New formulation of predictors for finite-dimensional linear control systems with input delay

Delphine Bresch-Pietri[a], Christophe Prieur[b], Emmanuel Trélat[c]

[a] *MINES Paristech, PSL Research University, CAS – Centre Automatique et Systèmes, 60 bd St Michel 75006 Paris, FRANCE, e-mail:* delphine.bresch-pietri@mines-paristech.fr
[b] *Université Grenoble Alpes, CNRS, GIPSA-lab, F-38000 Grenoble, France.*
[c] *Sorbonne Universités, UPMC Univ Paris 06, CNRS UMR 7598, Laboratoire Jacques-Louis Lions, F-75005, Paris, France*

Abstract

This paper focuses on a prediction-based control for linear time invariant systems subject to a constant input delay, also known as the Artstein reduction approach. Standardly, this method consists in considering a predicted delay-free system, on which one can design straightforwardly a stabilizing controller. The resulting controller is then defined through an implicit integral equation, involving both the original system state and past values of the input. We propose here an alternative formulation which allows to write explicitly the Artstein transformation, and thus the corresponding controller, in terms of past values of the state only. This formal explicit formulation is the main contribution of the paper.

Keywords: Time-delay systems, Finite Spectrum Assignment, Prediction-based controllers

1. Introduction

Even if voluntary delay introduction can sometimes benefit to the control action [25], most of the time, the appearance of delay in control loops is a source of substantial performance degradation, and even of instability if the controller has been designed neglecting this delay (see [6, 7, 24] for introductions to time-delay systems). Interestingly, these undesirable effects can be circumvented using a predictor-based approach [2, 13, 14] which enables to recover closed-loop performance similar to the delay-free case. The basic idea of this technique grounds on a the use of system state prediction instead of the current state in the control loop, thus compensating for the input delay.

This method has been first introduced for linear time-invariant dynamics subject to a constant input-delay. This is also the framework considered in this paper. It is worth mentioning that numerous improvements and extension of this technique have been proposed in the last decades, such as for nonlinear plants [8,

☆This research was partially supported by the ANR projects MACS-Drill ANR-15-CE23-0008-01 and Finite4sos ANR-15-CE23-0007-01.

10, 12], for various classes of non-constant delays [3, 19], for uncertain [5] or multiple delays [4, 23] or the design of alternative predictions to counteract the effect of integral discretization in the prediction (see the works of [17] on the addition of a low-pass filter or the ones of [29] on truncated predictors or again the ones of [15] on alternative recursive differential predictions).

In this paper, we aim at presenting an alternative formulation of the standard prediction-based technique for constant input delay, the so-called Arstein approach. Standardly, this prediction-based control law is obtained by solving an implicit integral equation involving past values of the input, namely, a Volterra equation of the second kind [22]. Here, we propose to inverse this transformation and obtain an expression of both the Artstein transform and the corresponding controller in terms of the state history only. This is the main contribution of the paper. We wish to emphasize that the novelty of this paper does not relate to implementation aspects, but rather to providing a new tool to study, e.g., implementation issues or robustness properties of standard prediction-based controllers, using the original Artstein transformation.

The paper is organized as follows. In Section 2, we briefly recall the Artstein approach before stating our main results, namely, the inversion of the Artstein reduction (see Theorems 1 and 2). Then, we illustrate the interest of this result in different technical applications in Section 3. Finally, Section 4 collects the proofs of all results, whereas some concluding remarks are given in Section 5.

2. Main results

2.1. Standard prediction – Artstein approach

In this section, we briefly recall the standard Artstein approach.

Consider the following input-delay finite-dimensional linear system

$$\dot{x}(t) = Ax(t) + Bu(t - D), \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, A is a real matrix of size $n \times n$, B is a real matrix of size $n \times m$ and D is a constant input-delay. In order to stabilize the control system (1), introduce the so-called Artstein model reduction (see [2], see also [13, 14, 24]), i.e., define, for $t \in \mathbb{R}$,

$$z(t) = x(t) + \int_{t-D}^{t} e^{(t-s-D)A} Bu(s)\, ds \tag{2}$$

which gives, from an easy computation,

$$\dot{z}(t) = Az(t) + e^{-DA} Bu(t), \tag{3}$$

that is, a delay-free linear system. Therefore, assuming controllability of the pair $(A, e^{-DA}B)$, this leads to the natural control choice[1]

$$u(t) = K_D z(t) = K_D \left(x(t) + \int_{t-D}^{t} e^{(t-s-D)A} B u(s)\, ds \right), \quad t \geqslant 0 \tag{5}$$

in which the gain matrix K_D is chosen such that $A + e^{-DA}BK_D$ is Hurwitz. Then, by construction, $t \mapsto z(t)$ converges exponentially to the origin, and hence both $t \mapsto u(t)$ and $t \mapsto \int_{t-D}^{t} e^{(t-D-s)A)} B u(s)\, ds$ converge exponentially to the origin as well. Then the equality (2) implies that $t \mapsto x(t)$ converges exponentially to the origin.

Theoretically, the predictor-based control (5) stabilizes exponentially the delay control system (1), whatever the value of the delay D may be, and without any restriction on the matrices of the system. This should be put in contrast with the use of a standard proportional controller $u(t) = Kx(t)$ which achieves closed-loop stabilization if sufficient conditions bearing on the feedback gain and involving both delay and dynamics matrices are satisfied. Yet, the prediction-based controller (5) is now infinite-dimensional as it involves an integral term depending on past values of the input, the implementation of which can generate serious computational issues [27].

2.2. Inversion of the Artstein transform

As emphasized previously, the Artstein transformation and, thus, the corresponding prediction-based control law depend on past values of the control input over a time-horizon $[t-D, t]$. In order to provide an alternative theoretical tool, we propose in this section to invert the Artstein transform (2), that is, to obtain an expression of it depending only on $x(\cdot)$ (and potentially the input over a fixed time-horizon). By expressing both the stabilization feedback law and a Lyapunov functional in terms of the state, we aim at potentially improving robustness margin but also provide new tools to study, e.g., the impact of the discretization of the integral in (5) in an implementation context.

In details, by inverting the Artstein transform, we mean to solve the fixed point implicit equality (5) or, equivalently, to invert the definition of the variable

[1] It is interesting to note that the approach (2)–(5) is formally equivalent to ones considering a pole placement in terms of the original dynamics matrices A, B as done, e.g., in [11], introducing

$$\begin{aligned} u(t) =& K \left[e^{DA} x(t) + \int_{t}^{t+D} e^{(t+D-s)A} B u(s-D) ds \right] \\ =& K e^{DA} z(t) \end{aligned} \tag{4}$$

Indeed, one formally obtains that the two control laws are similar with $K_D = Ke^{DA}$. Moreover, noting that

$$A + e^{-DA}BK_D = e^{-DA}(A + BK)e^{DA},$$

it follows that the closed-loop matrices $A + e^{-DA}BK_D$ and $A + BK$ (with $K = K_D e^{-DA}$) have the same eigenvalues and thus the same stability properties.

z, which, through (2) and (5), satisfies

$$z(t) = x(t) + \int_{\max(t-D,0)}^{t} e^{(t-s-D)A} B K_D z(s) ds + \int_{t-D}^{\max(t-D,0)} e^{(t-s-D)A} B u_0(s) ds \tag{6}$$

in which u_0 denotes the control values for time $t < 0$, i.e., $u(t) = u_0(t)$ for $t \in [-D, 0)$.

With this aim in view, for every function f defined on $\mathbb{R}$ and locally integrable, we define

$$(T_D f)(t) = K_D \int_{\max(t-D,0)}^{t} e^{(t-D-s)A} B f(s)\, ds\,, \tag{7}$$

$$(T_0 f)(t) = K_D \int_{t-D}^{\max(t-D,0)} e^{(t-D-s)A} B f(s)\, ds \tag{8}$$

It follows that (6) can be rewritten as $u(t) = K_D x(t) + (T_D u)(t) + (T_0 u_0)(t)$, for every $t \geqslant 0$. An explicit manual iteration leads to the following expression of the feedback u at time t,

$$\begin{aligned} u(t) = {} & K_D x(t) + K_D \int_{\max(t-D,0)}^{t} e^{(t-D-s)A} B K_D x(s)\, ds \\ & + K_D \int_{t-D}^{\max(t-D,0)} e^{(t-D-s)A} B u_0(s)\, ds \\ & + K_D \int_{\max(t-D,0)}^{t} e^{(t-D-s)A} B K_D \int_{\max(s-D,0)}^{s} e^{(s-D-\tau)A} B K_D x(\tau)\, d\tau\, ds \\ & + K_D \int_{\max(t-D,0)}^{t} e^{(t-D-s)A} B K_D \int_{s-D}^{\max(s-D,0)} e^{(s-D-\tau)A} B u_0(\tau)\, d\tau\, ds + \cdots \end{aligned} \tag{9}$$

We summarize more formally this relation in the following theorem (proved in Section 4.1).

Theorem 1. *There holds*

$$u(t) = \begin{cases} u_0(t) & \text{if } t \in [-D, 0)\,, \\ \displaystyle\sum_{j=0}^{+\infty} (T_D^j K_D x)(t) + \sum_{j=0}^{+\infty} (T_D^j T_0 u_0)(t) & \text{if } t \geqslant 0, \end{cases} \tag{10}$$

and the series is convergent, whatever the value of the delay $D \geqslant 0$ may be.

Note that, according to this result, the control law at time t depends on past values of x over the time interval $(0, t)$ and on the initial control values over the interval $(-D, 0)$. We reformulate this fact explicitly in the following result (proved in Section 4.2).

Theorem 2. *For every $t \in \mathbb{R}_+$, there holds*

$$x(t) = z(t) - \int_0^t \Phi_D(t,s)x(s)\,ds - \int_{-D}^0 \Phi_0(t,s)u_0(s)\,ds \tag{11}$$

where $\Phi_D = 0$ if $D = 0$ and, otherwise, is defined as, for $(t,s) \in \mathbb{R}_+^2$,

$$\Phi_D(t,s) = f_{\lfloor \frac{t-s}{D} \rfloor}\left(t - s - \lfloor \frac{t-s}{D} \rfloor D\right), \tag{12}$$

in which $\lfloor \cdot \rfloor$ denotes the integer part of a real number and the sequence of functions $f_i : [0,D] \to \mathcal{M}_n(\mathbb{R})$ is defined as follows:

- *f_0 is the solution of the fixed-point equation*

$$f_0(r) = \tilde{f}(r) + (\tilde{T}_0 f_0)(r)\,, r > 0 \tag{13}$$

with, for $r > 0$,

$$\tilde{f}(r) = e^{(r-D)A}BK_D \tag{14}$$

$$(\tilde{T}_0 f_0)(r) = \int_0^r e^{(r-\tau-D)A}BK_D f_0(\tau)d\tau \tag{15}$$

- *for $i \in \mathbb{N}$, f_{i+1} is the solution of the fixed-point equation*

$$f_{i+1}(r) = (\psi f_i)(r) + (\tilde{T}_D f_{i+1})(r)\,, r > 0 \tag{16}$$

with, for $r > 0$,

$$(\psi f_i)(r) = \int_r^D e^{(r-\tau)A}BK_D f_i(\tau)d\tau$$

$$(\tilde{T}_D f_{i+1})(r) = \int_0^r e^{(r-\tau-D)A}BK_D f_{i+1}(\tau)d\tau$$

and, finally, $\Phi_0 = 0$ if $D = 0$ and, otherwise, is defined as, for $(t,s) \in \mathbb{R}_+^2$,

$$\Phi_0(t,s) = g_{\lfloor \frac{t}{D} \rfloor}(t,s) \tag{17}$$

in which the sequence of functions $g_i : \mathbb{R}_+ \times [-D,0] \to \mathcal{M}_n(\mathbb{R})$ are given as follows

- *g_0 is the solution to the fixed-point equation*

$$g_0(t,s) = \begin{cases} \tilde{f}(t-s) + (\check{T}_{0,1} g_0)(t,s)\,, & \text{if } s \in [t-D, 0] \\ (\check{\psi}_0 g_0)(t,s) + (\check{T}_{0,2} g_0)(t,s)\,, & \text{if } s \in [-D, t-D] \end{cases} \tag{18}$$

with $\tilde{f}$ defined in (14) and

$$
\begin{aligned}
(\check{T}_{0,1}g_0)(t,s) &= \int_0^t e^{(t-\tau-D)A}BK_D g_0(\tau,s)d\tau \\
(\check{\psi}_0 g_0)(t,s) &= \int_0^{s+D} e^{(t-\tau-D)A}BK_D g_0(\tau,s)d\tau \\
(\check{T}_{0,2}g_0)(t,s) &= \int_{s+D}^t e^{(t-\tau-D)A}BK_D g_0(\tau,s)d\tau
\end{aligned}
$$

- *for $i \in \mathbb{N}$, g_{i+1} is the solution to the fixed-point equation*

$$
g_{i+1}(t,s) = (\check{\psi}_i g_i)(t,s) + (\check{T}_i g_{i+1})(t,s) \tag{19}
$$

with

$$
\begin{aligned}
(\check{\psi}_i g_i)(t,s) &= \int_{t-D}^{iD} e^{(t-\tau-D)A}BK_D g_i(\tau,s)d\tau \\
(\check{T}_i g_{i+1})(t,s) &= \int_{iD}^t e^{(t-\tau-D)A}BK_D g_{i+1}(\tau,s)d\tau
\end{aligned}
$$

Consequently, the prediction-based control law (5) can be formulated as

$$
u(t) = K_D x(t) + K_D \int_0^t \Phi_D(t,s)x(s)\,ds + K_D \int_{-D}^0 \Phi_0(t,s)u_0(s)ds\,, \quad t \geqslant 0 \tag{20}
$$

One can of course notice that the form of this last expression is consistent with the one provided in (10) and which was derived in Theorem 1.

2.3. Artstein transformation inversion: alternative expression of the kernels

In view of implementation, it is worth mentioning that (13)–(16) and (18)–(19) can equivalently be written under a differential form. We thus express the following dynamic reformulation of the fixed-point calculus of the kernel functions.

Corollary 1. *Consider $D > 0$. The functions f_i and g_i, $i \in \mathbb{N}$, introduced in Theorem 2 satisfy*

$$
\dot{f}_0(r) = (A + e^{-AD}BK_D)f_0(r)\,, \qquad f_0(0) = e^{-AD}BK_D \tag{21}
$$

$$
\dot{f}_1(r) = (A + e^{-AD}BK_D)f_1(r) - BK_D f_0(r)\,, \qquad f_1(0) = f_0(D) - BK_D \tag{22}
$$

$$
\dot{f}_{i+1}(r) = (A + e^{-AD}BK_D)f_{i+1}(r) - BK_D f_i(r)\,, \quad f_{i+1}(0) = f_i(D)\,, \quad i \in \mathbb{N}^* \tag{23}
$$

and

$$\partial_t g_0(t,s) = (A + e^{-DA}BK_D)g_0(t,s) \tag{24}$$

$$g_0(0,s) = e^{-(s+D)A}B\,, \quad g_0(t,(t-D)^-) = B + g_0(t,(t-D)^+) \tag{25}$$

$$\partial_t g_i(t,s) = (A + e^{-DA}BK_D)g_i(t,s) - BK_D g_{i-1}(t-D,s) \tag{26}$$

$$g_i(iD,s) = g_{i-1}(iD,s)\,, \quad i \in \mathbb{N}^* \tag{27}$$

The proof of this Corollary follows from a straightforward differentiation of the fixed point equations listed in Theorem 2.

Remark 1. Interestingly enough, (21)–(23) imply that $rank(\Phi_D(t,s)) \leqslant 1$ for all $(t,s) \in \mathbb{R}_+^2$, at least for the scalar control case ($m = 1$). Similarly to the inversion of (6) carried out in Theorem 1 and 2, if one wishes to invert the relation

$$u(t) = K_D x(t) + K_D \int_{t-D}^{t} e^{A(t-s-D)A} Bu(s)ds$$

under the form (assuming, without loss of generality, that $u_0 = 0$)

$$K_D x(t) = u(t) - \int_0^t \Psi_D(t,s) K_D x(s) ds$$

then, it follows that

$$K_D \Phi_D(t,s) = \Psi_D(t,s) K_D$$

that is, K_D^T is an eigenvector of $\Phi_D(t,s)^T$ associated with the eigenvalue $\Psi_D(t,s)$, in the scalar case $m = 1$. This geometrical property might be of interest for implementation purposes and thus could be worth investigating in future works.

3. Illustration and numerical simulations

In this section, we aim at illustrating the merits of the Artstein inversion proposed in Theorems 1–2 (and Corollary 1) by considering some cases of potential technical applications.

3.1. Explicit effect of integral discretization for the standard prediction-based controller

If we focus on the implementation of the standard expression (5) of a prediction-based controller, a critical issue concerns the effect of discretization of the integral in (5), that is, the use in practice of the discrete form

$$u(t) = K_D \left[x(t) + \sum_{i \in \mathcal{I}_n} h_i e^{A\theta_i} Bu(t - \theta_i) \right] \tag{28}$$

where $\mathcal{I}_n$ is a finite sequence of sets of length (h_i) mapping the interval $[-D, 0]$ and the scalars θ_i depend on the selected integration rule.

As was first shown in [27] and thoroughly investigated in [16, 17], the closed-loop system consisting of (1) and the discretized control law (28) may be unstable for *arbitrarily large values of* n. This striking fact can be understood using eigenvalue considerations. Indeed, when the theoretical control law (5) is replaced by the approximated form (28), the finite spectrum property is lost and the corresponding state-delayed differential equation possesses an infinite number of characteristic roots, some of them potentially tending to the right-half complex plane. However, determining these characteristic roots is potentially complex and one may want to rely on the addition of a low-pass filter, guaranteeing closed-loop convergence for any discretization scheme of the integral, as proposed in [17].

An alternative to the addition of this low-pass filter could be to use Theorem 2, which reveals helpful in this context as it allows to obtain directly the corresponding Delay Differential Equation and characteristic equations. Namely, assuming $u_0(\cdot) = 0$ for the sake of simplicity and using (11), one can straightforwardly rewrite (1) under the form

$$\dot{x}(t) = Ax(t) + BK_D x(t-D) + BK \sum_{i \in \tilde{\mathcal{I}}_n} h_i \Phi_D(t, t-\theta_i) x(t-\theta_i - D)\, ds\,, \quad t \geqslant D \tag{29}$$

where $\tilde{\mathcal{I}}_n$ is a finite sequence of sets of length (h_i) mapping the interval $[0, t]$, the characteristic equation of which can be studied numerically. Thus, Theorem 2 provides a constructive procedure to study the closed-loop stability resulting from a given integral discretization scheme.

3.2. Mixed implicit/explicit prediction controllers and corresponding robustness properties

A well-known fact about prediction-based techniques is that they may suffer from being sensitive to delay or plant parameters mismatch [21] and numerous works investigated the robustness of predictor-based controllers to such mismatch [1, 18, 20, 26].

A way to potentially increase this robustness could be to consider a mixed form between the standard implicit prediction and the explicit one proposed in this paper, such as

$$u(t) = \sum_{j=0}^{n} (T_D^j[K_D x + T_0 u_0])(t) + (T_D^j[T_D u + T_0 u_0])(t)\,, \quad t \geqslant 0 \tag{30}$$

for a given integer $n \in \mathbb{N}$ and in which T_D and T_0 are defined in (7)–(8). In details, this equation is neither explicit in the sense that it still corresponds to an integral equation in the input u, nor entirely implicit in the sense that only a final number of past control values was replaced by distributed terms in the state x. Including these direct state feedback terms could thus help in improving the practical properties of the plant[2].

To study the robustness of this alternative controller, one would have to rely on the inversion techniques carried out in Section 2, following the formalism of Theorem 2. This is a direction of future works.

3.3. Alternative proof of closed-loop stability

Finally, we show how the exponential stability of the Arstein's transform can be obtained independently from the arguments previously used in Section 2.1, using only the definitions and properties introduced in Theorem 2.

Corollary 2. *The variable z defined through* (11) *as*

$$z(t) = x(t) + \int_0^t \Phi_D(t,s)x(s)ds + \int_{-D}^0 \Phi_0(t,s)u_0(s)ds$$

satisfies

$$\dot{z} = (A + e^{-AD}BK_D)z\,, \quad t \geqslant 0 \tag{31}$$

and thus converges exponentially to the origin.

Proof. First, consider that $t \leqslant D$. Then, due the definitions of Φ_D, one simply has

$$z(t) = x(t) + \int_0^t f_0(t-s)x(s)\,ds + \int_{-D}^0 g_0(t,s)u_0(s)ds$$

Taking a time-derivative of (17)–(18), one can write

$$\begin{aligned}\dot{z}(t) =& Ax(t) + Bu(t-D) + f_0(0)x(t) + \int_0^t \dot{f}_0(t-s)x(s)ds \\ &+ \int_{-D}^0 \partial_t g_0(t,s)u_0(s)ds - Bu_0(t-D)\end{aligned}$$

Using Corollary 1, it follows that

$$\dot{z}(t) = (A + BK_D)z(t) + Bu(t-D) - Bu_0(t-D)\,, \quad t \in [0, D]$$

which gives (31) as $u(t) = u_0(t)$ for $t \leqslant 0$. Now, consider that $t > D$. We start by observing that, due to the definition of Φ_D, one can write

$$z(t) = x(t) + \sum_{i=0}^{\lfloor \frac{t}{D} \rfloor} \int_{\max\{0, t-(i+1)D\}}^{t-iD} f_i(t-s-iD)x(s)\,ds + \int_{-D}^0 g_{\lfloor \frac{t}{D} \rfloor}(t,s)u_0(s)ds$$

[2]Note that (30) only needs the knowledge of the state history x over a fixed horizon window (depending on the expansion order n), contrary to the complete inversion (20).

Taking a time-derivative of this last expression, one gets

$$
\begin{aligned}
\dot{z}(t) =& Ax(t) + Bu(t-D) + f_0(0)x(t) + \sum_{i=0}^{\lfloor \frac{t}{D}-1 \rfloor} \left[f_{i+1}(0) - f_i(D) \right] x(t-(i+1)D) \\
&+ \sum_{i=0}^{\lfloor \frac{t}{D} \rfloor} \int_{\max\{0, t-(i+1)D\}}^{t-iD} \dot{f}_i(t-s-iD)x(s)\, ds + \int_{-D}^{0} \partial_t g_{\lfloor \frac{t}{D} \rfloor}(t,s) u_0(s) ds
\end{aligned}
$$

Hence, with Corollary 1, it follows that

$$
\begin{aligned}
\dot{z}(t) =& (A + e^{-AD}BK_D)z(t) + Bu(t-D) - BK_D x(t-D) \\
&- BK_D \sum_{i=1}^{\lfloor \frac{t}{D} \rfloor} \int_{\max\{0, t-(i+1)D\}}^{t-iD} f_{i-1}(t-s-iD)x(s)\, ds \\
&- BK_D \int_{-D}^{0} g_{\lfloor \frac{t-D}{D} \rfloor}(t-D,s) u_0(s) ds \, .
\end{aligned}
$$

As the choice of the control law yields, for $t \geqslant D$,

$$
\begin{aligned}
& Bu(t-D) = BK_D z(t-D) \\
=& BK_D x(t-D) + BK_D \sum_{i=0}^{\lfloor \frac{t-D}{D} \rfloor} \int_{\max\{0, t-(i+2)D\}}^{t-(i+1)D} f_i(t-D-s-iD)x(s)\, ds \\
&+ BK_D \int_{-D}^{0} g_{\lfloor \frac{t-D}{D} \rfloor}(t-D,s) u_0(s) \\
=& BK_D x(t-D) + BK_D \sum_{i=1}^{\lfloor \frac{t}{D} \rfloor} \int_{\max\{0, t-(i+1)D\}}^{t-iD} f_{i-1}(t-s-iD)x(s) \\
&+ BK_D \int_{-D}^{0} g_{\lfloor \frac{t-D}{D} \rfloor}(t-D,s) u_0(s)\, ds
\end{aligned}
$$

it follows that

$$
\dot{z}(t) = (A + e^{-AD}BK_D)z(t) \, , \quad t \geqslant 0
$$

Thus, using the fact that K_D is chosen such that $A + e^{-AD}BK_D$ is Hurwitz, one concludes that z converges exponentially to the origin. □

3.4. Numerical example

Finally, we illustrate numerically the convergence properties obtained with a prediction-based controller expressed with the alternative Artstein inversion and discuss some related implementation issues.

Before detailing this numerical example, we wish to emphasize that the goal of this paper is to provide an alternative formulation of the Artstein transformation, to address related technical analysis, as emphasized above. At this stage, at least, we are not able to prove any robustness property necessary for practical applications of this explicit formulation. Actually, it is worth mentioning the fact that, as the feedback law (20) requires the knowledge of the state over the time horizon $(0, t)$, it is not implementable under the direct form (20) as this formula requires to store a number of information linearly increasing with time. Consequently, in all likelihood, implementation aspects can actually be considerably worsened using directly this reformulation, and this is why we believe it is of interest to discuss them in this section, in addition to illustrating numerically that the formulation provided in this paper is correct.

We consider the following system

$$\dot{x}(t) = Ax(t) + Bu(t - D)$$

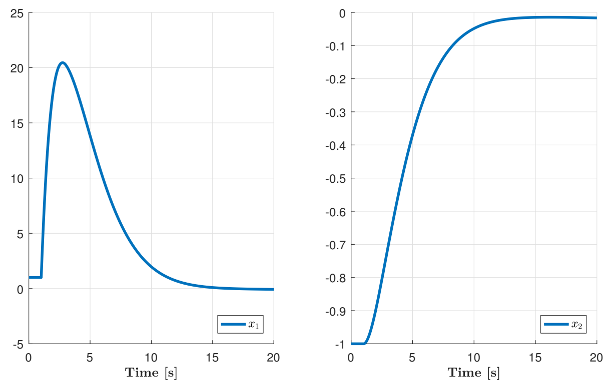
where A is the unstable matrix defined by $A = 0.01 \begin{pmatrix} 10 & 10 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $D = 1$ is a constant delay. The gain matrix $K_D = -(1.66 \quad 31.4)$ is chosen such that the delay-free closed-loop eigenvalues of $A + e^{-AD}BK_D$ are -0.5 and -0.6 respectively. The initial conditions are chosen as $x(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $u_0(\cdot) = 0$.

We chose to implement the control law (20) based on the differential expressions proposed in Corollary 1, using a discrete time step $h = 0.01$ s. Corresponding time-evolutions of the closed-loop system are given in Figure 1. Asymptotic convergence is well achieved and transient performances are identical to the ones that would have obtained using the original formulation (5), as was expected.

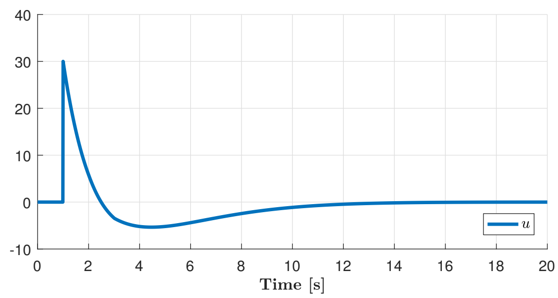
As mentioned above, the feedback law (20) requires the knowledge of the state over the time horizon $(0, t)$ and is thus not directly implementable. Furthermore, numerical approximations resulting from discretization result into a finite-time explosion, i.e., stability of the closed-loop discretized scheme only holds up to a reliability time $T^\star$ after which the system diverges (see Appendix for more details). This reliability time T^* can be taken arbitrarily large, but only at the expense of the computational burden by selecting the time step h arbitrarily small. This feature is in direct contradiction with the ISS property obtained in Theorem 4.2 in Chapter 4.4 in [9] and thus illustrates the very different implementation properties of the standard prediction scheme and the inverse one proposed in this paper.

Note that this interesting feature does not depend on the chosen implementation technique of the Artstein inversion. If one had chosen to rely on the fixed-point equations (13)–(19), an equivalent tradeoff exist between reliability time and the number of iterations used to solve the fixed-point equations. Similarly, if one implements the inversion with the infinite sum expression of Theorem 1, the tradeoff would have concerned the truncation order of the infinite sum.

Thus, to counteract this effect, in simulation, we virtually reset the initial time t_0 periodically (every 2 seconds here), with a period smaller than $T^\star$.



(a) State components x_1 (left) and x_2 (right).



(b) Control u given by (20) in Theorem 2.

Figure 1: Time-evolution of the closed-loop system consisting of the plant given in Section 3.1 and the control law (20). The gain matrix K_D is chosen such that the delay-free closed-loop eigenvalues are -0.5 and -0.6 respectively. The initial conditions are chosen as $x(0) = [1 \ -1]^T$ and $u_0(\cdot) = 0$.

Consequently, the implemented controller only requires the history of the state over a bounded time horizon (and still of the input over a fixed time horizon) and can be implemented accurately with a time step limiting the computational burden.

4. Proofs

4.1. Proof of Theorem 1

We define the functions φ_{D_j} iteratively by

$$\begin{aligned} \varphi_{D_1}(t,\tau) &= 1, \\ \varphi_{D_{j+1}}(t,\tau) &= \int_{\max(\tau,t-D)}^{\min(t,\tau+jD)} \varphi_{D_j}(s,\tau)\,ds, \quad j \in \mathbb{N}^*, \end{aligned} \tag{32}$$

for every $t \geqslant \tau$, and by $\varphi_{D_j}(t,\tau) = 0$ if $t < \tau$ and $j \in \mathbb{N}$.

Let us prove by induction that

$$\left|(T_D^j K_D x)(t)\right| \leqslant \|B\|^j \|K_D\|^{j+1} \int_{\max(t-jD,0)}^t \varphi_{D\,j}(t,\tau) e^{-(t-jD-\tau)\|A\|} \|x(\tau)\|\, d\tau, \tag{33}$$

for every $j \in \mathbb{N}^*$. This is clearly true for $j = 1$, since

$$\begin{aligned} |(T_D K_D x)(t)| &= \left| K_D \int_{\max(t-D,0)}^t e^{(t-D-s)A} B K_D x(s)\, ds \right| \\ &\leqslant \|B\| \|K_D\|^2 \int_{\max(t-D,0)}^t e^{-(t-D-s)\|A\|} \|x(s)\|\, ds. \end{aligned}$$

Assume that this is true for an integer $j \in \mathbb{N}^*$, and let us derive the estimate for $j+1$. Since

$$(T_D^{j+1} K_D x)(t) = K_D \int_{\max(t-D,0)}^t e^{(t-D-s)A} B (T_D^j K_D x)(s)\, ds,$$

we get

$$\begin{aligned} &\left|(T_D^{j+1} K_D x)(t)\right| \leqslant \|B\|^{j+1} \|K_D\|^{j+2} \times \\ &\quad \times \int_{\max(t-D,0)}^t e^{-(t-D-s)\|A\|} \int_{\max(s-jD,0)}^s \varphi_{D\,j}(s,\tau) e^{-(s-jD-\tau)\|A\|} \|x(\tau)\|\, d\tau\, ds, \end{aligned}$$

and, from the Fubini theorem, noting that (τ, s) is such that

$$\max(s-jD, 0) \leqslant \tau \leqslant s\,, \quad \max(t-D,0) \leqslant s \leqslant t,$$

if and only if

$$\max(t-(j+1)D, 0) \leqslant \tau \leqslant t\,, \quad \max(\tau, t-D) \leqslant s \leqslant \min(t, \tau + jD),$$

we get the estimate

$$\begin{aligned} &\left|(T_D^{j+1} K_D x)(t)\right| \leqslant \|B\|^{j+1} \|K_D\|^{j+2} \times \\ &\int_{\max(t-(j+1)D,0)}^t \left(\int_{\max(\tau,t-D)}^{\min(t,\tau+jD)} \varphi_{D\,j}(s,\tau)\, ds \right) e^{-(t-(j+1)D-\tau)\|A\|} \|x(\tau)\|\, d\tau, \end{aligned}$$

and the desired estimate for $j+1$ follows by definition of $\varphi_{D\,j+1}$.

Now, we claim that

$$0 \leqslant \varphi_{D\,j}(t,\tau) \leqslant \frac{(t-\tau)^{j-1}}{(j-1)!}, \tag{34}$$

for every $j \in \mathbb{N}^*$. Indeed, the nonnegativity is obvious, and the right-hand side estimate easily follows from the fact that $\varphi_{D\,j+1}(t,\tau) \leqslant \int_\tau^t \varphi_{D\,j}(s,\tau)\, ds$ and from a simple iteration argument.

Finally, from (33) and (34), we infer that

$$\left|(T_D^j K_D x)(t)\right| \leqslant \|B\|^j \|K_D\|^{j+1} \int_{\max(t-jD,0)}^t \frac{(t-\tau)^{j-1}}{(j-1)!} e^{-(t-jD-\tau)\|A\|} \|x(\tau)\|\, d\tau$$
$$\leqslant \|B\|^j \|K_D\|^{j+1} \frac{\left(te^{D\|A\|}\right)^j}{j!} \max_{0\leqslant s\leqslant t} \|x(s)\|,$$

whence the convergence of the series in (10).

Using similar arguments, one can obtain by induction that

$$\left|(T_D^j T_0 u_0)(t)\right| \leqslant \|B\|^{j+1} \|K_D\|^{j+1} D \frac{(te^{D\|A\|})^j}{j!} \max_{s\in[-D,0]} |u_0(s)|$$

and the convergence of the second series in (10) follows.

4.2. Proof of Theorem 2

When $D = 0$, as $x = z$ in view of Theorem 1, Theorem 2 straightforwardly holds.

When $D > 0$, let us search the kernels Φ_D and Φ_0 such that there holds

$$x(t) = z(t) - \int_0^t \Phi_D(t,s)x(s)\, ds - \int_{-D}^0 \Phi_0(t,s)u_0(s)\, ds\,,$$

postulating, in view of (9), that $\Phi_D(t,s) = 0$ whenever $s < 0$ or $s > t$ and that $\Phi_0(t,s) = 0$ for $s > 0$ or $s < -D$. When defining Φ_D and Φ_0 in the sequel, we do not consider sets of null Lebesgue measure, since it does not impact the integral in (11). Namely, in the following, we omit to define Φ_D and Φ_0 for $t - s = nD$, $n \in \mathbb{N}$. Using (2) and (5), we must have, for all $t \in \mathbb{R}_+$,

$$\int_0^t \Phi_D(t,s)x(s)\, ds = \int_{(t-D,t)\cap(0,+\infty)} e^{(t-s-D)A} BK_D \left(x(s) + \int_0^s \Phi_D(s,\tau)x(\tau)d\tau\right) ds \tag{35}$$

and

$$\int_{-D}^0 \Phi_0(t,s)u_0(s)\, ds = \int_{\max(t-D,0)}^t e^{(t-s-D)A} BK_D \int_{-D}^0 \Phi_0(s,\tau)u_0(\tau)\, d\tau ds$$
$$+ \int_{t-D}^{\max(t-D,0)} e^{(t-s-D)A} Bu_0(s)ds\,.$$

In a first move, we focus on (35) which, using Fubini theorem, rewrites

$$\int_0^t \Phi_D(t,s)x(s)\, ds = \int_{\max(t-D,0)}^t e^{(t-s-D)A} BK_D x(s)\, ds$$
$$+ \int_0^t \int_{\max(t-D,s)}^t e^{(t-\tau-D)A} BK_D \Phi_D(\tau,s)\, d\tau\ x(s)\, ds.$$

Since this equality should hold for every x,

$$\begin{aligned}\Phi_D(t,s) =& e^{(t-s-D)A}BK_D\chi_{(\max(t-D,0),t)}(s) \\ &+ \int_{\max(t-D,s)}^{t} e^{(t-\tau-D)A}BK_D\Phi_D(\tau,s)\,d\tau\end{aligned} \tag{36}$$

Let us now solve the implicit equation (36), following two cases depending on the value of t.

1. First of all, if $0 < t < D$ then $\max(t-D,0) = 0$ and (36) yields

$$\Phi_D(t,s) = e^{(t-s-D)A}BK_D\chi_{(0,t)}(s) + \int_s^t e^{(t-\tau-D)A}BK_D\Phi_D(\tau,s)\,d\tau\,. \tag{37}$$

   There are two subcases for the value of s.
   (a) If $s < 0$ or if $s > t$ then clearly $\Phi_D(t,s) = 0$ is a solution.
   (b) If $0 < s < t$ then

   $$\Phi_D(t,s) = e^{(t-s-D)A}BK_D + \int_0^{t-s} e^{(t-s-\tau-D)A}BK_D\Phi_D(\tau+s,s)\,d\tau$$

   and then setting $r = t-s$ (note that $0 < r < t < D$) we search $\Phi_D(t,s) = f_0(r)$ with

   $$f_0(r) = e^{(r-D)A}BK_D + \int_0^r e^{(r-\tau-D)A}BK_D f_0(\tau)\,d\tau,$$

   that is, $f_0(r) = \tilde{f}(r) + (\tilde{T}_0 f_0)(r)$ as stated in (13)–(15).
2. If $t > D$ then $\max(t-D,0) = t-D$ and (36) yields

$$\Phi_D(t,s) = e^{(t-s-D)A}BK_D\chi_{(t-D,t)}(s) + \int_{\max(s,t-D)}^{t} e^{(t-\tau-D)A}BK_D\Phi_D(\tau,s)\,d\tau$$

   and we have $\lfloor \frac{t}{D} \rfloor + 2$ subcases for the value of s.
   (a) If $s < 0$ or if $s > t$ then clearly $\Phi_D(t,s) = 0$ is a solution.
   (b) If $t-D < s < t$ then, following the exact same arguments as previously, one can show that $\Phi_D(t,s) = f_0(t-s)$ in which f_0 has been previously introduced as the solution of fixed-point equation $f_0 = \tilde{f} + \tilde{T}_0 f_0$ given in (13)–(15).
   (c) If $t-2D < s < t-D$, then

   $$\Phi_D(t,s) = \int_{t-D}^{t} e^{(t-\tau-D)A}BK_D\Phi_D(\tau,s)\,d\tau$$

and, from the previous subcase, one obtains

$$\Phi_D(t,s) = \int_{t-D}^{s+D} e^{(t-\tau-D)A} BK_D f_0(\tau - s)\, d\tau + \int_{s+D}^{t} e^{(t-\tau-D)A} BK_D \Phi_D(\tau, s)\, d\tau \tag{38}$$

Define $\Phi_D(t,s) = f_1(t-s-D)$ and $r = t-s-D \in [0, D]$. Then, (38) rewrites

$$f_1(r) = \int_r^D e^{(r-\xi)A} BK_D f_0(\xi) d\xi + \int_0^r e^{(r-\xi-D)A} BK_D f_1(\xi) d\xi \tag{39}$$

and, thus, f_1 is the solution of the following fixed-point equations

$$f_1(r) = (\psi f_0)(r) + (\tilde{T}_D f_1)(r)\,, r > 0 \tag{40}$$

with, for $r > 0$,

$$(\psi f_0)(r) = \int_r^D e^{(r-\xi)A} BK_D f_0(\xi)\, d\xi$$
$$(\tilde{T}_D f_1)(r) = \int_0^r e^{(r-\xi-D)A} BK_D f_1(\xi) d\xi$$

in which f_0 has been previously introduced.

(d) There remains $\lfloor \frac{t}{D} \rfloor - 1$ subcases for the value of s. Those can be straightforwardly investigated with the same arguments and an iteration procedure, as, in particular, the implicit equation (16) which is obtained does not depend on the index $i \in \mathbb{N}$. This concludes the definition of Φ_D.

We now have to investigate (36) which, using Fubini Theorem, rewrites

$$\int_{-D}^{0} \Phi_0(t,s) u_0(s)\, ds = \int_{-D}^{0} \int_{(t-D,t)\cap(0,+\infty)} e^{(t-\tau-D)A} BK_D \Phi_0(\tau, s) u_0(s)\, d\tau ds + \int_{t-D}^{\max(t-D,0)} e^{(t-s-D)A} Bu_0(s) ds\,.$$

As this equality should hold for every u_0, one gets

$$\Phi_0(t,s) = \int_{\max(t-D,0)}^{t} {}^{(t-\tau-D)A} BK_D \Phi_0(\tau, s)\, d\tau + e^{(t-s-D)A} B\chi_{(t-D,\max(t-D,0))}(s)\,.$$

Consider first that $t \in [0, D]$ and that $s \in [t-D, 0]$, then this last equality rewrites

$$\Phi_0(t,s) = \int_0^t e^{(t-\tau-D)A} BK_D \Phi_0(\tau, s)\, d\tau + e^{(t-s-D)A} B\,,$$

in which $s \geqslant \tau - D$ for $\tau \in [0, t]$ as $s \geqslant t - D$. This gives (18) which solution is unique and converges using the same arguments as previously.

Similar straightforward considerations on the kernel Φ_0 conclude the proof of Theorem 2.

4.3. Proof of Remark 1

In this subsection, we assume throughout that $m = 1$ and, for the sake of simplicity of the exposition, that $u_0 = 0$.

We first show that $\mathrm{rank}(f_i(r)) \leqslant 1$, $i \in \mathbb{N}$.

Indeed, B is a column and K_D is a row, then BK_D has rank one. Multiplying by an invertible matrix does not change the rank, thus $f_0(0) = e^{-DA}BK_D$ has rank one, and then $f_0(r) = e^{r(A+e^{-DA}BK_D)}f_0(0)$, according to Corollary 1, has rank one.

Now, from (21)–(22), one notices that

$$f_1(0) = \int_0^D e^{-sA}BK_D f_0(s)\,ds = \int_0^D e^{-sA}BK_D e^{s\tilde{A}}\,ds\, f_0(0) = C\, f_0(0),$$

where we have set $\tilde{A} = A + e^{-DA}BK_D$ and $C = \int_0^D e^{-sA}BK_D e^{s\tilde{A}}\,ds$. Hence[3], $\mathrm{rank}(f_1(0)) \leqslant 1$. Besides, integrating (22), we have

$$\begin{aligned} f_1(r) =& e^{r\tilde{A}}f_1(0) - \int_0^r e^{(r-s)\tilde{A}}BK_D f_0(s)\,ds \\ =& \left(e^{r\tilde{A}}C - \int_0^r e^{(r-s)\tilde{A}}BK_D e^{s\tilde{A}}\,ds\right) f_0(0) \end{aligned} \tag{41}$$

and thus $\mathrm{rank}(f_1(r)) \leqslant 1$. The desired conclusion follows by direct iteration using (23).

Now, if we consider

$$K_D x(t) = u(t) - \int_0^t \psi_D(t,s)K_D x(s)\,ds,$$

then, a reasoning similar to the one made in Section 4.2 leads to

$$\psi_D(t,s) = K_D e^{(t-s-D)A}B\chi_{(\max(t-D,0),t)}(s) + K_D\int_{\max(s,t-D,0)} e^{(t-\tau-D)A}B\psi_D(\tau,s)\,d\tau.$$

Comparing this last formula with (36), by uniqueness, one concludes that $K_D\phi_D(t,s) = \psi_D(t,s)K_D$.

5. Conclusion

The contribution of this paper was to inverse the Artstein transform and to derive an explicit expression of the corresponding stabilizing controller in terms of the history of the system state only. We illustrated how this new expression can be fruitful for theoretical analysis on linear systems with a constant delay in the input.

In a forthcoming work, we extend this inversion formula for infinite-dimensional linear systems, such as the heat equation with boundary delayed control.

[3] Recall that $\mathrm{rank}(AB) \leqslant \min(\mathrm{rank}(A), \mathrm{rank}(B))$ for two matrices A and B.

Appendix: Remarks on the numerical implementation

For the sake of comparison, let us consider the control law (2)–(5).

It is interesting to notice that a differential implementation of this control law using (3) exhibits a very different behavior from the one based directly on the integral expression (2). Indeed, defining the variable $y = x(t+D) - z(t)$, one concludes from (1) and (3) that y satisfies the Ordinary Differential Equation $\dot{y}(t) = Ay(t)$. Consequently, if the matrix A is unstable and if $y(0) \neq 0$ (due to discretization issues in implementation of $z(0)$ for example) y diverges numerically and, consequently, x diverges as well.

Conversely, if one wishes to implement directly (2),(5) under the integral form, there will still exist a robustness margin preserving numerical instabilities to occur, as established in the seminal paper [14]. Interestingly, in some sense, these differences are the twins of the ones existing between the original Smith Predictor and the Modified Smith Predictor, which is an equivalent representation of the Finite Spectrum Assignment as underlined in [28].

On the other hand, whatever the implementation method of (20) we follow (that is, using the fixed point equations (13)–(16) or their differential expression (21)–(23)) and the open-loop behavior of the plant (1), the stabilization properties of Theorem 2 only hold for a finite time T^* in practice. Indeed, consider the effect of a numerical error (either resulting from an approximation of the fixed point equations or from an Euler approximation of the differential equations) as

$$\hat{z}(t) = x(t) + \int_0^t \left[\Phi_D(t,s) + \epsilon(s)\right] x(s) ds = z(t) + \int_0^t \epsilon(s) x(s) ds \qquad (42)$$

which thus implies, with the control law $u(t) = K_D \hat{z}(t)$,

$$\dot{x}(t) = (A + e^{-AD} B K_D) x(t) + B K_D \int_0^t \epsilon(t,s) x(s) ds \qquad (43)$$

Then, except if one is able to reduce the numerical errors in a way that ϵ tend to zero in a time-increasing compact set (i.e., make the numerical errors vanish with time, which is quite an unrealistic assumption), asymptotic convergence is not feasible.

This is the reason why we proposed in the simulation section to virtually reset the inital time t_0 periodically to reset the controller as well.

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