Adaptive Eccentricity Compensation

Carlos Canudas de Wit and Laurent Praly

Laboratoire d'Automatique de Grenoble, UMR CNRS 5528
ENSIEG-INPG, BP 46, 38402, ST. Martin d'Hères, France
canudas@lag.ensieg.inpg.fr

Ecole des mines de Paris, Centre Automatique
35, rue Saint-Honoré, 77305 Fontainebleau France.
praly@cas.enmp.fr

Abstract

This paper is devoted to the problem of rejecting oscillatory position-dependent disturbances (eccentricity) with unknown frequency and unknown amplitude. Most of the previous works on eccentricity cancellation assume a time-varying oscillation, we instead assume that the oscillatory disturbance is position-dependent. This leads us to formulate and to globally solve the adaptive cancellation problem in the spatial domain coordinates (curvilinear abscissa associated to the trajectory of the motion), which is the rate-independent (or space) description of the position. An apparatus with rolling eccentricity has been build to test the controller.

Keywords: Adaptive compensation, Eccentricity sinusoidal disturbance rejection.

1. Introduction

We consider systems of the form:

\[ J\ddot{\theta} = u + d(x); \quad \dot{\theta} = \frac{dx}{dt} \] (1)

where \( x \) is the system angular position, \( J \) is the inertia, \( u \) is the control input and \( d(x) \) is the position-dependent oscillatory disturbance defined as:

\[ d(x) = A \cos(\omega x + \Phi) \] (2)
\[ = a_1 \cos \omega x + a_2 \sin \omega x \] (3)

It is assumed that the amplitude \( A \), the dimensionless frequency \( \omega \), and the phase \( \Phi \) (or equivalent, the parameters \((a_1, a_2, \omega)\)), of the disturbance \( d \), are unknown. The problem considered here is thus to cancel the effect of the disturbance \( d \) in the system (1).

This type of problem arises as a consequence of eccentricity in many mechanical systems where the center of rotation does not correspond with its geometric center. This is typically the case on drives with magnetic bearings. It also arises in systems with friction where the contact forces change as a function of the position \( x \).

The dependency on position of \( d(x) \) can be visualized in the following scenarios. It is known that the friction forces depend on the normal force acting between two surfaces. Inaccuracies in the geometric position of the rotating axis of a rolling mill (eccentricity), will produce position dependent disturbances. In gear boxes, friction will vary as a function of the effective surface in contact of the gear's teeth. The two dimensional rolling and spinning friction causes in ball bearings the frictional torque to be dependent on both position and velocity. Figure (1) shown some of these examples.

Many of the existing works consider \( d \) not as a position function, but as a time-dependent exogenous signal, of the form

\[ d(t) = A \cos(\omega t + \Phi) \] (4)

In the previous mentioned system this hypothesis is only valid if we assume that the system is operating and regulated, at constant velocity \( v_0 \) so that \( x(t) \) becomes proportional \( t \cdot v_0 \). Disturbances of the form (4) have been considered in problems such as active noise and vibration control. The noise \( d(t) \) is thus assumed to be generated by the rotating machinery and transmitted through the sensor path. Examples rate from engine noise in turbo-prop aircraft [7] to ventilation noise in HVAC system [8], passing through engine noise in automobiles [12].

The proposed solutions resort to "standard" adaptive algorithms if the frequency \( \omega \) is assumed to be known [4]. Repetitive control has also been used to compensate eccentricity in rolling [9]. For the general case where both amplitude and frequency are unknown, some approaches based on the phase-lock loop principle has been proposed [3], but without proof of stability. When formulating this problem in the time-domain, the main difficulty to show global stability properties of the adaptive algorithms comes from the fact that the unknown parameters appear...
it corresponds a non autonomous differential equation in the $t$-domain, i.e.

$$Dx = \dot{z} = |v(t)|f(x).$$

(9)

The converse may not hold in general with a problem when $v(t) = 0$.

This type of transformation has been used before in connection with hysteresis operators (see [10], [13]), and more recently as a mathematical tool to model dynamic friction [2]. In particular, authors in [2] and [1] shown how the nonlinear Dahl’s friction model can be transformed into a linear spatial invariant system.

**Internal $\nabla$-model for $d$.** 

Let

$$y = J\dot{v} - u = d(x)$$

(10)

with $d(x)$ given in (3). Computation of $\nabla y$ and $\nabla^2 y$, gives:

$$\left( \begin{array}{c} \nabla y/\omega \\ \nabla^2 y \end{array} \right) = \left( \begin{array}{cc} \cos \omega z & \sin \omega z \\ -\sin \omega z & \cos \omega z \end{array} \right) \left( \begin{array}{c} a_1 \\ a_2 \end{array} \right)$$

(11)

$$\nabla^2 y = -\omega^2 y$$

(12)

where the equation (12) describes the internal model for $y$. Let $z_1 = y$, and $z_2 = \nabla y$, a s-domain state space realization for (12) is:

$$\nabla z = \left( \begin{array}{cc} 0 & 1 \\ -\theta & 0 \end{array} \right) z$$

(13)

$$y = (1,0)z$$

(14)

with $\theta = b^2$, and $z = [z_1, z_2]^T$.

### 3. Control design

In this section the control law, the adaptive observer, and the adaptation law will be presented. Stability is studied at the end of the section. The control design philosophy consists in first designing an adaptive internal model predictor for $d$, directly in the s-domain, and then to study the stability properties of the coupled s-domain and t-domain equations.

We consider the problem of tracking the desired velocity $v_d(t)$ supposed to be bounded and continuous (for simplicity) as well as its derivative. To this aim we define the adaptive eccentricity control (AEC) control as:

$$u = J\dot{v}_d - k_v(v - v_d) - \dot{z}_1,$$

(15)

where $\dot{z}_1$ is an output of the following dynamic system (observer):

$$\nabla \dot{z}_1 = \dot{z}_2 + k_1(z_1 - \dot{z}_1)$$

(16)

$$\nabla \dot{z}_2 = -\theta \dot{z}_1 + k_2(z_1 - \dot{z}_1) - \lambda \nabla \theta$$

(17)

$$\nabla \dot{\theta} = -\gamma \nabla \theta$$

(18)

Also to any differential equation in the s-domain

$$\nabla x = f(x)$$

(8)
with
\[ y = z_1 = J\dot{v} - u = a_1 \cos(\omega x) + a_2 \sin(\omega x) \]  
(20)
and positive nonzero constants \( k_1, k_2, \lambda, \mu, \) and \( \gamma. \) As we shall see this representation of the controller is appropriate for analysis. But it cannot be implemented directly in this way unless the system acceleration, is assumed to be measurable. Alternatively, we can first transform the system (16)-(19) to the time-domain, and then show that measurement of \( \dot{v} \) is not needed. For the former, we have readily
\[
\dot{z}_1 = |v| [z_2 + k_1(J\dot{v} - u - \dot{z}_1)] \quad (21) \\
\dot{z}_2 = |v| [-\hat{\theta}z_1 + k_2(J\dot{v} - u - \dot{z}_1) - \lambda z_1 \nabla \hat{\theta}] \quad (22) \\
\dot{\hat{\theta}} = |v| [-\gamma \dot{z}_1(J\dot{v} - u - \dot{z}_1)] \quad (23) \\
\dot{\hat{\theta}} = |v| [-\gamma \dot{z}_1(J\dot{v} - u - \dot{z}_1)] \quad (24)
\]
For the latter, noting that
\[
\frac{d}{dt} \{ |v| v^i \} = (i + 1) |v| v^{i-1} \dot{v} \quad \forall i = 1, 2, 3, \ldots 
\]
(25)
and introducing
\[
\dot{\zeta}_1 = \dot{z}_1 - \frac{k_1 J}{2} |v| v \\
\dot{\zeta}_2 = \dot{z}_2 - \frac{k_2 J}{2} |v| v - \frac{\gamma \lambda J}{2} |v| v \dot{z}_1^2 \\
\dot{\zeta}_1 = \dot{z}_1 \\
\dot{\hat{\theta}} = \dot{\hat{\theta}} + \frac{\gamma J}{2} \dot{z}_1 |v| v
\]
We have
\[
\dot{\zeta}_1 = |v| [z_2 - k_1 (u + \dot{z}_1)] \\
\dot{\zeta}_2 = |v| [-k_2 \theta \dot{z}_1 - k_2 u - \gamma \lambda \dot{z}_1^2 (u + \dot{z}_1) + \gamma J |v| v \dot{z}_1 (\mu \dot{z}_1 - \dot{z}_1)] \\
\dot{\zeta}_1 = |v| [-\frac{1}{\lambda} (\mu \dot{z}_1 - \dot{z}_1)] \\
\dot{\hat{\theta}} = |v| [\gamma \dot{z}_1 (u + \dot{z}_1) + \frac{\gamma J}{2} |v| v (\mu \dot{z}_1 - \dot{z}_1)]
\]
(30)
(31)
(32)
(33)
This gives us a well defined t-domain state space realization depending only on the measurable velocity \( v. \) Note that with the assumption that \( u_d \) and \( v_d \) are bounded and continuous, with this controller, we get a closed loop system whose dynamics are described by an ordinary differential equation whose right hand side is continuous.

To analyze this closed-loop system, we introduce the error variables:
\[
\begin{aligned}
e & = v - v_d, \quad \bar{z} = z - \bar{z}, \quad \bar{\theta} = \theta - \bar{\theta}.
\end{aligned}
\]
(42)

Figure 2 shows the inter-block connection among these systems. Note that the closed-loop block connection, at the left of the figure -Equations (38-36)- involves signals that are parameterized in the t-domain, whereas the block at the right -Equation (34)- involves a LTI map having as input a signal parameterized in \( s. \) This results in a coupled s-domain / t-domain system.

**Stability analysis**  Note that the map \( G(\nabla) \) admits a state space representation of the form:
\[
\nabla \xi = A \xi - B \bar{z} \bar{\theta} \\
\bar{z}_1 = C \xi
\]
(38)
(39)
where:
\[
A = \begin{pmatrix} 0 & 1 \\ -(k_2 + \theta) & -k_1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad C = \begin{pmatrix} \lambda \\ \mu \end{pmatrix}
\]
(40)
and \( \xi = T \bar{z}, \) for some invertible linear transformation \( T. \) Since \( A \) in (40) is strictly Hurwitz for any positive value of \( k_2 \) (note that \( \theta > 0), \) and \( k_1, \) it follows that we can find \( P \) symmetric positive definite matrix such that:
\[
A^T P + P A = -I
\]
(41)
Then let \( \lambda \) and \( \mu \) in \( C \) in (40) be defined as:
\[
C = PB.
\]
(42)

**Theorem 1**  Consider the system (1)-(3). Consider the dynamic feedback defined by the equation set (15- 19). Let the control gains \( k_1 > 0, k_2 > 0, \) and \( \lambda, \mu \) be defined by (42). Then all the internal signal of the system are bounded and in addition, the velocity tracking error \( e \) tends
to zero, if the desired velocity time-profile is rich enough to meet
\[ \lim_{t \to +\infty} \frac{1}{t} \int_0^t |v_d(t)| \, dt = +\infty \]
else, \( v - v_d \) tends to some finite value.

**Proof:** From what we have observed on the closed loop system, to any initial condition corresponds a unique solution. Let \([0, T]\) be its right maximal interval of definition in the \( t \)-domain. It corresponds functions in the \( s \)-domain defined on an interval \([0, S_{\text{max}}]\) where:
\[
S_{\text{max}} = \lim_{t \to \infty} \int_0^t |v(r)| \, dr (\leq +\infty) \tag{43}
\]
Let us show that \( T \) must be infinite. To do this we consider the non negative function \( V \):
\[
V = \xi^T P \xi + \gamma^{-1} \dot{\theta}^2
\]
with \( P \) given in (41). With the help of (38)-(39), (36), (41) and (42), we obtain in the \( s \)-domain:
\[
\nabla V = -\xi^T \xi - 2(\xi^T P B z_1) \dot{\theta} + \gamma^{-1} (\nabla \dot{\theta}) \dot{\theta}
\]
\[
= -\xi^T \xi - 2 \dot{z}_1 \dot{\theta} - \gamma^{-1} (\nabla \dot{\theta}) \dot{\theta}
\]
\[
= -\xi^T \xi - 2 \left[ \dot{z}_1 + \gamma^{-1} \nabla \dot{\theta} \right] \dot{\theta}
\]
\[
= -\xi^T \xi
\]
where Equation (46) results from the adaptation law that cancels the terms in the square brackets in (45) (this is how its is designed). Since \( \nabla V \) is non positive, \( V \) it is bounded on \([0, S_{\text{max}}]\) and therefore on \([0, T]\) it follows that \( \xi, \dot{z}_1 \) and \( \dot{\theta} \) are bounded on \([0, T]\). But from (34), the same holds for \( e \). Since the external signals \( \dot{e}_d, v_d \) and \( z_1 \) (see (20)) are bounded, we conclude that all the functions are bounded. So \( T \) must be infinity.

From (47), we have also
\[
\int_0^s \xi(\tau)^T \xi(\tau) \, d\tau \leq V(0) \quad \forall s \in [0, S_{\text{max}}] \tag{48}
\]
Since from (38), \( \nabla \xi \) is bounded on \([0, S_{\text{max}}]\) and \( \xi \) is uniformly continuous. From Barbalat’s Lemma, this implies that, if \( S_{\text{max}} = \infty \), i.e. \( |v| \) is not summable in the time domain, then \( \xi \) and therefore \( \dot{z}_1 \) converge to 0 as \( t \) goes to infinity. From (34), the same holds for \( e \). So when \( |v| \) is not summable in the time domain, we have:
\[
\lim_{t \to +\infty} \{ v(t) - v_d(t) \} = 0 \tag{49}
\]
When \( S_{\text{max}} \) is finite, i.e. \( |v| \) is summable on \([0, \infty)\), from (21) and the boundedness of the functions, we have that \( \dot{z}_1 \) is summable on \([0, +\infty)\). This implies the existence of \( \dot{z}_{1_\infty} \) such that
\[
\lim_{t \to +\infty} \dot{z}_1(t) = \dot{z}_{1_\infty} \tag{50}
\]

4. Experimental results

This section describes the experimental evaluation of AEC-controller. More detailed description of the real-time system used for these experiments, as well as additional experiments, can be found in [11].

The schematic view of the apparatus build to study eccentricity is shown in Fig 3. The load cylinder of inertia \( J_l \) is driven by the motor of inertia \( J_m \). On the top of it, we have placed a rotating wheel (with neglected inertia), constrained by the force \( F_N \). The rotation wheel’s center is set to be different to its geometric center. The contact pressure at the point where the wheel radius is equal to \( R \), is larger than the contact pleasure at the point of radius \( r \), since \( R > r \). This produces an eccentricity effect changing the normal force acting on the wheel-to-cylinder contact surface.

The model for the motor drive under this setup is given as:
\[
J \ddot{\theta} = u - F - A \cos(\omega x + \Phi) \tag{53}
\]
with:
\[
F = \sigma_0 \eta + \sigma_1 \eta + \sigma_2 v
\]
\[
\dot{\eta} = v - \frac{\sigma_0 |v|}{F_C} \eta; \quad F_C \triangleq \frac{A_0}{n} \left( \frac{F_N}{n} + F_{Nm} \right) \tag{55}
\]
equations:

\[
F = \sigma_0 \dot{\theta} + \sigma_1 \ddot{\theta} + \sigma_2 v_d \\
\dot{\theta} = v_d - \frac{\sigma_0 |v_d|}{\mu_0} \dot{\theta}, \quad \dot{\theta}(0) = 0.
\]

Figure 4, shown the block diagram of the AEC control scheme with feedforward friction compensation. Locally, \( \dot{F} \approx F \), thus the closed-loop equation with this additional friction compensation term is similar to the frictionless system studied in the previous section.

The control parameters used for these experiments were: \( k_v = 40, k_1 = 1, k_2 = 0.25, \gamma = 2, \lambda = 2, \mu = 1 \).

\[
\begin{array}{|c|c|c|}
\hline
\text{Friction parameters} & \text{Motor parameters} & \text{Wheel & cylinder characteristics} \\
\hline
F_D = 0.38 \text{ [Nm]} & J_m = 0.00196 \text{ [kg.m^2]} & R_0 = 6 \text{ [cm]} \\
F_S = 0.42 \text{ [Nm]} & J_l = 0.0126 \text{ [kg.m^2]} & \text{R = 2.1 [cm]} \\
v_0 = 0.01 \text{ [rad/sec]} & K_1 = 0.352 \text{ [Nm/Amp]} & r = 1.9 \text{[cm]} \\
\sigma_0 = 200.0 \text{ [Nm.sec/rad]} & n = 15.5 & \text{Power = 200 [Watts]} \\
\sigma_1 = 0.6 \text{ [Nm/sec/rad]} & & \\
\sigma_2 = 0.011 \text{ [Nm/sec/rad]} & & \\
\hline
\end{array}
\]

Table 1: Motor, friction, and load parameters.

The AEC controller, with feedforward compensation is:

\[
u = J_0 \ddot{\theta} - k_v (v - v_d) + \tilde{F} - \tilde{z}_1,
\]

where \( \tilde{z}_1 \) is given by the set of equations (27)-(33), and \( \tilde{F} \) is a feedforward friction prediction obtained from the

Figure 4, Control block scheme of the adaptive eccentricity compensator (AEC), with feedforward friction compensation.

Figure 5: Velocity tracking error ((v - \dot{v})

Figure 6: control input time profile.

The experiments in Figures 5-8, show the tracking error, the prediction \( \tilde{z}_1(t) \), the estimate \( \hat{\theta} \), and the control signal. The experiments are realized at \( v_d = 30 \text{ Rad/sec} \). The eccentricity compensation is applied at \( t = 10\text{sec} \). It can be observed that the oscillatory disturbance affecting the tracking error is canceled when applying the term \( \dot{z}_1(t) \), in \( u \). The remaining error, which is about 12% of \( v_d \) before compensation, is reduced to 1.5% with the compensation. The time-profiles shown in all the presented figures has been filtrate for improve their visibility.

The time evolution of \( \dot{z}_1(t) \), has a shape similar to a sinusoidal wave (as it has been predicted). The imperfections may be attributed to the nonuniform deformation.
of the wheel contact surface (the wheel in contact is composed of an inner undeformable steel wheel, covered by a rubber 5mm o-ring). The estimated parameter \( \bar{\theta} \) is observed to converge (in average sense) to a value close to 0.055, which according to (3) implies that \( \omega = \sqrt{0.055} = 0.23 \). The theoretical value of \( \omega \) is given by the expression 56 as \( \omega = \frac{2\pi}{n(v+R)} = 3/15 = 0.2 \), which seems to correspond to the experimental found value (note also that the period of \( z_1(t) \) gives for a velocity of 30 Rad/sec, an experimental value of \( \omega = (2\pi)/(\nu T) \approx 0.2 \), for a \( T = 1.6\pi \).

5. Conclusions

We presented a method for compensating eccentricity in mechanical system. As a main difference with previous works, we have formulated the disturbance as a position-dependent periodic function. This formulation seems to be justified in most of the mechanical applications where eccentricity occurs.

As a results of this formulation, we have used a spatial description of the system (often used in system with hysteresis) which is more natural than the use of a time formulation. This did allows us to design of an adaptive predictor of the disturbance directly in the spatial domain. Experiments shown that the AEC controller improves over simpler linear controller without eccentricity compensation.

6. Acknowledgements

The first author would like to thanks Petra Posselius, and Holger Olofsson, for the help in performing the experiments, and the interesting discussion that results during their stay at Grenoble.

References