right][v-\vartheta(x)]+s\left[\Phi^{2}-\phi(x)^{2}\right] .
\end{align*}
$$

Assuming that $\Phi^{2}$ is a control variable, the invariant set may be stabilized by choosing:

$$
\begin{equation*}
\Phi^{2}=\phi(x)^{2}-\operatorname{sg}(x)[v-\vartheta(x)] \tag{28}
\end{equation*}
$$

where $g$ is an arbitrary function satisfying

$$
\begin{equation*}
g(x)+\left[\vartheta^{\prime}(x)+\frac{c_{f}}{m}\right]>0 \tag{29}
\end{equation*}
$$

In fact, since $\Phi$ is not a control variable, but, according to (5), the corresponding current is:

$$
\begin{equation*}
i^{2}=\frac{m}{\widehat{M}}(\widehat{N}-s x)^{2}\left(\phi(x)^{2}-\operatorname{sg}(x)[v-\vartheta(x)]\right) \tag{30}
\end{equation*}
$$

where $\widehat{M}$ and $\widehat{N}$ are estimates of $M$ and $N$. Since these estimates produce various consequences on the closed-loop behavior, we decompose the expression of $i^{2}$ as the sum of a constant, a linear term in $x-x_{1}$ and a linear term in $v$. From (11) we have:

$$
\begin{align*}
i^{2}= & s \frac{m}{\widehat{M}_{c}}(\widehat{N}-s x)^{2}\left(\vartheta^{\prime}(x) \vartheta(x)+\frac{k}{m} x\right.  \tag{31}\\
& \left.+\frac{c f}{m} \vartheta(x)-g(x)[v-\vartheta(x)]\right) .
\end{align*}
$$

Since, by construction, the function $\vartheta$ vanishes at $x_{1}$, we deduce that:

$$
\begin{align*}
& i^{2}=s \frac{k}{\hat{M}}\left(\widehat{N}-s x_{1}\right)^{2} x_{1} \\
& +\frac{s k\left(x-x_{1}\right)}{\widehat{M}}\left[(\widehat{N}-s x)^{2}-s x_{1}\left(2 \widehat{N}-s\left(x+x_{1}\right)\right)\right]  \tag{32}\\
& +\frac{s m(\widehat{N}-s x)^{2}}{\widehat{M}}\left[\left(\vartheta^{\prime}(x)+\frac{c_{f}}{m}\right) \vartheta(x)-g(x)(v-\vartheta(x)]\right.
\end{align*}
$$

In the first term of the right-hand side, the parameters $\widehat{M}$ and $\widehat{N}$ may be used to move the equilibrium point. In the other terms, their contribution is to change the position and velocity gains, and therefore the convergence rates. Therefore, we introduce two values $\widehat{M}_{1}$ and $\widehat{N}_{1}$ a priori distinct from $\widehat{M}$ et $\widehat{N}$ respectively. We therefore get:

$$
\begin{align*}
& \tilde{i}^{2}=s \frac{k}{\widehat{M}_{1}}\left(\widehat{N}_{1}-s x_{1}\right)^{2} x_{1} \\
& +\frac{s k\left(x-x_{1}\right)}{\widehat{M}}\left[(\widehat{N}-s x)^{2}-s x_{1}\left(2 \widehat{N}-s\left(x+x_{1}\right)\right)\right]  \tag{33}\\
& +\frac{s m(\widehat{N}-s x)^{2}}{\widehat{M}}\left[\left(\vartheta^{\prime}(x)+\frac{c_{f}}{m}\right) \vartheta(x)-g(x)(v-\vartheta(x))\right] .
\end{align*}
$$

Taking into account the constraint (8), we obtain:

$$
\begin{equation*}
i_{\mathrm{cons}}=\sqrt{\max \left\{0, \min \left\{i_{\mathrm{max}}^{2}, \tilde{i}^{2}\right\}\right\}} \tag{34}
\end{equation*}
$$

## 5 PHASE PORTRAIT ANALYSIS AND TUNING PROCEDURE

Let us assume for a moment that the current loop is perfect, i.e. $i=i_{\text {cons }}$, that the constraint on $i^{2}$ is satisfied, i.e. $i_{\text {cons }}=$ $\tilde{i}$, and that the mass $m$ and the coefficient $M$ are perfectly known: $\widehat{M}_{1}=\widehat{M}=M$.

To study the closed-loop behavior in presence of estimation errors on $N$, we rewrite $\dot{v}$ as follows:

$$
\begin{equation*}
\dot{v}=-U^{\prime}(x)-k(x) v, \tag{35}
\end{equation*}
$$

where $U^{\prime}$ is the derivative of a function $U$, interpreted as a potential, with the total energy defined by:

$$
\begin{equation*}
E(x, v)=\frac{1}{2} v^{2}+U(x) \tag{36}
\end{equation*}
$$

or

$$
\begin{equation*}
\dot{E}=-k(x) v \tag{37}
\end{equation*}
$$

Thus, the energy is dissipated by the system if the function $k$ is non negative. According to the Legendre-Dirichlet Theorem, if the critical points of $U$, i.e. the points that are solutions of $U^{\prime}(x)=0$, are equilibrium points and they are stable and attractive if they correspond to a local minimum of $U$. If such a point is unique in the domain of interest, i.e. on $(-\infty, N)$ for $s=1$ and $(-N,+\infty)$, for $s=-1$, and if the potential is proper on this domain, this point is a globally asymptotically stable equilibrium.

For $k$, we directly have:

$$
\begin{equation*}
k(x)=\frac{c_{f}}{m}+g(x) \frac{(\widehat{N}-s x)^{2}}{(N-s x)^{2}} . \tag{38}
\end{equation*}
$$

The system is thus always dissipative if $g$ is non negative, which implies 29.

The potential $U(x)$ may be directly determined by computing $\dot{v}$. These computations are ommitted here. We just


Figure 3: The potential $U(x)$.


Figure 4: Equilibrium points and associated eigenvalues locus.
indicate that there exist values of $\widehat{M}, \widehat{M}_{1}, \widehat{N}, \widehat{N}_{1}$ such that the potential $U$ is given by the curve of Figure 3 .

The critical points of the obtained $U$ are the roots of a 3 rd degree polynomial. The left column of Figure 4 shows the root locus function of $\widehat{N}$ for 5 different values of $\widehat{N}_{1}$. The right column shows the real part of the eigenvalues of the tangent linearization at the largest equilibrium. The vertical and horizontal lines are respectively the true values of $N$, $x_{1}$ and of the eigenvalues. We observe that for $\widehat{N}>N$, we only have one globally asymptotically stable equilibrium point, all the closest of $x_{1}$ since $\widehat{N}$ is large. When $\widehat{N}_{1}>N$, the equilibrium point is beyond the cylinder head, which means that a shock occurs. When $\widehat{N}_{1}<N$, the equilibrium point is below the cylinder head, which means levitation.

To conclude, we may choose $\widehat{N}_{1}$ as close as possible to $N$ and $\widehat{N}>N$ large, such that $N^{2}+4 N \widehat{N}-\widehat{N}^{2}<0$,

Also, $g$ can be chosen as $g(x)=-g_{\vartheta} \vartheta^{\prime}(x)$, with $g_{\vartheta}$ not too large to avoid saturations.

## 6 SPEED AND FLUX OBSERVER DESIGN AND ESTIMATOR OF $N$

To estimate the velocity $v$ from position and current and voltage measurements without using the constant $N$, we set:

$$
\begin{equation*}
z=v-k_{v} x \tag{39}
\end{equation*}
$$

where $k_{v}$ is an arbitrary positive real. Differentiating, we get

$$
\begin{equation*}
\dot{z}=-\left(\frac{k}{m}+k_{v}\left(\frac{c_{f}}{m}+k_{v}\right)\right) x-\left(\frac{c_{f}}{m}+k_{v}\right) z+s \Phi^{2} \tag{40}
\end{equation*}
$$

Thus, if we set

$$
\begin{equation*}
\widehat{z}=\widehat{v}-k_{v} x \tag{41}
\end{equation*}
$$

where $\widehat{v}$ is the required estimate of $v$ and $\widehat{z}$ the one of $z$, with

$$
\begin{equation*}
\dot{\hat{z}}=-\left(\frac{k}{m}+k_{v}\left(\frac{c_{f}}{m}+k_{v}\right)\right) x-\left(\frac{c_{f}}{m}+k_{v}\right) \widehat{z}+s \widehat{\Phi}^{2} \tag{42}
\end{equation*}
$$

we obtain, using the fact that $\widehat{z}-z=\widehat{v}-v$ and $\overparen{z-z}=\overparen{v}-v$, the estimation error

$$
\begin{equation*}
\overparen{\widehat{v}-v}=-\left(\frac{c_{f}}{m}+k_{v}\right)(\widehat{v}-v)+s\left(\widehat{\Phi}^{2}-\Phi^{2}\right) . \tag{43}
\end{equation*}
$$

In addition, this observer may be completed by using the dynamics of $\Phi, u$ and $i$ being measured. We set

$$
\begin{equation*}
\zeta=\Phi-k_{\Phi} x . \tag{44}
\end{equation*}
$$

thus

$$
\dot{\zeta}=\frac{1}{2 \sqrt{m M}}(u-r i)-k_{\Phi} v
$$

and, noting $\widehat{\zeta}$ the estimate of $\zeta$, the following system is an observer

$$
\begin{equation*}
\dot{\hat{\zeta}}=\frac{1}{2 \sqrt{m M}}(u-r i)-k_{\Phi} \widehat{v} \tag{45}
\end{equation*}
$$

with

$$
\begin{equation*}
\widehat{\Phi}=\widehat{\zeta}+k_{\Phi} x \tag{46}
\end{equation*}
$$

Thus, the error dynamics satisfies:

$$
\begin{equation*}
\dot{\zeta}-\dot{\zeta}=\dot{\Phi}-\dot{\Phi}=-k_{\Phi}(\hat{v}-v) \tag{47}
\end{equation*}
$$

To summarize, the following reduced observer has been constructed:

$$
\left\{\begin{array}{l}
\dot{\hat{z}}=-\left(\frac{k}{m}+k_{v}\left(\frac{c_{f}}{m}+k_{v}\right)\right) x-\left(\frac{c_{f}}{m}+k_{v}\right) \widehat{z}+s \widehat{\Phi}^{2}  \tag{48}\\
\dot{\zeta}=\frac{1}{2 \sqrt{m M}}(u-r i)-k_{\Phi} \widehat{v} \\
\widehat{v}=\widehat{z}+k_{v} x \\
\widehat{\Phi}=\widehat{\zeta}+k_{\Phi} x
\end{array}\right.
$$

and the estimation error on $v$ and $\Phi$ is given by

$$
\left\{\begin{array}{l}
\dot{\hat{v}}-\dot{v}-\left(\frac{c_{f}}{m}+k_{v}\right)(\hat{v}-v)+s(\widehat{\Phi}+\Phi)(\widehat{\Phi}-\Phi)  \tag{49}\\
\stackrel{\grave{\Phi}}{ }-\dot{\Phi}-k_{\Phi}(\widehat{v}-v) .
\end{array}\right.
$$

It can be made stable by choosing $k_{v}>0$ and $k_{\Phi}>0$. More precisely, for every solution of the closed-loop system such that $\widehat{\Phi}+\Phi \leq \Phi_{\max }$, it suffices that $k_{v}$ and $k_{\Phi}$ satisfy:

$$
\begin{equation*}
\left(\frac{c_{f}}{m}+k_{v}\right)^{2}>2\left(s k_{\Phi}\right) \Phi_{\max } \tag{50}
\end{equation*}
$$

In addition, an estimate of $N$, given $\widehat{M}$ and $\widehat{m}$, is given by:

$$
\begin{equation*}
\widehat{N}=-x+\sqrt{\frac{\widehat{M}}{\widehat{m}}\left|\frac{i}{\widehat{\Phi}}\right|} \tag{51}
\end{equation*}
$$

## 7 SIMULATION IN THE OPENING PHASE

In Figure 5, we give a simulation displaying a typical opening behavior. The red curves are the references obtained by $\vartheta$ and the associated current and voltage. The blue ones are the real ones. The magenta ones describe the flux and velocity estimates.

The function $\vartheta$ is the one of Figure 2, obtained with the parameters: $x_{0}=-2 \mathrm{~mm}, v_{0}=-2.91 \mathrm{~m} / \mathrm{s}, u_{0}=0 \mathrm{~V}, x_{1}=$ $-4.0018 \mathrm{~mm}, v_{1}^{\prime}=-28001 / \mathrm{s}$.

The other parameters used to stabilize the set $v=\vartheta(x)$ are: $g_{\vartheta}=7$ and $\left.g(x)=-g_{\vartheta} \vartheta^{\prime}(x)\right), k_{v}=10000, k_{\Phi}=$ $10000 * k_{v} / 120, \widehat{N}=4.25 \mathrm{~mm}, \widehat{N}_{1}=4.084 \mathrm{~mm}, \widehat{M}=$ $2.4726710^{-7}$.

The phase portrait in the right below graph of Figure 5 shows, on the right of the point marked by a star, the function $\vartheta$ and the real trajectory. When the coil is activated, the error $v-\vartheta(x)$ is significant but has the right sign and the controller can react by increasing the current (second graph on the left). Such a current cannot be delivered at once since the voltage is saturated (3rd graph on the left). However, the flux increases and catches up with its reference. The velocity and flux are given in the two first graphs on the right, showing a satisfactory convergence of the observer.

## 8 CONCLUSION

We have presented a nonlinear controller able to track a reference trajectory corresponding to soft landing and playing the role of an invariant manifold. This control law is completed by a reduced observer of the velocity and flux. However, the final precision might be improved by taking into account the magnetic losses, as in [4].


Figure 5: Example of valve opening behavior (Simulation)

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[^0]:    *Work supported by PSA SEE-CITN

[^1]:    ${ }^{1}$ We indeed have $s^{2}=1$.

