

# Investigating the ability of various buildings in handling load shiftings

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**Abstract**—In modern constructions of residential buildings, several energy saving technologies exist. Therefore, when such buildings are renovated, various investments can be considered. The contribution of this article is a method for evaluating the ability of several renovating configurations to keep the inhabitants in a comfortable situation during load shifting periods. This question is of importance in the relationship, and then in the price setting, between the users (inhabitants) and the energy provider who uses these load shifting periods to optimize his production on a regional or national scale.

The evaluation is performed as follows. We consider a house equipped with electric heaters. Then an optimization method is used to compute, in a dynamical context, best heating strategies. Gradually, the load shifting period is made longer, and, thus, the weather conditions and the comfort constraints serve to characterize the actual ability of the building to guarantee an acceptable comfort during load shifting periods.

One conclusion is that for poorly insulated houses (which represents approximately 58% of the French stock) it is impossible to shift the load more than 20 minutes in winter time, even when using advanced regulation strategy of the electric heating system. On the other hand, other configurations are much better suited as illustrated in this article.

## I. INTRODUCTION

Recently, significant efforts have been made to reduce electricity peak-demand. In Europe, these peaks mostly occur in winter time, and are, for the main part, due to heating systems. To guarantee the grid stability, numerous studies have been focused on the overall load reduction. At the level of the individual houses, this reduction can be achieved thanks to a careful architectural design to efficiently capture and restore solar gains [1]. An advanced heating control strategy can also be a solution. Such control can be based on power tariff [2] or use the building thermal mass as an asset to shift the building consumption, reducing the peak consumption [3], [4], [5], [6] in turn.

This article follows such an approach and studies the impact of load shifting on five thermal models ranging from poorly to well insulated houses. The method of analysis consists in solving dynamic optimization problems under constraints to accurately compute optimal trajectories following a continuous-time approach similar to those presented in [7], [8], [9]. Gradually, considering the duration of the load shift as a parameter, one can determine the maximum allowable duration of a complete heating load shifting while maintaining an acceptable level of comfort. The results obtained in this

study show that the thermal mass of a poorly insulated building is not sufficient to perform load shiftings superior to twenty minutes. Thus, the use of houses inertia as power stocks appears to be relevant only in the case of sufficiently insulated buildings (which can actually handle load shiftings of several hours). Practical cases of interest are presented.

In Section II, a description of the considered building is given, together with a description of the discretization scheme yielding a high order linear model of the system. In Section III, this model is reduced and constraints are formulated on its input and outputs. In Section IV, the algorithm serving to solve the obtained constrained dynamical optimization is presented. In Section V, the results on the abilities of the different considered systems are presented together with the maximum bearable duration of daily load shiftings for each model. Finally in Section VI the conclusion and the perspectives of the study are presented.

## II. BUILDING MODEL

### A. Building description

The building under study is a single-family house. It corresponds to an actual experimental passive house being part of the INCAS platform built in Le Bourget du Lac, France (see Figure 1). For our study, five low performance versions of the building are considered. The reference version corresponds to a house built prior to the introduction of the first French thermal regulation (1975). This reference version used to represent 58% of the french stock in 2008. The four other versions correspond to various renovation levels on this reference. In this paper, they serve to study the beneficial effects of renovation efforts on the peak load management. The house has two floors for a total living area of 89 m<sup>2</sup>. 34% of its South facade surface is glazed while the North facade has only two small windows. All the windows are single-glazed. The South facade is also equipped with solar protections for the summer period. The external walls are made of a 30 cm-thick layer of concrete blocks and the floor consists in 20 cm of reinforced concrete. There is no insulation in the building except for the 10 cm of glass-wool in the attic. According to thermal simulation results using the Pléiades+COMFIE software [10], the heating load is 253 kWh/(m<sup>2</sup>.year) which is typical for such type of house. Comparisons have been performed during the design phase on

the passive house version of this building with other simulation tools like Energy Plus and TRNSYS [11] and have shown similar results.



Fig. 1. 3D view of the house (west and south facades)

Four different renovations of this building are presented in Table I:

TABLE I  
VERSIONS OF THE CONSIDERED BUILDING THROUGHOUT RENOVATIONS

Version	Renovation applied	Heating consumption (kWh/m <sup>2</sup> .year)
Reference (1 <sup>st</sup> )	none	253
Roof insulation (2 <sup>nd</sup> )	(1) + 30 cm of glass-wool in the attic	246
Triple glazing (3 <sup>rd</sup> )	(2) + Triple glazed windows	215
Insulation of external walls (4 <sup>th</sup> )	(3) + 15 cm of glass-wool in external walls	93
Heat recovery ventilation (HRV) (5 <sup>th</sup> )	(4) + HRV with an efficiency of 0.5 (accounting for air infiltration)	80

### B. Thermal model

The building is modeled as spatial zones of homogenous temperature. For each zone, each walls are divided in fine meshes small enough to also have a homogeneous temperature in each mesh point. There is one more mesh for the air and furniture in the zone. Eventually, a thermal balance is performed on each mesh within the building. It takes into account:

- $P_{\text{cond}}$ : the losses (or gains) by conduction in walls, floor and ceiling.
- $P_{\text{sol}}$ : the gains due to solar irradiance through the windows
- $P_{\text{conv}}$ : the losses (or gains) due to convection at walls surface
- $P_{\text{in}}$ : the internal gains due to heating, occupancy and other loads (only for zone air mesh)
- $P_{\text{bridges}}$ : heat losses through thermal bridges, not associated to thermal mass
- $P_{\text{ventil}}$ : heat losses due to air exchange.

When applied to the air of each zone, the thermal balance equation reads:

$$C_{\text{air}} \dot{T}_{\text{air}} = P_{\text{in}} + P_{\text{cond}} + P_{\text{bridges}} + P_{\text{ventil}} + P_{\text{sol}} + P_{\text{conv}} \quad (1)$$

with  $C_{\text{air}}$  the thermal capacity of the node air (including furniture) and  $T_{\text{air}}$  the temperature of the mesh. For each zone, repeating equation (1) for each mesh point and including an output equation leads to the following continuous linear time-invariant system.

$$\left. \begin{aligned} C\dot{T}(t) &= AT(t) + EU(t) \\ Y(t) &= JT(t) + GU(t) \end{aligned} \right\} \quad (2)$$

with:

- $T$  mesh temperatures vector
- $U$  driving forces vector (climate parameters, heating, etc.)
- $Y$  outputs vector (here, temperature of the air nodes)
- $C$  thermal capacity diagonal matrix
- $A, E, J, G$  matrices relating the vectors of the dynamics.

For representative simulations, it is important to know the occupancy of the building, which partly defines  $P_{\text{in}}$  with the emission of heat by the inhabitants and the appliances. The second part of heat emission in  $P_{\text{in}}$  is due to the heating system. Another important factor is the weather model. It defines the losses due to heat transfer with the ambient temperature and the gains with solar irradiance. All the data of the house occupancy and weather models are included in the input vector  $U$ .

## III. MODEL REDUCTION AND CONSTRAINTS

### A. Model reduction

The high order linear model (2) is now reduced. In view of application of optimization methods, its state dimension (order 33) is too large to allow a fast convergence of the optimization algorithm. A reduction method is applied to lower the state dimension. To reduce the dimension of the dynamics, several methods exist such as, e.g., singular perturbations [12], and identification methods [13]. In our case, an efficient method is the balanced truncation [14]. Indeed, this truncation consists in removing the state variables which receive the least effort from the input and contribute the less to the variations of the output, that is to say, the state variables that are easily negligible from an energetic view-point<sup>1</sup>. To determine the order of the reduced model, one can compare the error between the high order model and the reduced ones over one year and quantify it in terms of mean and standard deviation. Here, we decide to take the minimal order such that these statistical properties are both inferior to 0.1. In this case, all models are at least third order, and one of them is fourth order.

In Table II, the various time constants of the considered models are reported. One shall notice that these thermal building models clearly have three well separated time scales [12]. Interestingly, it shall be noticed that the main effects of the renovation is to enlarge the slow time constants. This is particularly true for the adjunction of insulation.

<sup>1</sup>We refer the interested reader to [14], for mathematical definitions of the considered approximation

TABLE II  
VALUE OF THE TIME CONSTANTS OF THE FIVE DIFFERENT MODELS.

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>
Time constants	8 min 13 h 95 h	7 min 13 h 98 h	8 min 2 h 8 h 91 h	9 min 13 h 160 h	9 min 18 h 180 h

### B. Model and constraints

1) *Model notations*: In the following, we use the classical linear state space representation to represent the model:

$$\dot{x}(t) = Ax(t) + Bu(t) + d(t) \quad (3)$$

$$y(t) = Cx(t) \quad (4)$$

where  $x$  is the state of the model,  $y$  is the inside temperature,  $d$  represents the influence of the outside temperature and the solar fluxes on the heating of the house and  $u$  represents the heating flux on the air node and is the control variable.

#### 2) Constraints:

a) *Inside temperature constraints*: The temperature constraints are 24 hours periodic and are:

- $y \leq 24^\circ C$  at all times.
- $y \geq 14^\circ C$  between 9 a.m. and 5 p.m.
- $y \geq 20^\circ C$  otherwise.

To simplify the notations, we write these temperature constraints as follows:

$$y^- \leq y \leq y^+ \quad (5)$$

b) *Control constraints*: The control constraints are not the same for all systems:

- $0 \leq u \leq 20$  kW for the buildings whose walls have not been insulated.
- $0 \leq u \leq 10$  kW for the buildings whose walls have been insulated.

To simplify the notations in the algorithm, we write the control constraints as follows:

$$0 \leq u \leq u^+ \quad (6)$$

c) *Load shifting*: In our case the load shiftings consist in a daily time period when the heating of the house is not allowed to consume any energy. These shiftings start everyday at 5 p.m.. The objective of this study is to determine the maximum duration of these load shiftings beyond which it becomes impossible to satisfy both (5) and (6).

## IV. METHOD AND ALGORITHM

### A. Method

In order to characterize the duration of load shifting which allows the inside temperature to satisfy (5) while the heating power satisfies (6), a state constrained optimal control approach is used. Indeed, when there does not exist any solution for the constrained optimal control problem, it is deduced that the load shiftings are too long. This property is independent of the temperature control system, and solely stems from the ability of the building to store energy.

To determine the maximum allowable duration of the load shifting, a sequence of continuous time optimal control problems is solved with increasing load shifting periods until there exists no solution satisfying the constraints (5) and (6).

### B. Algorithm

For this problem, the constraints are (5) and (6) and the considered criterion is the energy consumed over the whole week. To solve the state constraint optimal control problem a maximum principle-based interior method is used (following interior point approaches in [7], [8], [9], [15]), which has been adapted for the energy consumption problem. The criterion is given by the following (to minimize energy consumption):

$$J = \min_{u \in [0, u^+]} \int_0^T u(t) dt \quad (7)$$

with the dynamics and the state constraint  $y \in [y^-, y^+]$  seen above, and where  $T = 7$  days.

The first step, in this algorithm is to operate the following change in variable<sup>2</sup> on the control variable  $u$ :

$$u \triangleq \phi(\nu) = u^+ \left( \frac{e^{k\nu}}{1 + e^{k\nu}} \right) \quad (8)$$

This change in variable is such that  $\nu$  is an unconstrained variable and the optimization problem is now  $J = \min_{\nu \in \mathbb{R}} \int_0^T \phi(\nu) dt$ .

To solve this problem, an indirect method [17] using an adjoint vector  $p$  is used. This adjoint vector  $p$  has the same dimension as the state  $x$  of the reduced dynamical system (3). It satisfies the following differential equation

$$\frac{dp}{dt}(t) = -A^t p(t) - C^t \gamma'_y(Cx(t)) \quad (9)$$

where  $\gamma'_y$  is the derivative of the following function

$$\gamma_y(y(t)) = \left( \frac{y^+(t) - y^-(t)}{\sqrt{(y^+(t) - y(t))(y(t) - y^-(t))}} - 1 \right)^{2.1} \quad (10)$$

In (10), the expression serves to keep the output away from the constraint (see [16]). The power 2.1 guarantees the well-posedness of the method (see [9]). Now, to compute the optimal control, an interior method is used. It consists in solving a sequence of optimal control problems, depending on a parameter  $\epsilon_n$ , converging to the optimal solution of the problem while producing trajectories lying strictly in the interior of the constraints. The iterative algorithm is the following:

- Step 1: Initialize the functions  $x(t)$  and  $p(t)$  such that the initial unknown  $Cx(t) \in (y^-(t), y^+(t))$  for all  $t \in [0, T]$ , and set  $\epsilon = \epsilon_0$ . Simply,  $p$  can be chosen identically equal to zero at first step.
- Step 2: Compute  $\nu_\epsilon^* = \sinh^{-1} \left( -\frac{1+p^t B}{\epsilon} \right)$ . Thus the optimal solution  $u_\epsilon^* = \phi^{-1}(\nu_\epsilon^*)$  is given using equation (8) with  $k = 0.8$ .

<sup>2</sup>Similar to the saturation function approach considered in [16]

- Step 3: Solve the two differential systems of equation  $\frac{dx}{dt} = Ax + Bu_\epsilon^*$  and  $\frac{dp}{dt} = -A^t p - C^t \gamma'_y(Cx)$  forming a two point boundary value problem (e.g. using `bvp4c` see [18]), with the following boundary constraints  $x(0) = x_0$  and  $p(T) = 0$ .
- Step 4: Decrease  $\epsilon$ , initialize  $x(t)$  and  $p(t)$  with the solutions found at Step 3 and restart at Step 2.

In our case, the sequence  $(\epsilon_n)$  has been chosen such that  $\epsilon_n = 10^{-\frac{n}{10}}$  with  $n = 0 \dots 40$ . The proof of convergence of this algorithm can be found in [9]. Details on its derivation can also be found in [16].

## V. SIMULATIONS AND RESULTS

### A. Simulations

The considered optimization takes place in winter over one particularly cold week. The ambient temperature history is reported on Figure 2. For each version of the building, indoor temperatures (see Figure 3) and energy consumption over the week have been computed, first without load shifting and then, with maximal bearable load shifting (see Figure 4).

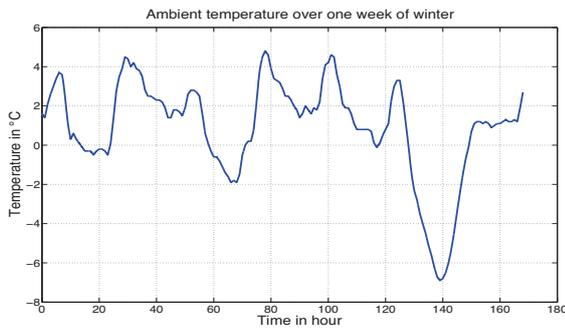


Fig. 2. Ambient temperature over one week of winter

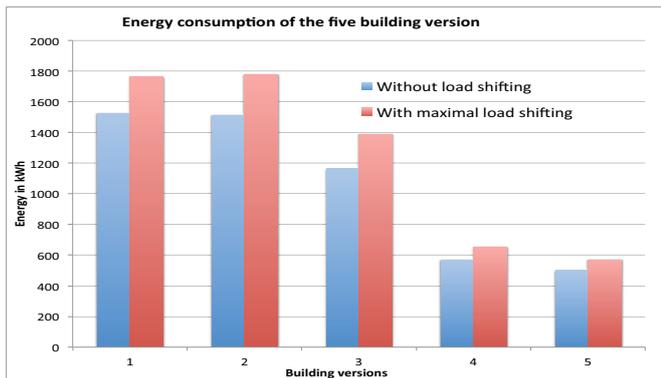


Fig. 4. Energy consumption over one week for the five versions of the building. For each building the consumed energy is displayed without load shifting and with the maximal bearable one.

### B. Summary of the results

In terms of energy consumption the first and second versions of the building are quite similar (Fig. 4). The adjunction

of triple glazed windows (3<sup>rd</sup> version) induces a significant decrease of energy consumption ( $\approx 30\%$ ). The insulation of the external walls (4<sup>th</sup> version) clearly allows to reduce the energy consumption further ( $\approx 50\%$ ). The most effective renovation strategy in terms of energy consumption seems to be the increasing of insulation.

Now, considering the ability in handling load shiftings, Table III and Figure 5 illustrate that the three first versions of the building cannot handle load shifting superior to 20 minutes. Interestingly, the adjunction of triple glazed windows (3<sup>rd</sup> version) does not improve the load shifting ability whereas it is efficient for energy savings. Actually, handling large load shifting periods becomes possible solely when the insulation is sufficiently increased.

TABLE III  
VALUE OF THE MAXIMUM LOAD SHIFTING DURATION FOR EACH VERSION OF THE BUILDING.

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>
Load shifting duration	15 min	20 min	20 min	4 h	6 h

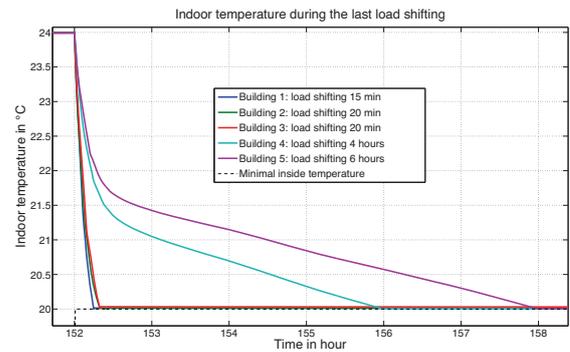


Fig. 5. Comparison of the optimal indoor temperature during the load shifting of the last day of the week

### C. Explanation of the results: the role of zeros in the transfer function

As reported on Table II, the renovation mainly influences the slow time constant of the systems. It is particularly noticeable that once some insulation has been added to the building the slow time constant is almost twice as large as the one of the non insulated versions. Nevertheless, even for the first version of the building, the slow and medium time constants are large compared to the duration of the maximum bearable load shifting (15 min). Thus, this modification of the slow and medium time constants through insulation does not satisfyingly explain the increase of maximum bearable load shifting duration (from 15 min to 6 h).

Actually, a role is played by the zeros of the transfer function between the heating flux on the air node and the indoor temperature. Table IV exhibits that the slow (resp. medium) zero and pole of the first building version are closer from each other than the ones of the fifth version. This increased

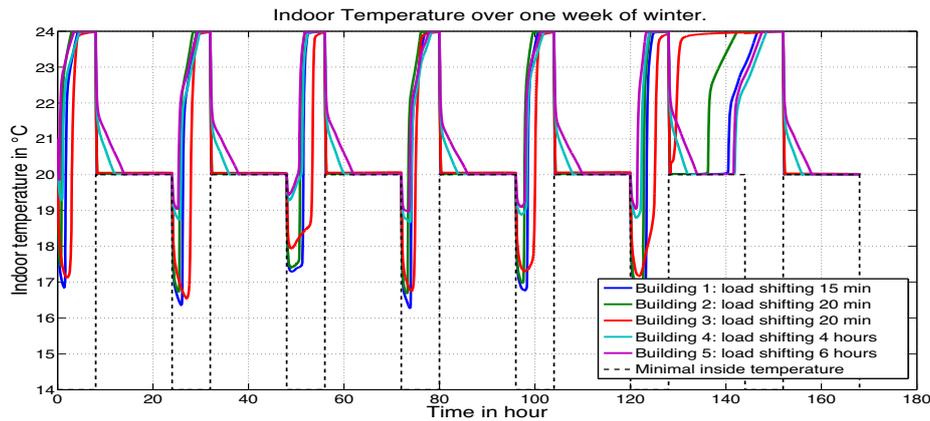


Fig. 3. Comparison of the optimal indoor temperature of each building with the maximum bearable load shiftings duration. The behavior of the indoor temperature is different on the last day because of the particularly cold ambient temperature.

distance between poles and zeros results in an increased control authority on the slow time scale state variables. This improvement allows to store energy within 24 hours in the slow state variables. By contrast, the non-insulated building slow time scale are not controllable enough to store energy within 24 hours in the slow time scale state variables, resulting in an energy storage in the fast time scale state variable and thus in a small load shifting period. This point will be a topic of future investigations.

TABLE IV  
VALUE OF THE POLES AND THE ZEROS OF THE DIFFERENT TIME SCALES FOR THE FIRST AND THE FIFTH VERSION.

	1 <sup>st</sup> version	5 <sup>th</sup> version
slow pole	$P = -0.0105$	$P = -0.00558$
slow zero	$Z = -0.0178$	$Z = -0.0174$
medium pole	$P = -0.0789$	$P = -0.0548$
medium zero	$Z = -0.268$	$Z = -0.251$

## VI. CONCLUSION

On the methodological side, it appears that the state constrained dynamic optimization is an effective tool to study properties of the buildings. The existence of feasible trajectories only depends on the characteristics of the buildings. The presented time continuous method yields precise results even when considering fast time scales phenomenon. On the applicative side, we have emphasized that a non-insulated residential house cannot handle load shifting superior to 20 minutes even if an advanced strategy of regulation is used. To allow these buildings to handle long load shiftings, their thermal mass is not sufficient, the buildings must be insulated enough or have auxiliary energy storage capacity. But, the regulation system itself is not sufficient to achieve long load shifting duration.

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