A Simple Algorithm for the Construction of Flat Outputs

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Since the introduction of flatness for lumped parameter systems into control about 15 years ago, this approach has become quite popular in theory of lumped parameter systems. Also extensions to distributed parameter systems are known, particularly for trajectory planning. Apart from many progresses the "simple" problem, how to construct a flat output, has remained open. Although several special results are known, a general method to determine the flat output for nonlinear systems with more than one input, if it exists, is currently not available.

This contribution proposes an elimination theory based approach to construct flat outputs for nonlinear lumped parameter systems with an arbitrary number of inputs. The main concept is the gradual reduction of the system to simpler ones such that one can decide for the final system, whether it admits a flat output. If the final system admits one, then a flat output for the initial system can be constructed in a straightforward manner. Since elimination of variables leads to implicit systems in general, this contribution deals with implicit dynamic systems. The proposed algorithm starts with a system of the type

$$f(t, u, x, \dot{x}) = 0 \ ,$$

where the time derivatives $\dot{u}$ of u do not enter the equations f. The elimination of u leads to a system

$$g(t, x, \dot{x}) = 0$$

with a reduced number of equations. In the reduction step, we look for a coordinate transformation $(t, x) = \phi(\bar{t}, \bar{u}, \bar{x})$ with $t = \bar{t}$ such that the equations in the new coordinates take the form

$$\bar{g}(t, \bar{u}, \bar{x}, \dot{\bar{x}}) = g \circ \phi(\bar{t}, \bar{u}, \bar{x}) = 0 \ .$$

If one is able to construct ϕ then the repeated application of this procedure will end with a system simple enough to decide whether it admits flat outputs. But it will be shown that the transformation ϕ exists only, if certain conditions are met. From a geometric point of view, these conditions ensure that a certain distribution, restricted to the submanifold defined by $g = 0$, is projectable there. These conditions are rarely met, but they can be weakened by a dynamic extension h of the form

$$g(t, x, \dot{x}) = 0 \ , \quad h(t, x, \dot{x}) = p \ ,$$

for a choice of the functions h and new coordinates p, such that the conditions from above are met.

To complete the talk some examples will demonstrate, how the proposed approach can be used to construct flat outputs. Furthermore, the well known VTOL problem can be downloaded from
http://regpro.mechatronik.uni-linz.ac.at/about/staff/schoeberl/.