Preprints of the 18th IFAC World Congress
Milano (Italy) August 28 - September 2, 2011

Handling parameter ranking, equalities and bounds in adaptive control of blending systems

Mériam Chèbre * Yann Creff ** Nicolas Petit ***

* Advanced Process Control Department, Technical Direction TOTAL Refining & Marketing, Le Havre, France.
** Technology Division, Control Department, IFP Énergies nouvelles Lyon, France.
*** Centre Automatique et Systèmes, Unité Mathématiques et Systèmes, MINES ParisTech, France, (e-mail: nicolas.petit@mines-paristech.fr)

Abstract: This paper presents solutions to handle ranking, equalities and bounds for the parameter estimates of a linear adaptive controller scheme used in the production of commercial fuels by blending. The control problem under consideration is a multi-variable output regulation problems with large uncertainties in the plant parameters. It can be solved using a specifically designed adaptive controller which combines constrained optimization and a closed-loop estimator of the plant parameters. As in numerous applications of adaptive control, while output convergence is usually guaranteed under feasibility assumptions, little is known about the asymptotic behavior of the parameter estimates themselves. Yet, from an application view-point, it is desired that these estimates satisfy some physical properties. In particular, parameters ranking, equalities and bounds are of practical importance and assert the consistency of the estimation. In this paper we expose techniques that guarantee this desired behavior.

INTRODUCTION

The problem under consideration in this article is the adaptive control of blending systems as they are found in refining applications. These systems are used to produce a mixture (commercial fuels) having some desired properties. This problem is relatively general, as similar issues can be formulated in various applications where non-reactive components are blended and linearly impact on the properties of the blend (see e.g. the early references by Bay et al. [1969], Feld et al. [1968]).

A challenge which is encountered in practice is that the properties of the components are vastly uncertain. The culprits usually are the upstream units which produce these components with time-varying, unmeasured, unknown or poorly known properties.

In most applications, and in refining in particular, the blending objectives are to produce a mixture having some prescribed properties while minimizing production costs. Considering the previously mentioned uncertainties, this is a real challenge.

Over the past 40 years, such blending control problems have attracted much attention (see Walton and Swart [2003], Hi-Spec [2003], Perkins [2000], Le Febre and Lane [1995], Singh et al. [2000]). There has been significant research effort to propose closed-loop strategies using signals from on-line analysers located downstream the blender. It is worth mentioning that, usually, only downstream measurements are considered. The main reason for this choice is to minimize the number, and thus the cost, of required analysers. Basic strategies use single-variable controllers (mostly integral effect) in a single-input single-output modeling approach. A priori estimates of the components properties are used to assign the feedback loops. Recently, a new method has been presented in Chèbre et al. [2010], Bernier et al. [2006]. It uses a genuine approach in which measurements are used to update knowledge on the components properties. This is a multi-variable adaptive scheme. As is required in the applications under consideration, this algorithm directly addresses constraints on the blend properties flows limitations and pumping constraints by solving a constrained optimization problem. This algorithm, which has been installed from 2001, is now being used on 17 blenders located in 6 refineries within the TOTAL group.

The main algorithm consists of two distinct, though connected, layers: an optimization problem and a feedback loop with an observer. The optimization problem permits to account for the various discussed constraints and production cost minimization. The observer is used to partially estimate the components properties in a spirit of adaptive control methods. Both layers are required to provide convergence and guarantee a successful blend. To provide convergence of the blend properties to a prescribed target, the observer needs not to converge to the actual unknown values of the components properties. This behavior can be analyzed using LaSalle's invariance principle (see e.g. Khalil [1992]) for the underlying dynamical system. This might be the case though, but may not be so common in practice. Such behavior is well documented for multi-variable adaptive controllers (see e.g Ioannou and Sun

Copyright by the
International Federation of Automatic Control (IFAC)

[1996], Åström and Wittenmark [1995]). Most of the time, the blend target is reached before accurate estimates of the components properties are obtained. Very often, this is not a concern, because blend properties are definitely the primary target.

So far, nothing more was expected from the properties estimates than providing output convergence, i.e. a successful blend. Yet, from the end-user perspective, and also for diagnosis purposes (e.g. to detect possible malfunctions in upstream units), it is desirable that the estimates show a sensible behavior. Among the possible requirements, two are often formulated by process engineers: ranking and constraints (bounds or equalities) on certain of the properties. Ranking constraints express the fact that some components are known to have higher or lower properties values than others. Bounds are usually accounting for the fact that absolute minimum and maximum expectable values are known. Equalities are implied by upstream coupling of components or branching particularities in the fluids routing. By contrast with the values of the parameters which are vastly uncertain, these features are certain: no matter what happens in the blending process, the formulated ranking, equalities and bounds must hold.

In this paper, we present some solutions to force the adaptive scheme proposed in Chèbre et al. [2010] to generate parameter estimates which satisfy these parameters ranking, equalities and bounds. In details, we propose updates in the adaptation law that take the form of additional terms in the right-hand-side of the observer differential equations, or, as an alternative, periodically reset the observer state at discrete times. The first solution guarantees that the ranking and bounds are satisfied asymptotically, i.e. when the time t goes to $+\infty$. This fact is guaranteed by a careful study of LaSalle's invariance set (see Khalil [1992]) which is eventually reached by the system. The second solution considers an optimization problem to periodically compute corrections to the estimator state. Such optimization problems are proven to possess unique solutions by means of Farkas' lemma. Two typical cases are considered.

The paper is organized as follows. In Section 1, we present the control problem under consideration: the blending process, the actuators and the available sensors are briefly exposed. The solution introduced in Chèbre et al. [2010], Bernier et al. [2006] is detailed along with its convergence properties. Then, we formulate the desired additional features. In Section 2, we expose a first solution to guarantee ranking and bounds. The method is investigated using a Lyapunov function and LaSalle's invariance principle. In Section 2, we expose the second method which invokes an additional optimization problem to address ranking, equalities, and bounds. The mathematical feasibility of this constrained optimization problem is proven. Finally, we briefly compare the merits of the two methods and draw some conclusions in Section 4.

1. PROCESS DESCRIPTION, CONTROL PROBLEM, AND PROPOSED SOLUTION

1.1 Process description

In this section, we recall the basics of blending operations in refining. This process is used to obtain finished (or semi-finished) products from transformed petroleum cuts and upstream units flows. The main operational problem is that, for cost and reliability reasons (in particular, sensors drift over time), the components properties are usually not measured on line while they are vastly uncertain. Among the sources of uncertainty are the drifts in the operation of upstream units, and slicing phenomena in storage tanks. On the other hand, the output of the system, i.e. the (m) properties of the blend, are analyzed on line. Mixing the various (n) components with the right proportions provides the final *blend*, with properties required by the m specifications of interest. Usually, m is larger than n.

The blend properties can be controlled with n blender motorized inlet valves. Given a blender outlet total volume flow rate, the valves openings define a control vector consisting of n volume ratios $u = (u_1, \ldots, u_n)^T$, referred to as *the recipe* [1].

1.2 Control problem

The primary goal of any blending system is to produce a mixture having some specified properties. In other words, the blending system has to find a recipe u such that the properties of the mixture satisfy some objective.

The instantaneous blend properties are considered as the output of the system. They are denoted by the $m-$dimensional vector y. The components properties are grouped in a $m \times n$ matrix B.

Following a common practice (see Chèbre et al. [2010]), the blending models are assumed to be linear. This assumption is not restrictive, because, up to some change of variables, numerous properties actually satisfy this linearity assumption. Therefore, preliminary vector coordinate-wise non linear mappings can be used to validate this assumption. In particular, at steady-state, the following relation holds

$$y = Bu \qquad (1)$$

Several constraints on the recipe u need to be considered. For mathematical consistency, the recipe vector u coordinates must all lie in $[0, 1]$ and satisfy $\sum_{i=1}^{n} u_i = 1$. From an operational and economical point of view, u should remain close to a recipe of interest u^{opt}. Further, hydraulic constraints (physical limitations of the pumps and pipes) and components availability impose upper and lower bounds on the coordinates u_i, $i = 1, ..., n$.

Various constraints on the blend properties y need to be considered as well. A reference y^{r} and/or upper and lower bounds are associated to each coordinate of y. From a practical point of view, these bounds can be considered as *hard bounds* (related to commercial specifications) or *soft bounds*, which can be violated at the expense of profit losses (also referred to as "give-away").

The matrix of the components properties B is poorly known. Yet, $\hat{B}$ an initial estimate for it (most frequently given by laboratory samples) and some on-line blend properties measurements are available.

The control problem to solve is as follows: given $\hat{B}$ an initial estimate of B, given real time measurements of

[1] low-level flow controllers (named *ratio control system*) guarantee that this vector tracks any reference signal

the output blend properties y, find a closed-loop control scheme, acting on u, such that y converges to y^{r} and remains between pre-specified bounds. At all times, u must satisfy the operational constraints, and preferably be close to a recipe of interest u^{opt}.

1.3 Proposed solution

To solve the previously presented control problem, a twofold approach was proposed in Chèbre et al. [2010]. The constraints and the various control objectives are formulated in an optimization problem. Simultaneously, an observer reconstructs an estimate of the components properties. These two parts of the control law closely interact. Under some mild simplifying assumptions, theoretical convergence of this strategy is guaranteed as discussed below.

Optimization problem First consider that $\hat{B}$, an estimate of the components properties matrix B, is given. Not every blend property needs to match a specified reference. Some of them must simply remain within some prescibed bounds. Values of blend properties associated to specified references y^{r} ($\dim y^{\mathrm{r}} = r \leq m$) can be estimated, using (1), through an $(r \times n)$ sub-matrix $\hat{B}^{\mathrm{r}}$ of $\hat{B}$. Similarly, the blend properties associated to hard and soft bounds can be computed using the sub-matrices $\hat{B}^{\mathrm{h}}$ and $\hat{B}^{\mathrm{s}}$. Lower and upper bounds vector on the hard and soft constraints are noted $y^{\mathrm{h,lb}}$, $y^{\mathrm{h,ub}}$, $y^{\mathrm{s,lb}}$, and $y^{\mathrm{s,ub}}$, respectively. Vector lower and upper bounds on the control vector are noted u^{lb} and u^{ub}. Taking into account the consistency equation $\sum_{i=1}^{n} u_i = 1$, the recipe of interest u^{opt}, and, most importantly, the blending objectives, one can formulate the control problem under the form of the following optimization problem

$$\min_u \|u - u^{\mathrm{opt}}\|_Q^2$$
$$\begin{cases} 0 \leq u^{\mathrm{lb}} \leq u \leq u^{\mathrm{ub}} \leq 1 \\ \displaystyle\sum_{i=1}^{n} u_i = 1 \\ \hat{B}^{\mathrm{r}} u = y^{\mathrm{r}} \\ y^{\mathrm{h,lb}} \leq \hat{B}^{\mathrm{h}} u \leq y^{\mathrm{h,ub}} \\ y^{\mathrm{s,lb}} \leq \hat{B}^{\mathrm{s}} u \leq y^{\mathrm{s,ub}} \end{cases} \tag{2}$$

where a symmetric definite matrix Q is used to weight the Euclidian norm, i.e. $\|u\|_Q^2 = u^T Q u$. This matrix can be chosen to promote or to penalize the use of some components. The optimization problem (2) is a quadratic programming problem. It can be very effectively handled with various software packages such as IMSL. [2006]. Its solution gives an open-loop control u.

Feedback On-line blend properties measurements y are used to update the open-loop control law by means of updates of $\hat{B}$ the estimate of the matrix B, which generates a feedback into the optimization problem (2). The measurements, which are assumed to be done continuously, are related to the current values of the control variable by the model

$$y = Bu$$

Then, the estimation $\hat{B}$, which is assumed to be constant, is updated as follows. Considering its j^{th} row $\hat{B}_j$, the continuous-time update law is

$$\frac{d\hat{B}_j^{\mathrm{T}}}{dt} = -\beta_j H u \left(\hat{B}_j u - y_j \right), \tag{3}$$

where H is the following diagonal scaling matrix ($\bar{u}$ being a reference recipe, e.g. a constant value close to u^{opt})

$$H = \frac{1}{\|\bar{u}\|} \begin{pmatrix} \frac{1}{\bar{u}_1} & & 0 \\ & \ddots & \\ 0 & & \frac{1}{\bar{u}_n} \end{pmatrix}$$

and β_j is a strictly positive parameter. This update law is analogous to those found in adaptive control (see e.g. Ioannou and Sun [1996], Åström and Wittenmark [1995]). Considering the output relation (1), the essential idea is the comparison of the observed system response Bu with the model output $\hat{B}u$.

Finally, the feedback control law is as follows: sequentially, the optimization problem (2) is solved and the estimate of the components properties $\hat{B}$ are updated, when new measurements are available, according to (3).

The proposed solution combines an on-line parameter estimator (3) and a control law which is defined as the solution of the optimization problem (2). From this description, it can be viewed as an (indirect) adaptive controller (see Ioannou and Sun [1996]), . As will now appear, $\hat{B}$ is continuously adjusted so that $\hat{B}u(t)$ approaches $Bu(t)$ as $t \to +\infty$. Yet, no particular effort is made to design the input $u(t)$ so that $\hat{B}$ converges toward B, as would normally be desired in an on-line parameter estimation technique. This is not one of the objectives, as it could cause large variations of the input signal $u(t)$ (e.g. to satisfy some persistency of excitation property, see Ioannou and Sun [1996], Åström and Wittenmark [2008], Khalil [1992]).

Consider, for any property $j = 1, \ldots, m$, the scalar function (Lyapunov function candidate)

$$\Psi(\hat{B}_j) = \frac{1}{2}(\hat{B}_j - B_j) H^{-1} (\hat{B}_j - B_j)^{\mathrm{T}}$$

This function is strictly positive away from B_j, where it equals 0. Its time-derivative along the trajectories of (3) is

$$\frac{d\Psi}{dt}(\hat{B}_j) = -\beta_j \left(\hat{B}_j u - y_j \right)^2 \leq 0$$

Therefore, $\Psi(\hat{B}_j)$ is a Lyapunov function for system (3) (see Khalil [1992]). From LaSalle's invariance principle, for any initial condition, the solution of system (3), $\hat{B}_j(t)$, converges when $t \to +\infty$ towards the largest invariant set of (3), included in the subset $\{\hat{B}_j \text{ s. t. } d\Psi/dt(\hat{B}_j) = 0\}$. Therefore, $\hat{B}_j(t)$ converges in a way such that $B_j u = \hat{B}_j u$. Yet, by definition of the optimisation problem (2), which is assumed to possess a solution (which is necessarily unique), $\hat{B}_j u$ satisfies the blend objectives. Therefore, so does $y_j(t) = B_j u(t)$, in the limit as $t \to +\infty$. The same reasoning applies to all the blend properties. In summary, the blend is successful, even though $\hat{B}_j$ does not converge to B_j. In details, one has

$$\lim_{t\to+\infty} \hat{B}^{\mathrm{r}}(t)u(t) = y^{\mathrm{r}},$$
$$y^{\mathrm{h,lb}} \leq \lim_{t\to+\infty} \hat{B}^{\mathrm{h}}(t)u(t) \leq y^{\mathrm{h,ub}},$$
$$y^{\mathrm{s,lb}} \leq \lim_{t\to+\infty} \hat{B}^{\mathrm{s}}(t)u(t) \leq y^{\mathrm{s,ub}}$$

while the equality $\lim_{t\to+\infty} \hat{B} = B$ *might not hold.*

1.4 Desired additional features

As emphasized above, the control algorithm presented in Chèbre et al. [2010] and briefly recalled, guarantees that the blend is successful, even though $\hat{B}_j$ does not converge to B_j. It may converge to a point that is not B_j. A point that is of importance for process engineers who monitor the blend operations and the operators themselves, is that the calculated estimates should be sensible. In other words, it would be a plus to guarantee that the intermediate variables in the adaptive control law $\hat{B}_j$ satisfy some consistencies with real world. Yet, no persistency of excitation assumption can be formulated, in other words, it is not acceptable to consider input signals having time variations rich enough so that all the components of B_j, for all j, can be identified [2]. Unfortunately, this is precisely the kind of conditions which would guarantee that $\hat{B}_j$ would converge to B_j, and therefore, asymptotically reach sensible values. A less demanding goal is that the estimates should satisfy the following two requirements:

(1) all (or some of) the coordinates of $\hat{B}_j = (\hat{B}_j^1, ..., \hat{B}_j^m)$ should remain sorted in a certain order, and stay within some bounds [3]. Without loss of generality, up to a re-ordering, it is desired that
$$B_j^{\min} \leq \hat{B}_j^1 \leq ... \leq \hat{B}_j^m \leq B_j^{\max} \tag{4}$$
$$B_j^{\min} \leq \hat{B}_j^{m+1} \leq B_j^{\max}, ..., B_j^{\min} \quad \leq \hat{B}_j^n \leq B_j^{\max} \tag{5}$$
where m is a given integer.

(2) because some of the components are routed together in the networks of pipes, some properties of the components are necessarily equal, and their estimates should account for this fact and be equal for all times.

The first requirements is suggested by the a-priori knowledge on the nature of the components, and bounds on their properties. The ranking may bear on some of the components only, as some of the ranking relations might be uncertain. The second requirement can bear on numerous components depending on the flow network under consideration. For sake of clarity, we start by presenting solutions for the first requirements only. The second ones are treated next. Numerous combinations are possible, depending on variations of the optimisation problem under consideration (2) which can be relaxed. The interested reader can refer to Chèbre et al. [2010], Bernier et al. [2006].

2. CHANGES IN THE UPDATE LAW

We now expose a first solution to guarantee ranking and bounds. The idea is to introduce a new term in the adaptation law (3) to penalize estimates that would be sorted in the wrong order, or leave the prescribed bounds. For this, a simple comparison function is considered

$$h(x) = \begin{cases} 0, \text{ if } x \leq 0 \\ x, \text{ if } x > 0 \end{cases}$$

and used to define the following vector

$$f(B_j) = \begin{pmatrix} h(B^{\min} - B_j^1) - h(B_j^1 - B_j^2) \\ h(B_j^1 - B_j^2) - h(B_j^2 - B_j^3) \\ \vdots \\ h(B_j^{m-2} - B_j^{m-1}) - h(B_j^{m-1} - B_j^m) \\ h(B_j^{m-1} - B_j^m) - h(B_j^m - B^{\max}) \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

In fact, this function composed of comparisons functions representing the bounds (4) plays a key role in the convergence of $\hat{B}_j$ toward a feasible region when used to change the update law as follows

$$\frac{d\hat{B}_j^{\mathrm{T}}}{dt} = -\beta_j H u \left(\hat{B}_j u - y_j\right) + \lambda H f(\hat{B}_j) \tag{6}$$

This fact is readily seen using a Lyapunov analysis using the same Lyapunov function candidate as before. Here, one has

$$\begin{aligned} \frac{d\Psi}{dt}(\hat{B}_j) &= -\beta_j \left(\hat{B}_j u - y_j\right)^2 + \lambda f(\hat{B}_j)(\hat{B}_j - B_j) \\ &= -\beta_j \left(\hat{B}_j u - y_j\right)^2 \\ &+ \lambda(h(B^{\min} - B_j^1) - h(B_j^1 - B_j^2))(\hat{B}_j^1 - B_j^1) \\ &+ \lambda(h(\hat{B}_j^1 - B_j^2) - h(B_j^2 - B_j^3))(\hat{B}_j^2 - B_j^2) + ... \\ &+ \lambda(h(B_j^{m-1} - B_j^m) - h(B_j^m - B^{\max})(\hat{B}_j^m - B_j^{\max}) \\ &= -\beta_j \left(\hat{B}_j u - y_j\right)^2 \\ &+ \lambda \sum h(\hat{B}_j^i - \hat{B}_j^{i+1}) \left(B_j^1 - B_j^{i+1} + \hat{B}_j^{i+1} - \hat{B}_j^i\right) \end{aligned}$$

Yet, from its definition, $h(\hat{B}_j^i - \hat{B}_j^{i+1}) \geq 0$, $B_j^1 - B_j^{i+1} \geq 0$, and $h(\hat{B}_j^i - \hat{B}_j^{i+1})(\hat{B}_j^{i+1} - \hat{B}_j^i) \leq 0$. It can easily be deduced from this that $\frac{d\Psi}{dt}(\hat{B}_j) \leq 0$. Applying LaSalle's invariance principle, we conclude that the $\hat{B}_j$ converges to the set such that $\hat{B}_j u = B_j u$ and (4) holds. As before, the blend is therefore successful and, further, the desired bounds are satisfied, *asymptotically*, as $t \to +\infty$. Formally, this yields the following result.

Proposition 1. Consider the update law (6), for all j. Used in the closed-loop control algorithm consisting of repetitively solving the optimization problem (2), this algorithm guarantees convergence to a successful blend as $\lim_{t\to\infty} \hat{B}_j(t)u(t) = B_j(t)u(t)$ and asymptotically, the estimates $\hat{B}_j$ satisfy the additional constraints (4).

3. RESET OF THE ESTIMATE MATRIX

3.1 Handling ranking and bounds

The preceding method provides satisfaction of the desired inequalities on the estimates $\hat{B}_j$ for all j, but only *asymp-*

[2] the estimate will eventually stop being updated in the case where no new information is available
[3] these bounds are the same for a given property j

totically. As will now be shown, it is possible to make punctual changes so that the estimate satisfies *after this update* all the desired requirements. Mathematically, the update is performed by computing a vector δ_j such that $\hat{B}_j + \delta_j$ satisfies the constraints and such that $(\hat{B}_j + \delta_j)u = \hat{B}_j u$. Formally, the following proposition holds, which shows that such δ_j can be found.

Proposition 2. Consider a (recipe) vector $\sum u_i = 1$, $u_i \geq 0$, for all i, such that $B_j^{\min} \leq \hat{B}_j u \leq B_j^{\max}$. Then, there exists $\delta = (\delta^1, ..., \delta^n) \in \mathbb{R}^n$ such that

$$B_j^{\min} \leq \hat{B}_j^1 + \delta^1 \leq ... \leq \hat{B}_j^m + \delta^m \leq B_j^{\max} \tag{7}$$

$$B_j^{\min} \leq \hat{B}_j^{m+1} + \delta^{m+1} \leq B_j^{\max}, ..., B_j^{\min} \leq \hat{B}_j^n + \delta^n \leq B_j^{\max} \tag{8}$$

$$\text{and } \delta u = 0 \tag{9}$$

In other words, the vector $\hat{B}_j + \delta$ satisfies both the ranking and the bound constraints. The set of all such δ is a convex subset of $\mathbb{R}^m$.

The preceding proposition is an existence result. In particular, the choice of δ can be made unique by invoking the minimization of a convex function such as $\|\delta\|^2$. The proof of this result is given below.

Proof. The constraints bearing on the vector δ can be represented under the affine form

$$A\delta \leq b$$

where A is a $(2n - m + 3) \times n$ matrix composed as follows. Its first two lines are u and $-u$ respectively, then its lines are composed by a band diagonal matrix using $[-1, 1]^T$ column vectors to represent the inequalities (7), and columns of zeros on the right of this matrix, then the last lines of A contains the opposite of the identity matrix (of size $n - m$) with columns of zeros on the left, and the identity matrix of the preceding size with columns of zeros on its left. The identity matrices are used to represent (8). The vector b is defined following the same procedure. The set $A\delta \leq b$ is a convex set (polytope). To prove that it is non-empty, we use Farkas' lemma in a form which is now given (see e.g. Nering and Tucker [1993]).

Lemma 3. (Farkas's lemma). For any matrix A and any vector b, one and only one of the following two properties hold: i) there exists a vector δ such that $A\delta \leq b$, ii) there exist a vector having only positive coordinates $\nu \geq 0$ such that $\nu A = 0$ and $\nu b < 0$.

We now use this lemma, by contradiction, to show that no vector ν can be found that satisfies the requirements of ii), which gives the conclusion that i) holds.

Consider the matrix A and b defined above, and assume that there exists a vector $\nu = (\nu^1, ..., \nu^{2n-m+3})$ having only positive coordinates $\nu \geq 0$ such that $\nu A = 0$ and $\nu b < 0$. By summing up all the n equations obtained from $\nu A = 0$, we have

$$\nu^1 - \nu^2 = \nu^3 - \nu^{m+3} + \sum_{i=m+1}^{n} (\nu^{i+3} - \nu^{i+3+n-m})$$

Yet, from the relation $\nu b < 0$, which writes

$$0 > \nu b = \nu^3(\hat{B}_j^1 - B_j^{\min}) + \sum_{i=1}^{m-1} \nu^{i+3}(\hat{B}_j^{i+1} - \hat{B}_j^i)$$
$$+ \nu^{m+3}(B_j^{\max} - \hat{B}_j^m) + \sum_{i=1}^{n-m} \nu^{i+m+3}(\hat{B}_j^{i+m} - B_j^{\min})$$
$$+ \sum_{i=1}^{n-m} \nu^{i+n+3}(B_j^{\max} - \hat{B}_j^{i+m})$$

some calculations yield that the the following holds

$$- B_j^{\min}(\nu^3 + \sum_{i=m+1}^{n} \nu^{i+3}) + (\nu^1 - \nu^2)(\hat{B}_j u)$$
$$+ B_j^{\max}(\nu^{m+3} + \sum_{i=m+1}^{n} \nu^{i+3+n-m}) < 0$$

This gives the following inequality

$$(\hat{B}_j u - B_j^{\min}) \left(\nu^3 + \sum_{i=m+1}^{n} \nu^{i+3}\right)$$
$$< (\hat{B}_j u - B_j^{\max}) \left(\nu^{m+3} + \sum_{i=m+1}^{n} \nu^{i+3+n-m}\right) \tag{10}$$

This inequality can not be true, because, by assumption $\hat{B}_j u - B_j^{\min} > 0$, $\hat{B}_j u - B_j^{\max} < 0$ while all the coordinates of ν are positive. Therefore, by Farkas' lemma, the set $A\delta \leq b$ is non-empty, which concludes the proof. □

Proposition 2 guarantees existence of a vector δ that can serve to update the estimates $\hat{B}_j$ periodically. In between the updates, the usual adaptation law (3) can be used as before.

3.2 Handling equalities and bounds

We now expose a solution to update the estimates $\hat{B}_j$ for all j such that they satisfy equality constraints and lie within some prescribed bounds. Again, the update is performed by computing a vector δ_j (noted δ below for simplicity) such that constraints, listed below in (13)-(14)-(15) are satisfied by $\hat{B}_j + \delta_j$. Additionally, it is required that $\delta_j u = 0$ to avoid disturbing the output of the system (as discussed in § 3.1). Note m the number of (multiple) equalities and, for all $i = 1, ..., m$ note n_i the number of components having the same value (i.e. the multiplicity). Note also $N_i = \sum_{k=1}^{i-1} n_k$, and $N = N_{m+1} = n_1 + ... + n_m$, $N_1 = 0$. Without loss of generality, up to a re-ordering, it is required to find a vector $\delta = (\delta^1, ..., \delta^n)$ such that

$$\begin{cases} \hat{B}_j^1 + \delta^1 = \hat{B}_j^2 + \delta^2 = ... = \hat{B}_j^{n_1} + \delta^{n_1} \\ \hat{B}_j^{n_1+1} + \delta^{n_1+1} = ... = \hat{B}_j^{N_3} + \delta^{N_3} \\ \vdots \\ \hat{B}_j^{N_m+1} + \delta^{N_m+1} = ... = \hat{B}_j^N + \delta^N \end{cases} \tag{13}$$

$$\begin{cases} B_j^{\min} \leq \hat{B}_j^{n_1} + \delta^{n_1} \leq B_j^{\max} \\ B_j^{\min} \leq \hat{B}_j^{N_3} + \delta^{N_3} \leq B_j^{\max} \\ \quad\vdots \\ B_j^{\min} \leq \hat{B}_j^N + \delta^N \leq B_j^{\max} \end{cases} \tag{14}$$

$$\nu_1 - \nu_2 = \sum_{i=1}^{m} (\nu_{N_i+2i+n_i+1} - \nu_{N_i+2i+n_i+2}) + \sum_{k=1}^{n-N} (\nu_{N+2m+k+2} - \nu_{N+2m+k+2+n-N}) \quad (11)$$

$$-B_j^{\min} \left(\sum_{i=1}^{m} \nu_{N_i+2i+n_i+1} + \sum_{k=1}^{n-N} \nu_{N+2m+k+2} \right) + (\nu_1 - \nu_2)\hat{B}_j u + B_j^{\max} \left(\sum_{i=1}^{m} \nu_{N_i+2i+n_i+2} + \sum_{k=1}^{n-N} \nu_{N+2m+k+2+n-N} \right) < 0 \quad (12)$$

$$\begin{cases} B_j^{\min} \leq \hat{B}_j^{N+k} + \delta^{N+k} \leq B_j^{\max} \\ \text{for all } k = 1, ..., n-N \end{cases} \quad (15)$$

Formally, the following proposition holds

Proposition 4. Consider a (recipe) vector $\sum u_i = 1$, $u_i \geq 0$, for all i, such that $B_j^{\min} \leq \hat{B}_j u \leq B_j^{\max}$. Then, there exists $\delta = (\delta^1, ..., \delta^n) \in \mathbb{R}^n$ such that $\delta u = 0$ and such that (13)-(14)-(15) hold.

Proof. Again, as in § 3.1, the set of constraints bearing on the vector δ can be written under the affine form $A\delta \leq b$, which defines a polytope. Proposition 4 states the non-emptiness of this convex set.

Each of the m lines of equalities in (13), say the ith without loss of generality, can be decomposed into a series of $n_i - 1$ equalities on the differences $\delta^{N_i+1} - \delta^{N_i+2}$, ...$\delta^{N_i+n_i-1} - \delta^{N_{i+1}}$. For convenience, a redundant equation bearing on $\delta^{N_{i+1}} - \delta^{N_i+1}$ is considered as well.

Consider A_i the $(n_i + 2) \times n_i$ matrix composed of a $(n_i - 1) \times n_i$ band-diagonal matrix using $[1, -1]$ line vectors to represent the (first) $n_i - 1$ equalities followed by a line [-1, 0, ..., 0, 1]. It is complemented by $[0, ..., 0, -1]$ and $[0, ..., 0, 1]$ to account for (14).

Consider A the $(2+2(n+m)-N) \times n$ matrix, represented below in (16), which is composed of two first lines being u and $-u$ respectively, followed by a block-diagonal matrix built using the collection of A_1, ..., A_m, and eventually followed by A_b which is the $(2(n-N)) \times n$ matrix used to represent the bounds (15). This last matrix A_b contains an identity matrix of size $(n-N)$ below its opposite.

On the other hand, the vector b is defined following the same procedure. To prove that the convex set $A\delta \leq b$ is non-empty, we use Farkas's lemma under the form of Lemma 3. As in § 3.1, we proceed by contradiction. Assume that there exists a vector having only positive coordinates $\nu \geq 0$ such that $\nu A = 0$. Considering that A has the form

$$A = \begin{pmatrix} u_1 & ... & ... & u_n \\ -u_1 & ... & ... & -u_n \\ A_1 & & & \\ & \ddots & & \\ & & A_m & \\ & & & A_b \end{pmatrix} \quad (16)$$

we obtain, by exploiting, through the matrix A, the ith out of m lines of equalities in (13), and its corresponding inequality in (14)

$$\begin{cases} (\nu_1 - \nu_2)u_{N_i+1} + \nu_{N_i+2i+1} - \nu_{N_i+2i+n_i} = 0 \\ (\nu_1 - \nu_2)u_{N_i+2} + \nu_{N_i+2i+2} - \nu_{N_i+2i+1} = 0 \\ \quad \vdots \\ (\nu_1 - \nu_2)u_{N_i+n_i-1} + \nu_{N_i+2i+n_i-1} - \nu_{N_i+2i-n_i-2} = 0 \\ (\nu_1 - \nu_2)u_{N_i+n_i} + \nu_{N_i+2i+n_i} - \nu_{N_i+2i+n_i-1} \\ \qquad + \nu_{N_i+2i+n_i+2} - \nu_{N_i+2i+n_i+1} = 0 \end{cases} \quad (17)$$

Besides, from the lines of (15), we simply obtain, through the matrix A,

$$(\nu_1 - \nu_2)u_{N+k} - \nu_{N+2m+2+k} + \nu_{N+2m+2+n-N+k} = 0$$
$$\text{for all } k = 1, ..., n-N \quad (18)$$

Now, by summing up all the lines of (17) for all $i = 1, ..., m$, with all the lines of (18), and noting that $\sum u_i = 1$, one simply obtains (11).

On the other hand, we now wish to formulate the second relation in the statement of Farkas' lemma, namely $\nu b < 0$. This scalar relation can be factorized under the form (12).

Consider the two relations (11) and (12). By introducing four factors $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$, these actually take the form

$$\nu_1 - \nu_2 = \mathcal{A} - \mathcal{B} + \mathcal{C} - \mathcal{D}$$

and

$$-B_j^{\min}(\mathcal{A} + \mathcal{C}) + (\nu_1 - \nu_2)\hat{B}_j u + B_j^{\max}(\mathcal{B} + \mathcal{D}) < 0$$

These yields the necessary condition

$$(\hat{B}_j u - B_j^{\min})(\mathcal{A} + \mathcal{C}) + (B_j^{\max} - \hat{B}_j u)(\mathcal{B} + \mathcal{D}) < 0$$

which, considering the assumed positiveness of the coordinates of ν and the implied positiveness of all the factors $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$, along with the assumption $B_j^{\min} \leq \hat{B}u \leq B_j^{\max}$, is impossible. This yields the conclusion by contradiction. □

4. CONCLUSIONS

In this article, two methods have been proposed to impose ranking, equalities and bounds on the parameters of the adaptive controller of blending systems. Blending systems are relatively particular processes, due to the implicit constraint on the actuator (the sum of its coordinates being constant) but the control method, and the conclusions raised in this article are relatively general. This contribution addresses one practical limitation of this multivariable method, which, is well known in the theory of adaptive control (see Ioannou and Sun [1996]). The first solution, which relies on a carefully designed change of the adaptation law, provides asymptotic satisfaction of these requirements of practical interest. In practice, this solution is very light to implement, does not require any particular tuning effort, but on the other side, its asymptotic convergence can be considered as too slow for the end-user, especially in a context of frequently changing plant parameters (the true value of the plant parameters being time-varying). Then, it appears that the second solution, which requires to solve a convex optimization problem (more precisely a quadratic program), is a very good choice. It is also relatively easy to implement since standard optimization routines (such as IMSL. [2006]) can be incorporated in the existing code. The corresponding computational burden remains reasonably small, and most importantly, the corrective terms have a direct effect on the parameters. After the update, they are sorted in the right order, all lie within the prescribed bounds, and all equalities are satisfied. In practice, this is the solution which has been included in the industrial software ANAMEL.

ACKNOWLEDGEMENTS

The authors wish to thank Vincent Bachelot, MINES ParisTech intern at TOTAL, Eléna Braceras, advanced process control engineer at the Grandpuits refinery, Guillaume Pitollat and Edith Di-crescenzo for their technical support on the industrial application, Stefan Janaqi for fruitful discussions, Philippe Ricoux from TOTAL Scientific Division who brought financial support for this project.

REFERENCES

K. J. Åström and B. Wittenmark. A survey of adaptive control applications. In *Decision and Control, 1995., Proceedings of the 34th IEEE Conference on*, volume 1, pages 649–654 vol.1, Dec 1995. doi: 10.1109/CDC.1995.478986.

K. J. Åström and B. Wittenmark. *Adaptive control*. Dover, second edition, 2008.

T. Bay, T.-S. Chen, and J. W. Schwartzenberg. System for feed blending control. Patent, Oct. 1969. Patent number 3,473,008.

M. Bernier, N. Petit, Y. Creff, and M. Chèbre. Method and device for controlling production of a mixture of components, in particular a mixture with pre-mix dead volumes. Patent, May. 2006. Dépôt 06.04868, extended may 2007 PCT/FR2007/000886, GCCP/2007/8408, Thailand 0701002593.

M. Chèbre, Y. Creff, and N. Petit. Feedback control and optimization for the production of commercial fuels by blending. *Journal of Process Control*, 20(4):441 – 451, 2010.

R. T. Feld, N. J. Pitman, T. C. Jr. Cattrall, N. Y. Huntington, H. F. Hoffmann, C. N. Smith, W. Smith, and N. J. Woodbury. Fluid blending system. Patent, May 1968. Patent number 3,385,680.

Honeywell Hi-Spec. *Blend Property Control Release 55.0 Revision 2*. Honeywell Hi-Spec Solutions, 2003.

IMSL. *IMSL C Numerical Library User's guide*. Visual Numerics, San Ramon, CA, USA, 2006.

P. A. Ioannou and J. Sun. *Robust adaptive control*. Prentice-Hall, Englewood Cliffs, 1996.

H. K. Khalil. *Nonlinear Systems*. MacMillan, 1992.

D. A. Le Febre and L. M. Lane. Process for controlling blending. Patent, Jul. 1995. Patent number 5,430,295.

E. D. Nering and A. W. Tucker. *Linear programs and related problems*. Academic Press Professional, Inc., San Diego, CA, USA, 1993. ISBN 0-12-515440-2.

J. H. Perkins. Method for predictive intrinsic properties of a mixture. Patent, Dec. 2000. Patent number 6,159,255.

A. Singh, J. F. Forbes, P. J. Vermeer, and S. S. Wood. Model-based real-time optimization of automotive gasoline blending operations. *Journal of Process Control*, 10: 43–58, 2000.

P. J. Walton and C. J. Swart. The Boss of blending. *Hydrocarbon Engineering*, July 2003.