Impact of imprecise dating of measurements for bulk material flow network monitoring

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Abstract: This paper discusses the negative impact of errors in the time datation of data as they are gathered accross a distributed newtork of sensors to be treated by a centralized monitoring algorithm. An example of flow monitoring serves as basis for the analysis. Using a simple probability model, we establish the variance of an estimator serving for loss detection and show how it varies with the time uncertainty. A mitigating solution is proposed. Further extensions are discussed.

1. INTRODUCTION

In various industrial fields, conveyor systems are used to transport bulk material from one inlet point to an outlet point, over possibly long distances, see Perry and Green [2007]. For several reasons, including hardware ageing and economic interests, these systems are subject to product losses or thefts. Besides the obvious economical losses, these are causes for other numerous issues such as environmental disasters (e.g. oil leaks) or unreliability of downstream production process (e.g. in a supply chain).

To address these risks, loss detection systems have been developed and installed. A detection method commonly employed uses the law of conservation of mass to relate measurements from sensors distributed along the transport path. The resulting "balancing methods" evaluate the deviation in each part of the path between measured inlet and outlet mass or volume flow. In the oil industry, this method, also known as *compensated mass balance* (when the variation of density is accounted for, see Geiger [2006]), is popular. Its main advantages are the relative simplicity for actual implementation, the ease of tuning, the ability to discover small leaks over long time periods, its fast reaction to major leaks, see Dudek [2005]. Considering the available sensing technologies for bulk material flow measurement (laser or ultrasonic based, e.g. Sonbul et al. [2012], Fraden [2010]), this approach can be generalized to a wide class of bulk material conveyors.

Applying the conservation of mass principle using spatially distributed sensors requires a good synchronization of data. The return of experience from several remarkable installations reported in the literature has served to formulate recommendations concerning the data acquisition technology, see Dudek [2005]: *i)* the employed remote terminal units retrieving informations from in-situ instruments should allow fast data transfer to the centralized master monitoring system, *ii)* the data should be carefully time-stamped with accurate and synchronized clocks (e.g. using GPS clock or Rugby clocks which, unfortunately, can be hardly available and subjected to jamming in many areas, or even SMS over cellular networks). Unfortunately, these recommendations are far from the current status observed in installed systems. This is not surprising, as the problem of clock synchronization over a network is a complex one, even under the assumption of perfect two-way communications across the network, see Freris et al. [2009], Freris et al. [2011]. It has been identified as a bottleneck in several control and monitoring architectures as described in **?**, Hokayem and Spong [2006], Bars et al. [2008], Zezulka et al. [2010].

As detailed in Institute [2002], the variation in reporting times from one data acquisition device (DAD) situated in one field location to the distant centralized supervisory control and data acquisition system (SCADA) can be quite large. The discussed time-stamping is usually not performed at the level of the DAD but at the centralized SCADA level. The SCADA creates the time-stamp when the data is received. This procedure yields uncertainty on the age of data that are collected at the centralized level.

In this paper we investigate the effect of timing uncertainty on flow monitoring problem. Aiming at producing an analysis of the observations formulated by field practitioners, we conduct investigations on a "toy problem". As will appear, the model we propose shows that time uncertainty can produce false alarms in loss detection algorithms, which are usually considered as particularly annoying for production engineers. This underestimated problem can be as troublesome as the usual noises corrupting measurements.

The paper is organized as follows. In Section 2, we formulate a simple transport model of a bulk material conveyor, the flow of which is monitored thanks to measurements produced by one inlet flow sensor and one outlet flow sensor. The measurements are produced almost periodically, due to the effect of a varying and unknown lag impacting each sample. Based on a probabilistic model of the lag value distribution, we establish the variance of the error introduced in the balance equation in Section 3. This balance relation serves as criterion for product-loss detection, and we relate the probability of detection and of false alarms to it. In Section 4 we determine flow pattern allowing one to minimize this variance. Simulations are presented in Section 5 stressing the role of data timing uncertainty, which, essentially increases the likelihood of false alarms.

2. MODEL AND PROBLEM STATEMENT

Consider a bulk material conveyor with one input sensor DAD_I and one output sensor DAD_O as represented in Figure 1. Each DAD gives a local measurement (q_I or q_O) of the material flow. The sensors could be volume or mass flow meters (in the case of a pipeline as in the oil industry), or ultrasonic sensor (in the case of solid material, see Sonbul et al. [2012], as in the mining or process industry, Ensminger and Bond [2010]). The conveyor is supposed to generate a flow q having constant velocity v with respect to a fixed reference frame. Under this simplifying assumption, q is solution of a simple delay equation. This hypothesis is typically satisfied by a conveyor belt used for solid material. The situation is more complex for a liquid pipeline, for which the water-hammer equation is usually considered, see Begovich et al. [2007] and Rahiman et al. [2007], or for multiphase flow, see Falcimaigne and Decarre [2008]. However, the approach advocated in this article can be extended, at the expense of including physics-based transformations.

For this "toy problem", we wish to detect the occurrence of product losses by monitoring the sensor values. Note l the length between the two measurement locations. As discussed earlier, our simple model states that, in the absence of any product loss, q_I and q_O are related by the delay equation

$$q_O(t+\frac{l}{v}) = q_I(t)$$

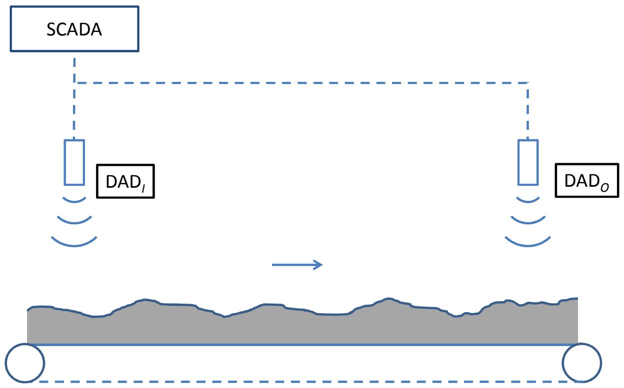

Fig. 1. Bulk material conveyor: two networked sensors communicate information to the SCADA.

We assume that a loss has a linear effect on q_O, so that a fraction of the input flow is lost. The delay equation becomes

$$q_O(t+\frac{l}{v}) = \lambda q_I(t)$$

where λ is a parameter in $]0,1]$ representing the product loss. Considering that a loss can randomly appear along the transport path, λ is the realization of a random variable Λ with values in $]0,1]$, the occurrence of a loss being equivalent to the random event $\Lambda < 1$.

As discussed earlier, a simple way to detect the occurrence of losses is to monitor the mass imbalance over a time window of width T

$$\int_0^T (q_I(t) - q_O(t+\frac{l}{v}))\, dt \qquad (2.1)$$

which equals 0 if $\lambda = 1$. The input and output measurements are sampled at a frequency $\nu_s = \frac{T}{N}$, transmitted to and processed by the SCADA. Due to-ill synchronisation of sampling dates, buffering and various other sources of network processes, the sampling time at which the measurements q_O and q_I are processed by the SCADA may differ. Taking $T = 1$ (without loss of generality) and the clock of the DAD_I as a reference, the inlet sensor provides N input measurements of the form

$$y_I[i] = q_I(\frac{i+\frac{1}{2}}{N}) + n_i,\ i = 0,\ldots,N-1$$

where n_i represents measurement noise. The synchronization discrepancies are modeled by a biased random time-shift (jitter) on the DAD_O measurements. Namely, the output measurements have the form

$$y_O[i] = \lambda q_O(\frac{i+\frac{1}{2}}{N} - \mu + w_i + C) + n_i',\ i = 0,\ldots,N-1$$

where n_i' represents measurement noise, μ represents a known constant bias (average communication lag), w_i is the realization of a zero-mean random variable W_i and C is a constant that can be chosen. Choosing

$$C = \frac{l}{v} + \mu$$

we then have

$$y_O[i] = \lambda q_I(\underbrace{\frac{i+\frac{1}{2}}{N} + w_i}_{=t_i}),\ i = 0,\ldots,N-1$$

A loss detection algorithm would typically compare a discrete version of (2.1) such as

$$b \triangleq \frac{1}{N}\sum_{i=0}^{N-1} y_I[i] - y_O[i] \qquad (2.2)$$

to a threshold value b^*, rising a loss flag when $b \geq b^*$.

In the following, we study in details the effects of the law of the time uncertainty random variables W_i on this detection algorithm.

3. IMBALANCE ESTIMATOR

3.1 Preliminary assumptions

We make the two following assumptions.

Assumption 1. q_I is continuous on $[0,1]$ and affine on every $[\frac{i}{N}, \frac{i+1}{N}]$.

Assumption 2. The W_i have support in $[-\delta, \delta]$ with $\delta < \frac{1}{2N}$. Thus, for any realization of the W_i, one has $0 < t_0 < \ldots t_{N-1} < 1$.

The parameter δ scales the time uncertainty magnitude. Note a_i the slope of $\frac{q_I}{N}$ on $[\frac{i}{N}, \frac{i+1}{N}]$. We have, for all i

$$y_O[i] = \lambda y_I[i] + N\lambda a_i w_i + n_i'$$

Without loss of generality, the total amount of bulk material entering the conveyor between times 0 and 1 is unitary, i.e.

$$1 = \int_0^1 q_I(t)dt$$

As q_I is affine on every $[\frac{i}{N}, \frac{i+1}{N}]$, we have, exactly,

$$1 = \frac{1}{N}\sum_{i=0}^{N-1} y_I[i]$$

and the imbalance estimator is

$$b = 1 - \lambda - \lambda \sum_{i\in\mathscr{I}} a_i w_i + \frac{1}{N}\sum_{i=0}^{N-1} n_i - n_i' \qquad (3.1)$$

where

$$\mathscr{I} = \{i = 0, \ldots, N-1 \mid a_i \neq 0\}$$

In the following, we assume that $\mathscr{I}$ is non empty, so that the flow is not constant on the considered time-window.

3.2 Probability law of the estimator

To emphasize the effect of time uncertainty, we first consider a noise-free case where $n_i = n'_i = 0$. Then, b appears as the realization of the random variable

$$B = 1 - \Lambda - \Lambda \sum_{i \in \mathscr{I}} a_i W_i \tag{3.2}$$

To establish the probability law of B, we assume that

- Λ and the W_i are jointly independent
- the W_i are identically distributed (IID) and have a continuous probability density function (*pdf*) f^W (with support $[-\delta, \delta]$)
- to account for the likelihood of a no-loss scenario, Λ has a mixed-law comprising a Dirac at $\lambda = 1$ and a continuous density h on an interval $[\alpha, \beta] \subset]0, 1]$, so that the *pdf* of λ is of the form
$$f^\Lambda(\lambda) = ph(\lambda) + (1-p)\delta_1(\lambda)$$
where $p = P(\Lambda < 1)$ is the probability of occurrence of a loss.

By the formula of total probability, the *pdf* f^B of B can be recovered as

$$f^B(b) = pf_L(b) + (1-p)f_{\bar{L}}(b) \tag{3.3}$$

where f_L (respectively $f_{\bar{L}}$) is the *pdf* of B conditional to the loss event $\Lambda < 1$ (respectively the loss-free event $\Lambda = 1$). For $\lambda = 1$, we have $b = -\sum_{i \in \mathscr{I}} a_i w_i$. Hence,

$$f_{\bar{L}}(b) = \circledast_{i \in \mathscr{I}} \frac{1}{|a_i|} f^W\left(\frac{\cdot}{-a_i}\right)(b) \tag{3.4}$$

where $\circledast$ designates multiple convolution products. On the other hand, for any $\lambda \in [\alpha, \beta]$, $b = 1 - \lambda - \lambda \sum_{i \in \mathscr{I}} a_i w_i$. Hence,

$$f_L(b) = \int_\alpha^\beta \frac{1}{\lambda} f_{\bar{L}}\left(\frac{b - 1 + \lambda}{\lambda}\right) h(\lambda) d\lambda \tag{3.5}$$

Example 1. We take

$$N = 10,\ p = 0.5,\ a_i = \sin\left(\frac{i}{N}\right)$$

We assume that the W_i are independent Beta variables, see Montgomery and Runger [2010], of parameter $(2, 2)$ with support in $[-\delta, \delta]$, namely

$$f^W(w) = \frac{3}{4\delta^3}(\delta^2 - w^2)\mathbb{1}_{[-\delta,\delta]}(w)$$

and that h is uniform on $[\alpha, \beta] = [0.6, 0.9]$. In Figure 2, we represent $f_{\bar{L}}$, f_L and eventually f^B for $\delta = 0.01$ and $\delta = 0.04$. The larger the time uncertainty δ, the more the two modes of the *pdf* of B overlap.

3.3 Probability of occurrence of a loss conditional to the measurement b

Consider the accuracy of measurement ε. A measured value b guarantees that $B \in I_\varepsilon \triangleq]b - \varepsilon, b + \varepsilon[$. We note

$$p_L(b) \triangleq P_{B \in I_\varepsilon}(\Lambda < 1)$$

the probability of a loss conditional to $B \in I_\varepsilon$.

As illustrated in Figure 2, the supports of $f_{\bar{L}}$ and f_L depend on the value of δ (and of α, β and the a_i). Indeed, note

$$|a|_1 = \sum_{i=0}^{N-1} |a_i|$$

According to (3.4) and (3.5), the respective supports $S_{\bar{L}}$ and S_L of $f_{\bar{L}}$ and f_L are, assuming $\delta|a|_1 \leq 1$

$$S_{\bar{L}} = [-\delta|a|_1, \delta|a|_1]$$
$$S_L = [1 - \beta(1 + \delta|a|_1), 1 - \alpha(1 - \delta|a|_1)]$$

and we have

$$p_L(b) = 1,\ \forall b \in S_L \cap S_{\bar{L}}^c$$
$$p_L(b) = 0,\ \forall b \in S_{\bar{L}} \cap S_L^c$$

Hence, if $S_L \cap S_{\bar{L}}$ is empty, the value of $p_L(b)$ is either 0 or 1 and the measure of b indicates a loss without any ambiguity. Both supports are disjoint if and only if

$$\beta(1 + \delta|a|_1) < 1 - \delta|a|_1 \tag{3.6}$$

This is illustrated in Figure 3.2. Condition (3.6) fails to be met when the time uncertainty becomes too large, namely when

$$\delta \geq \frac{1 - \beta}{|a|_1(1 + \beta)}$$

In such a case (illustrated in Figure 3.2), a measure of $b \in S_L \cap S_{\bar{L}}$ is ambiguous. The Bayes formula (Montgomery and Runger [2010]) yields

$$\begin{aligned} p_L(b) &= \frac{pP_{\Lambda<1}(B \in I_\varepsilon)}{pP_{\Lambda<1}(B \in I_\varepsilon)p + (1-p)P_{\Lambda=1}(B \in I_\varepsilon)} \\ &= \frac{p\int_{I_\varepsilon} f_L(x)dx}{p\int_{I_\varepsilon} f_L(x)dx + (1-p)\int_{I_\varepsilon} f_{\bar{L}}(x)dx} \\ &= \frac{pf_L(b)}{pf_L(b) + (1-p)f_{\bar{L}}(b)} + \mathscr{O}(\varepsilon^2) \end{aligned}$$

This probability is represented in Figure 3 for the parameter values of Example 1 and various values of δ. The probability p_L varies from 0 to 1 with a steep slope, which softens as δ grows.

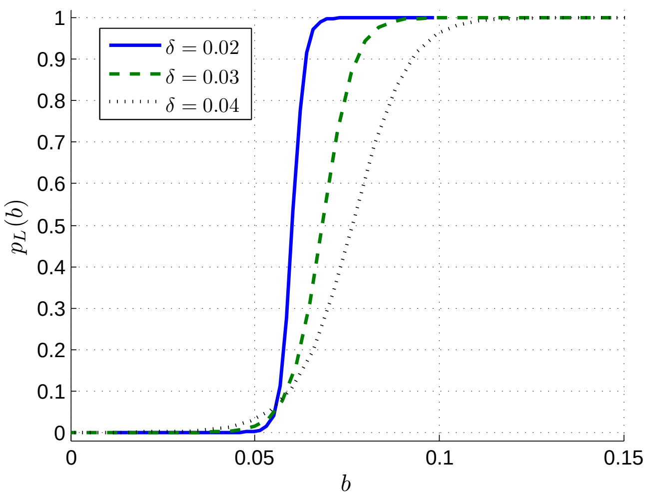

Fig. 3. Probability of a loss for values of b in the ambiguous zone

3.4 Loss detection and false alarms

For a given threshold value b^*, we are interested in

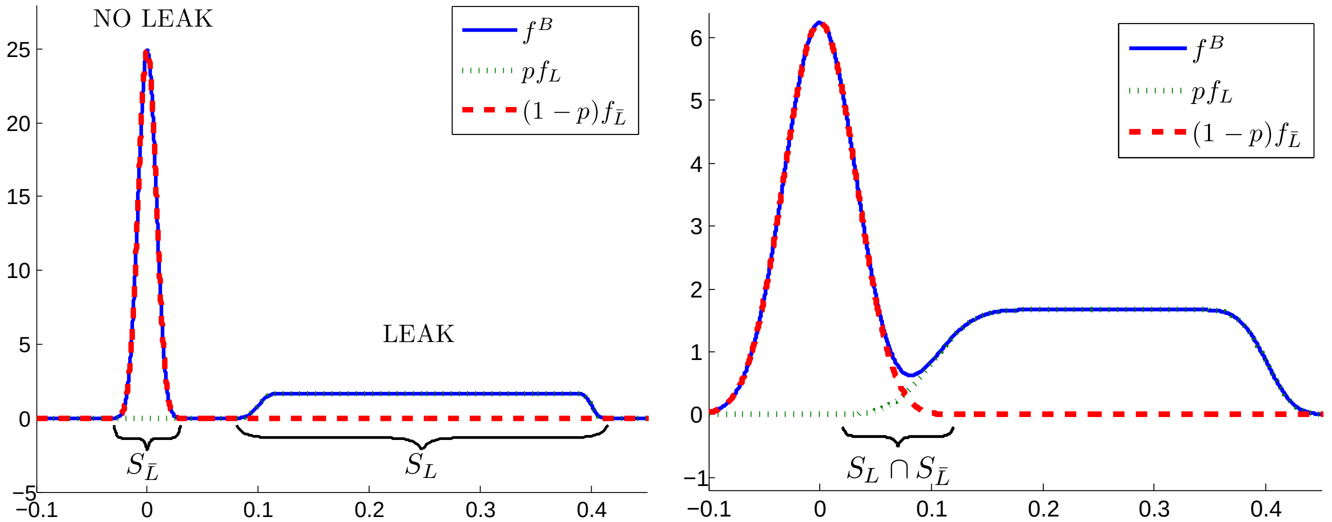

Fig. 2. *pdf* of B with parameters of Example 1 for $\delta = 0.01$ (left) and $\delta = 0.04$ (right). On the right, the overlap generated by the timing uncertainty causes some ambiguity.

- the probability p_D of detecting a loss
- the probability p_F of an alarm being false.

We have

$$p_D = P_{\Lambda<1}(b \geq b^*) = \int_{b^*}^{\infty} f_L(b)db$$

$$p_F = P_{b\geq b^*}(\Lambda = 1) = \frac{(1-p)\int_{b^*}^{\infty} f_{\bar{L}}(b)db}{\int_{b^*}^{\infty} f^B(b)db}$$

Ideally, one wants p_D as close to 1 as possible to detect most losses, and p_F as low as possible to avoid false alarms. These probabilities decrease as the threshold grows. Thus, the tuning of b^* results from a trade-off between the detection capabilities and the desired reliability of the loss-detection algorithm. This tuning is however difficult, as p_D and p_F also depend on the time uncertainty scaled by δ. We represent p_D and p_F in Table 1 for various values of these parameters. Clearly, the performance of the loss-detection algorithm deteriorates with δ.

Table 1. The performance of loss-detection algorithm deteriorates with time uncertainty δ. p_D: probability of loss detection. p_F: probability of false alarm. Case without noise.

Threshold	$\delta = 0.02$		$\delta = 0.03$		$\delta = 0.04$	
	p_D	p_F	p_D	p_F	p_D	p_F
$b^* = 0.03$	1.000	0.024	1.000	0.086	1.000	0.137
$b^* = 0.07$	0.999	0.000	0.996	0.001	0.991	0.011
$b^* = 0.11$	0.951	0.000	0.944	0.000	0.936	0.000

3.5 Impact of measurement noise

For realism, we now add noise to the model and assume that, for all i, n_i (respectively n_i') is the realization of a random variable N_i (respectively N_i') and that the N_i (respectively N_i') are IID zero-mean Gaussian variables with standard deviation σ_0 (respectively σ_1). We also assume that all the random variables of the problem are jointly independent. With these assumptions, (3.2) becomes

$$B = 1 - \Lambda - \Lambda \sum_{i \in \mathscr{I}} a_i W_i + \frac{1}{N} \sum_{i=0}^{N-1} N_i - N_i'$$

where

$$\frac{1}{N} \sum_{i=0}^{N-1} N_i - N_i'$$

is a zero-mean Gaussian variable with standard deviation

$$\sigma = \sqrt{\frac{\sigma_0^2 + \sigma_1^2}{N}}$$

Hence, the *pdf* f^B computed in Section 3.2 gets simply convolved with a Gaussian *pdf*, and the same applies to $f_{\bar{L}}$ and f_L. As a result, p_D and p_F are altered as reported in Table 2, where the parameters are the same as in Table 1, with $\sigma_0 = 0.1$ and $\sigma_1 = 0.05$. The same conclusions can be drawn.

Table 2. The performance of loss-detection algorithm deteriorates with time uncertainty δ. p_D: probability of loss detection. p_F: probability of false alarm. Case with noise.

Threshold	$\delta = 0.02$		$\delta = 0.03$		$\delta = 0.04$	
	p_D	p_F	p_D	p_F	p_D	p_F
$b^* = 0.03$	0.999	0.167	0.998	0.180	0.996	0.194
$b^* = 0.07$	0.984	0.031	0.981	0.043	0.976	0.059
$b^* = 0.11$	0.927	0.002	0.922	0.004	0.917	0.009

4. CHOICE OF A SUITED INPUT FLOW PATTERN

Clearly, the time uncertainty has no effect under steady flow conditions, as the a_i are all zero and, irrespective of the time uncertainty, the measurements will be identical (up to noise) The detrimental effect of time uncertainty on the detection algorithm performance will appear for transient flow patterns. Then, a natural question is to determine a way to alleviate these effects. As will appear, an active control strategy brings a possible solution. In this section we assume that the conveyor has an actuator at the input point so that one can chose the input pattern q_I. The problem we consider is the scheduling of the flow for a sudden overload consisting of a unitary amount of bulk material spread over a unitary time. We investigate the choice of an "optimal" input pattern q_I with respect to loss detection.

Consider a fixed value for λ. The expectancy of B conditional to $\Lambda = \lambda$ is exactly the imbalance. Indeed, one has

$$\mathbb{E}(1-\lambda-\lambda\sum_{i\in\mathscr{I}} a_i W_i + \frac{1}{N}\sum_{i=0}^{N-1} N_i - N_i') = 1-\lambda$$

The variance of B conditional to $\Lambda = \lambda$ is

$$\sigma_\lambda^2 \triangleq \operatorname{var}\left(1-\lambda-\lambda\sum_{i=0}^{N-1} a_i W_i + \frac{1}{N}\sum_{i=0}^{N-1} N_i - N_i'\right)$$
$$= \left(\lambda\sigma_W\sqrt{\sum_i |a_i|^2}\right)^2 + \frac{\sigma_0^2+\sigma_1^2}{N} \tag{4.1}$$

where σ_W is the standard deviation of the IID W_i. Decreasing σ_λ seems promising for loss-detection, as the bias estimator will all the more accurately represent the true imbalance $1-\lambda$. We have, for all i,

$$q_I(t) = q_I(0) + Na_i(t-\frac{i}{N}) + \sum_{j=0}^{i-1} a_j, \forall t \in [\frac{i}{N}, \frac{i+1}{N}]$$

Starting from and returning to a steady flow q_n, we consider equations

$$q_I(0) = q_I(1) = q_n,\ \int_0^1 (q_I(t)-q_n)dt = 1$$

which directly translate into two affine constraints bearing on the a_i

$$\sum_{i=0}^{N-1} a_i = 0, \quad N + \sum_{i=0}^{N-1} i a_i = 0 \tag{4.2}$$

Thus, we consider the following problem.

Problem 1. Find $a = (a_0, \dots, a_{N-1})$ minimizing

$$|a|_2 = \sqrt{\sum_i |a_i|^2}$$

under affine constraints (4.2).

As $|.|_2$ is convex and radially unbounded, it reaches a unique minimum under the constraints. The minimum point $a^\#$ is easily found using the Lagrangian equation (Boyd and Vandenberghe [2004]). It satisfies

$$a_i^\# = \frac{6}{N+1} - \frac{12i}{N^2-1}, \quad \forall i = 0, \dots, N-1 \tag{4.3}$$

Note $q^\#$ the corresponding flow. The associated variance is

$$\sigma_\lambda^2 = \frac{12N}{N^2-1}\lambda^2\sigma_W^2 + \frac{\sigma_0^2+\sigma_1^2}{N}$$

Interestingly, note that Problem 1 is equivalent to minimizing $\int_0^1 \dot{q}_I(t)^2 dt$ under Assumption 1. If this assumption is relaxed and smooth flow patterns are considered, one needs to solve the following straightforward calculus of variations problem (Bryson and Ho [1975]).

Problem 2. Minimizing $\int_0^1 \dot{q}_I(t)^2 dt$ under constraints

$$q_I(0) = q_I(1) = q_n, \quad q_I(1) - q_I(0) = 1$$

This problem is easily solved by means of the corresponding Euler-Lagrange equation (on $\int^t q_I$). The corresponding input pattern is

$$q_I(t) = q^*(t) \triangleq 6t(1-t) \tag{4.4}$$

5. SIMULATION RESULTS

We now study the performance of the loss detection algorithm on a theft scenario simulation for various input patterns. In

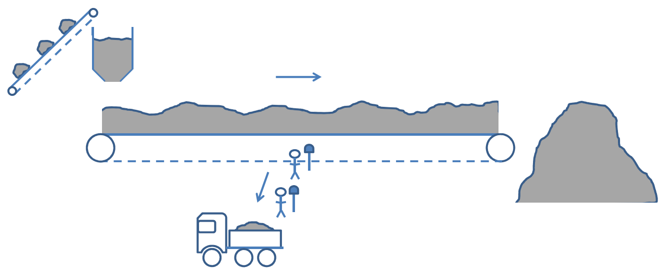

Fig. 4. Theft scenario

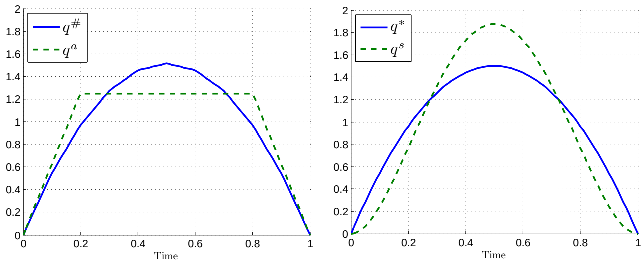

Fig. 5. Optimal and reference flow input patterns, piecewise affine (left) and smooth (right).

Figure 4, we represent a conveyor belt connecting a production site to a storage facility. The flow of bulk material is steady with nominal value q_n except for punctual overloads (of unitary time, without loss of generality) randomly appearing with probability p_1. In such a case, the shape of the overloads is controlled by a flow input pattern q_I. Somewhere along the conveyor, a group of thieves may steal bulk material. One would like to detect this robbery.

We simulate N_{simu} unitary windows. On every window, we consider two theft scenarios

- basic theft: the thieves act randomly (for example whenever they are free of surveillance) with probability p. When they do so, they reroute a fraction $1-\lambda$ of the flow (steady or overload), λ following the *pdf* f^Λ.
- smart theft: the thieves act with the same *modus operandi* but only on the overloaded windows.

We use the following parameters

$$N = 20,\ \delta = \ 0.08,\ \alpha = 0.6,\ \beta = 0.9,\ p = 0.3$$
$$\sigma_0 = \sigma_1 = 0.1,\ b^* = 0.09,\ N_{simu} = 100000,\ p_1 = 0.2$$

We compare the performance of the loss detection algorithm on those two cases for four different overload controlled input pattern represented in Figure 5. The considered flows are the optimal patterns computed in Section 4, as well as a reference piecewise affine pattern q^a and a reference smooth pattern q^s. As expected, we observe a strong similarity between both optimal patterns, $q^\# \simeq q^*$. It is easy to show that $q^\#$ converges uniformly to q^* as N goes to infinity.

The rate of loss detection and of false alarms for the four flow patterns and the two theft strategies are gathered in Table 3. The smart theft strategy is clearly more efficient from the thieves viewpoint. For all the input patterns the losses are less detected, and the rate of false alarms is much higher, which, in turn, deteriorates the reliability of the theft surveillance. Also, the optimal patterns yield better overall performance than the reference ones, especially regarding false alarms. This difference is however smaller than the impact of theft strategy. The false alarms are partly due to measurement noise and partly to time uncertainty δ. To emphasize the contribution of time uncertainty on the algorithm performance, the same simulation has been run with $\delta = 0$, see Table 4.

Table 3. Impact of the input flow pattern and the theft strategy on the algorithm performance - timing uncertainty $\delta = 0.08$

	random theft		smart theft	
	detections	false alarms	detections	false alarms
q^a (ref.)	96.8 %	3.1 %	95.3 %	11.0 %
$q^\#$ (opt.)	97.0 %	1.5 %	95.9 %	5.9 %
q^s (ref.)	96.8 %	2.2 %	95.7 %	8.4 %
q^* (opt.)	97.2 %	1.6 %	96.6 %	5.8 %

Table 4. Impact of the input flow pattern and the theft strategy on the algorithm performance - timing uncertainty $\delta = 0$

	random theft		smart theft	
	detections	false alarms	detections	false alarms
q^a (ref.)	97.3 %	0.5 %	97.3 %	2.2 %
$q^\#$ (opt.)	97.2 %	0.6 %	97.6 %	2.5 %
q^s (ref.)	97.1 %	0.5 %	97.4 %	2.4 %
q^* (opt.)	97.2 %	0.5 %	97.4 %	2.2 %

6. CONCLUSIONS AND PERSPECTIVES

The study conducted in this article has shown that the problem of accurate data timing, which surprisingly has not generated much theoretical studies before, is of importance for online monitoring. The analysis has been performed on a very simple model, allowing one to derive explicit formulas for estimator variance and probabilities of detection and false alarms. Solutions to mitigate this malicious effect have been derived. For the “naive” average imbalance estimator, the variance is scaled by the L^2 norm of the input flow. In the future, we will investigate if this conclusion holds for more advanced estimation methods such as (extended) Kalman filtering or state observers. Applying the same methodology to more complex flow dynamics such as water hammer equation for liquid pipelines calls for further investigations.

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