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An example of robust internal model control under variable and uncertain delay☆

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ABSTRACT

This paper proposes a particular study of the classic internal model control algorithm for a sampled-data system in a generalized context of uncertainty. Besides the usually considered model mismatch, the particularity of the case under consideration is that the measurements available to the control algorithm suffer from large, varying and uncertain delays. The presented study considers a simple SISO nonlinear system. The control algorithm is a sampled nonlinear model-based controller with successive model inversion and bias correction. The main contribution of this article is its proof of global convergence and robustness despite time-varying delays and uncertain measurement dating. In particular, the model error, the varying delays and measurements dating error are treated using monotonicity of the system and a detailed analysis of the closed-loop behaviour of the sampled dynamics, in an appropriate norm.

1. Introduction

In this article, we investigate the effects of delay variability and uncertainty on the internal model controller (IMC, see e.g. [1]) of a single-input single-output (SISO), static, nonlinear, sampled-data process with delayed measurements whose dating is uncertain. As is well-known, the uncertainty and the variability of delays lead to challenging control problems that may jeopardize closed-loop stability, see [2,3] and references therein. It is also known, see [4], that metrology delays coupled with inaccurate process models could lead to closed-loop instability. Interestingly, the general treatment of these issues is still an open problem.

The process and its controller constitute a sampled-data system (following the terminology employed in e.g. [5,23]) which can be reformulated using a classic discrete time representation. The specific case under consideration is actually also formally very similar to a scalar run-to-run controller, the robustness of which is not trivial. Run-to-run control is a popular and efficient class of techniques, originally proposed in [6], specifically tailored for processes lacking in situ measurement for the quality of the production (see [7]). Numerous examples of implementations have been reported in the semiconductor, and materials industry, in particular, see e.g. [7,8] and references therein. Indeed, the field of run-to-run control encounters two of the practical problems addressed in this article: nonlinear model uncertainty and variable metrology delays. While these issues have often been reported (see, e.g. [4,9–11]), they have not received any definitive treatment from a theoretical viewpoint.

In the problem considered here, model uncertainty stems from the interactions between the input and the system states which can be rather complex, and, in turn, cause some non-negligible uncertainty on the quantitative effects of the input. On the other hand, the measurements are available after a long time lag covering the various tasks of sample collection, receipt, preparation, analysis and transfer of data through an information technology (IT) system to the control system. Measurements are thus impacted by large delays, which can be varying to a large extent, and in some applications be state- or input-dependant. This variability of the delay builds up with the intrinsic IT dating uncertainty, because, in numerous implementations, no reliable timestamp can be associated to the measurements, see [12] and references therein. The delay variability cannot be easily represented by Gaussian models (e.g. additive noise on the measurement), nor can it be fully described as deterministic input or state dependant delay, nor known varying delays that could be exactly compensated for by predictor techniques (as done in e.g. [13–17]).

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In the absence of measurement dating uncertainty, robust stability in the presence of model mismatch can be readily established, using the monotonicity of the system and model which is formulated here as an assumption. The study of measurement dating uncertainty effects is more involved. Once expressed in the sampled time-scale, the control scheme exhibits a variable delay discrete-time dynamics. No straightforward eigenvalues or Nyquist criterion analysis (see [9]) can be used to infer stability. A complete stability analysis in a space of sufficiently large dimension, with a well chosen norm, yields a proof of robust stability under a small gain condition. Interestingly, the small-gain bound is reasonably sharp, so that it can serve as guideline for practical implementation. The novelty of the approach presented in this article lies in the proof technique. It does not treat the uncertainty of the delay using the Padé approximation approach considered in [18], but directly uses an extended dimension of the discrete time dynamics. In future works, it is believed that these arguments of proof could be extended to address more general problems, in particular to higher dimensional forms (lifted forms) usually considered to recast general iterative learning control into run-to-run as is clearly explained in [7].

The paper has two objectives. Firstly, it establishes robust stability results with respect to model mismatch when measurements are delayed but exactly dated. Secondly, it extends robust stability to small model errors when measurement are delayed and their dating is uncertain. Those results are illustrated through simulations.

2. Notations

Given $\mathcal{I}$ an interval of $\mathbb{R}$, and $f:\mathcal{I}\to\mathbb{R}$ a smooth function, let us define

$$\|f\|_\infty = \sup_{x\in\mathcal{I}}|f(x)|$$

For any vector X, note $\|X\|_1$, $\|X\|_2$ and $\|X\|_\infty$ its 1-norm, its Euclidean norm and its infinity norm, respectively. Note $\|\cdot\|_*$ any of the vector norms above. For any square matrix A, note $\|A\|_*$ the norm of A, subordinate to $\|\cdot\|_*$. Classically (e.g. [19]), for all A, B

$$\|AB\|_* \le \|A\|_*\|B\|_*$$

We note $\lfloor x\rfloor$ the floor value of x, mapping x to the largest previous integer.

For any matrix of dimension s, define E_i the matrix of general term $e_{k,l}$

$$\forall(k,l),\ \ e_{k,l} = \delta_{k,s}\delta_{l,i} \tag{1}$$

where δ is the Kronecker delta $\delta_{i,j}=1$ if $i=j$ and 0 otherwise.

3. Problem statement

3.1. Plant (delay-free)

We note y the controlled variable (output) of the considered plant and u the control variable (input). It is assumed that there exists f_p a strictly monotonous smooth function such that

$$y = f_p(u) \tag{2}$$

Although f_p is unknown, we can use a model of it, f, which is also smooth and monotonous,[1] such that $f_p(0)=f(0)$. Usually, f is a rough estimate of f_p. Typical models are represented in Fig. 1. For the simulations considered in this article, the model error can be as large as 20–40%, which is representative of industrial applications requirements.

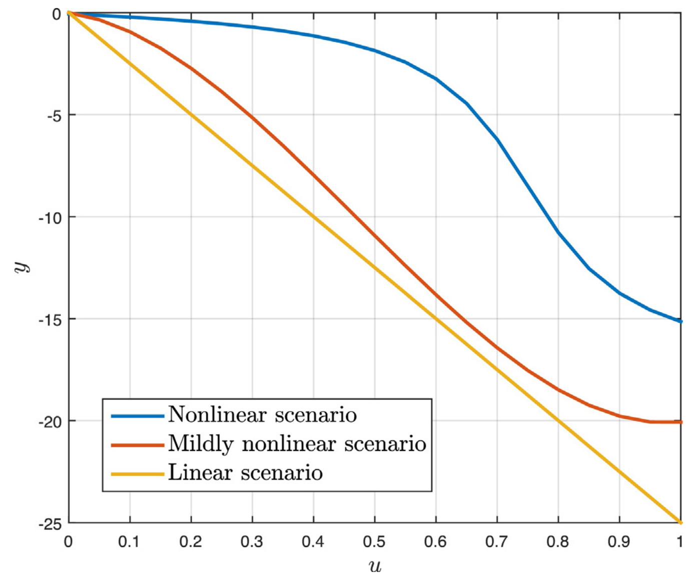


Fig. 1. Examples of possible monotonic and smooth input-output mappings f, courtesy of TOTAL.

The target value c for the controlled variable is assumed to be reachable by both the system and the model, i.e. there exists u_c and $\tilde{u}_c$ verifying

$$f_p(u_c) = c,\ \ f(\tilde{u}_c) = c \tag{3}$$

3.2. Measurement delay

A measurement system provides estimates of y with some time delay in a sampled manner. In many cases, this delay is time varying. Depending on the IT structure, measurements dating is usually done either using timestamping or an *a priori* estimation of the measurement delay. Either way, exact measurement dating is usually impractical, and some uncertainty on the measurement delay must be considered.

In the system considered in this article, the measurements available for feedback in a control loop thus have two specificities. They are delayed and the measurement delay $0\le D$ itself is varying and uncertain. With $0\le\hat{D}$ the available estimation of D, we note $\Delta \triangleq \hat{D}-D$ the mismatch.

Assumption 1. There exits D_{max} such that $D\le D_{max}$.

Assumption 2. There exits Δ_{max} such that $\Delta\le\Delta_{max}$. If Assumption 1 holds, it is clear from definition that $-D_{\max}\le\Delta$.

3.3. Control problem

A closed-loop controller can be designed for the system. Each time a measurement is received, the control is updated and the value of the control is kept constant until the next measurement is received, creating piece-wise constant control signals (with varying step-lengths). Repetitive application of this process generates a sequence of inputs and outputs. The delay results in shift of index in the measurement sequence.

Formally, the control design should aim at solving the following problem.

Control problem. *Create a sequence* (u_n) *using the approximate model* f *and the delayed measurements* $(f_p(u_{n-D_n}))_{n\in\mathbb{N}}$ *of* y_n *such that*
$\lim_{n\to+\infty} f_p(u_n) = c$

[1] In practice, it can result from the analysis of sensitivity look-up tables obtained from experiments and derivation of interpolating models.

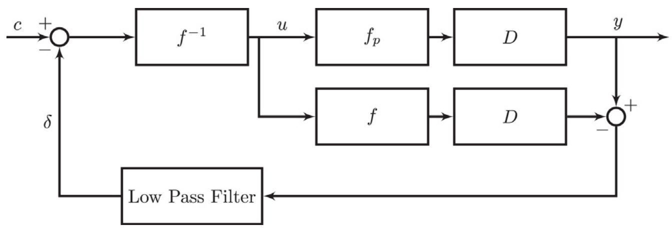


Fig. 2. Idealized closed-loop control scheme.

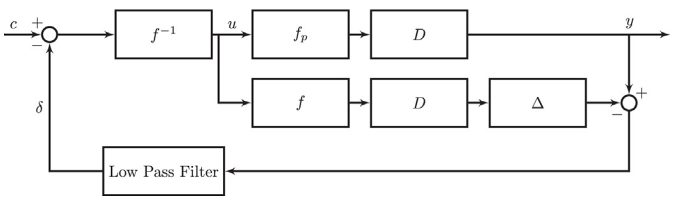


Fig. 3. Realistic closed-loop control scheme.

We propose a simple nonlinear IMC algorithm to address the problem. This algorithm adapts a bias term used in a model inversion. Assuming that one could estimate exactly the measurement delay D_n, the implementation of such an algorithm would be

$$\begin{cases} u_0 = 0, \ \delta_0 = 0, \ \alpha \in]0;1] \\ n \geqslant 0, \ u_{n+1} = f^{-1}(c - \delta_n) \\ \delta_{n+1} = \delta_n + \alpha(y_{n-D_n} - f(u_{n-D_n}) - \delta_n) \end{cases} \tag{4}$$

which can be wrapped up in the following usual block diagram of Fig. 2.

However, the uncertainties in measurements dating have an impact on the controller dynamics. Instead of (4), one is able to implement the following

$$\begin{cases} u_0 = 0, \ \delta_0 = 0 \\ n \geqslant 0, \ u_{n+1} = f^{-1}(c - \delta_n) \\ \delta_{n+1} = \delta_n + \alpha(y_{n-D_n} - f(u_{n-\hat{D}_n}) - \delta_n) \end{cases}$$

When $\Delta_n = \hat{D}_n - D_n \neq 0$, this becomes

$$\begin{cases} u_0 = 0, \ \delta_0 = 0 \\ n \geqslant 0, \ u_{n+1} = f^{-1}(c - \delta_n) \\ \delta_{n+1} = \delta_n + \alpha(y_{n-D_n} - f(u_{n-D_n-\Delta_n}) - \delta_n) \end{cases} \tag{5}$$

where Δ_n is a dating uncertainty term. The situation is pictured in Fig. 3. To show that (5) constitutes a viable solution to our control problem, it is necessary to investigate the closed-loop stability of the controller in this case.

4. Convergence with model mismatch and delay, without measurement dating uncertainty

In the analysis, three problems must be treated: model mismatch, delayed measurements and measurement dating uncertainty.

We first consider the system without the later. Used in closed loop, controller (4) gives

$$\begin{cases} u_0 = 0, \ \delta_0 = 0, \ \delta_1 = \alpha_0(f_p(0) - f(0)) \\ n \geqslant 0, \ u_{n+1} = f^{-1}(c - \delta_n) \\ \delta_{n+2} = (1-\alpha)\delta_{n+1} + \alpha(\delta_{n-D_{n+1}} - c + f_p \circ f^{-1}(c - \delta_{n-D_{n+1}})) \end{cases} \tag{6}$$

The asymptotic behaviour of (6) is determined by the extended dynamics of (δ_n) since convergence of (δ_n) clearly implies convergence of (u_n). If (u_n) and (δ_n) converge towards the limits u and δ respectively, then, necessarily,

$$u = u_c \text{ and } \delta = c - f(u_c)$$

We now define the sequence $(d_n \triangleq \delta_n - \delta, n \geq 0)$. Equivalently, the error dynamics is represented by the equation

$$d_{n+2} = (1-\alpha)d_{n+1} + \alpha(d_{n-D_{n+1}} + f_p \circ f^{-1}(f(u_c) - d_{n-D_{n+1}})) - \alpha c$$

Applying the mean value theorem to the function $x \mapsto x + f_p \circ f^{-1}(f(u_c) - x)$, one easily deduces that there exists

$$a_n \in [\min(0, d_{n-D_{n+1}}); \max(0, d_{n-D_{n+1}})]$$

such that

$$d_{n+2} = (1-\alpha)d_{n+1} + \alpha\left(1 - \frac{f_p' \circ f^{-1}(f(u_c) - a_n)}{f' \circ f^{-1}(f(u_c) - a_n)}\right) d_{n-D_{n+1}}$$

Gathering past values of d_n over the range $n - D_{\max} \ldots n+1$, the system can be written as a linear time varying system (LTV) of dimension $D_{max} + 2$

$$X_{n+1} = A_n X_n \tag{7}$$

where

$$X_n = \begin{pmatrix} d_{n-D_{\max}} & \cdots & d_{n+1} \end{pmatrix}^T$$

with

$$A_n = C + \alpha h(a_n) F_n \tag{8}$$

where

$$C = \begin{pmatrix} 0 & 1 & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & & \vdots \\ \vdots & & & \ddots & \ddots & \ddots & \vdots \\ \vdots & & & & \ddots & \ddots & 0 \\ \vdots & & & & & 0 & 1 \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & 1-\alpha \end{pmatrix}$$

using the notation (1),

$$F_n = E_{D_{\max}+1-D_{n+1}}$$

and

$$h(a_n) = 1 - \frac{f_p' \circ f^{-1}(f(u_c) - a_n)}{f' \circ f^{-1}(f(u_c) - a_n)} \tag{9}$$

Interestingly, h can be interpreted as a metric of the model error: if $f \equiv f_p$, we do indeed get $h \equiv 0$. Since (7) is a LTV system, establishing its convergence is non-trivial. Establishing the asymptotic (not to say exponential) convergence of a general LTV discrete time system is usually a difficult task. In particular, it is not sufficient to study its eigenvalues (see [20]). Some results have long been available for slowly varying systems and have recently been refined in [21], in particular. However, in our present case, it is not necessary to use them. The particular structure of the varying term allows more straightforward investigations.

Define the (infinite) set of possible transition matrices (8)

$$\mathcal{A} = \{C + \alpha h(x) E_i, \ x \in \mathbb{R}, \ i \in [\![1; D_{\max}+1]\!]\} \tag{10}$$

Let us assume that $\|h\|_\infty < 1$. This implies that the set $\mathcal{A}$ is bounded. Consider a sequence of n transition matrices $(A_k)_{k\in[\![0;n-1]\!]} \in \mathcal{A}^n$ and for $i \in [\![0;n-1]\!]$ note

$$A_k = C + \alpha h_k E_{i_k}$$

Define

$$\forall k \in [\![1;n]\!],\ \ \Pi_k = \prod_{i=1}^{k} A_{k-i} = \begin{pmatrix} L_1^k \\ \vdots \\ L_p^k \end{pmatrix} \tag{11}$$

where L_i^k designates the ith row of the product of the k matrices. Calculating the product Π_{k+1}, it is clear that

$$L_{D_{\max}+2}^{k+1} = (1-\alpha)L_{D_{\max}+2}^{k} + \alpha h_{k+1} L_{i_{k+1}}^{k}$$

While for $j \in [\![1;D_{\max}+1]\!]$

$$L_{k+1}^{j} = L_k^{j+1}$$

it follows that

$$\|L_{D_{\max}+2}^{k+1}\|_1 \le \underbrace{((1-\alpha)+\alpha\|h\|_\infty)}_{<1} \max(\|L_{D_{\max}+2}^{k}\|_1, \|L_{i_{k+1}}^{k}\|_1)$$

Recursively, it is then straightforward to show that

$$\|\Pi_{D_{\max}+1}\|_\infty = \max_{j\in[\![1;D_{\max}+2]\!]} \|L_j^{D_{\max}+1}\|_1 \le (1-\alpha)+\alpha\|h\|_\infty \tag{12}$$

Preliminary results on discrete linear time-varying systems. Consider a discrete linear time-varying system (13) of dimension s, and $\mathcal{A}$ a bounded set of possible transition matrices in $\mathcal{M}_s(\mathbb{R})$ and initial condition X_0

$$\forall n \ge 0, \quad X_{n+1} = A_n X_n,\ A_n \in \mathcal{A} \tag{13}$$

For any vector norm $\|\cdot\|_*$ and any $N \in \mathbb{N}^*$, we define

$$M_{N,*} \triangleq \sup_{A_{N-i}\in\mathcal{A}} \|\prod_{i=1}^{N} A_{N-i}\|_* = \sup_{A_i\in\mathcal{A}} \|\prod_{i=0}^{N-1} A_i\|_* \tag{14}$$

Proposition 1 (Suff. cond. for exp. stab.). *Consider the system (13). If there exists $N_0 \in \mathbb{N}^*$ such that $M_{N_0,*} < 1$, then the system (13) (globally) exponentially converges to 0. One has, for some $K > 0$,*

$$\forall n \in \mathbb{N}^*,\ \|X_n\|_* \le K\|X_0\|_* \left(M_{N_0,*}\right)^{\lfloor \frac{n}{N_0} \rfloor} \tag{15}$$

Proof 1. See Appendix A.

As a consequence, using the notation (14)

$$M_{D_{\max}+1,\infty} = \sup_{(A_i)\in\mathcal{A}^{D_{\max}+1}} \|\Pi_{D_{\max}+1}\|_\infty < 1$$

which, according to Proposition 1, means that the (X_n) sequence is exponentially convergent. This, in turn, allows us to formulate the following result

Theorem 1 (Global convergence without measurement dating uncertainty). *Let $\Delta = 0$. Consider any $0 < \alpha \le 1$.*

If $\|h\|_\infty < 1$, then the closed loop error (6) converges exponentially and

$$\lim_{n\to+\infty} f_p(u_n) = c$$

Remark 1. In particular, one can notice that f' and f_p' must have the same sign so that the condition $\|h\|_\infty < 1$ can be verified. In this case, if

$$0 < \|\frac{f_p'}{f'}\|_\infty < 2 \tag{16}$$

then the sufficient condition is satisfied.

Remark 2. The result derived in Theorem 1 is a sufficient condition for the controller stability. We can still get some additional insight into the controller's behaviour by studying the particular case in which $D_{\max} = \Delta_{\max} = 0$. Then, we derive a necessary stability condition from a straightforward eigenvalue analysis showing that the equilibrium point of the system is locally stable if and only if

$$0 \le \frac{f_p'(u_c)}{f'(u_c)} \le 1 + \frac{1}{\alpha} \tag{17}$$

If $\alpha = 1$, this shows that the sufficient condition previously derived is also necessary. Otherwise, this shows that taking $\alpha < 1$ small enough may allow one to stabilize systems where the ratio $\frac{f_p'}{f'}$ is greater than 2 (actually, this is indeed observed in simulations).

5. Convergence with measurement dating error

We now consider the implementation of the same controller with measurement dating uncertainty causing the discussed mis-synchronization between measurement and prediction with $\Delta \neq 0$.

Using the same transformation as in Section 4, we establish the closed-loop error

$$\begin{aligned} d_{n+2} &= (1-\alpha)d_{n+1} + \alpha(f_p(f^{-1}(f(u_c) - d_{n-D_{n+1}})) - f(u_c) + d_{n-D_{n+1}-\Delta_{n+1}}) \\ &\quad -\alpha(c - f(u_c)) \end{aligned}$$

and, applying the mean value theorem, we get

$$d_{n+2} = (1-\alpha)d_{n+1} - \alpha\rho(a_n)d_{n-D_{n+1}} + \alpha d_{n-D_{n+1}-\Delta_{n+1}}$$

where

$$\rho = 1 - h$$

and

$$a_n \in [\min(0, d_{n-D_{n+1}}); \max(0, d_{n-D_{n+1}})]$$

The system can be written as a LTV system of dimension $p \triangleq D_{max} + \Delta_{max} + 2$

$$X_{n+1} = A_n X_n \tag{18}$$

where

$$X_n = \begin{pmatrix} d_{n-D_{\max}-\Delta_{\max}} & \cdots & d_{n+1} \end{pmatrix}^T$$

with

$$A_n = C + \alpha F_n - \alpha\rho(a_n)F_n' \tag{19}$$

and, with the notation (1),

$$F_n = E_{p-1-D_{n+1}-\Delta_{n+1}}$$

and

$$F_n' = E_{p-1-\Delta_{n+1}}$$

5.1. Convergence analysis without model error

Let us first assume that there is no model error. Under this assumption

$$\rho = 1$$

and the transition matrices A_n of the dynamics (18) all belong to the finite set

$$\mathcal{A} = \{C + \alpha E_k - \alpha E_{k'},\ (k, k') \in [\![1; p-1]\!] \times [\![p-1-D_{\max}; p-1]\!]\} \tag{20}$$

Consider a sequence of n transition matrices $(A_i)_{i \in [\![0;n-1]\!]} \in \mathcal{A}^n$. Similar to Section 4, define

$$\forall k \in [\![1; n]\!],\ \Pi_k = \prod_{i=1}^{k} A_{k-i} = \begin{pmatrix} L_1^k \\ \vdots \\ L_p^k \end{pmatrix} \tag{21}$$

where L_i^k designates the ith row of the product of the k matrices. The convergence analysis is built recursively upon the fact that there exists $K > 0$ such that for all $n \in \mathbb{N}^*$

$$M_{n,\infty} = \|\prod_{i=1}^{n} A_{n-i}\|_\infty \leq K\eta^{\lfloor \frac{n-1}{p+D_{\max}} \rfloor} \tag{22}$$

where

$$\eta \triangleq \max_{r \in [\![1; D_{\max}+1]\!]} 1 - \frac{\alpha}{(1-\alpha)^r} + \frac{2\alpha}{1-\alpha}(1-(1-\alpha)^r) \tag{23}$$

For all $n \geq 2$, it is clear that

$$\forall j \in [\![1; p-1]\!],\ L_j^n = L_{j+1}^{n-1} \tag{24}$$

and

$$\exists (r_n, m_n) \in [\![1; p-1]\!]^2,\ L_p^n = (1-\alpha)L_p^{n-1} - \alpha L_{p-r_n}^{n-1} + \alpha L_{p-m_n}^{n-1} \tag{25}$$

We wish to prove that there exists K such that the following relation holds for all $n \geq 0$

$$\forall j \in [\![1; p]\!],\ \|L_j^n\|_1 \leq K\eta^{\lfloor \frac{n+j-2}{p+D_{\max}} \rfloor} \tag{26}$$

Let us define

$$K \triangleq \max_{(A_0, \ldots, A_{n-1}) \in \mathcal{A}^n} \left(\max_{i \in [\![1; D_{\max}+1]\!]} \|L_p^i\|_1 \right)$$

It is clear that (25) is true for all indexes from 2 to $D_{\max}+1$. Given $n \geq D_{\max}+1$, let us assume that the property is true for this rank. One has

$$\Pi_{n+1} = \prod_{i=1}^{n+1} A_{n+1-i} = \begin{pmatrix} L_1^{n+1} \\ \vdots \\ L_p^{n+1} \end{pmatrix}$$

with

$$\forall j \in [\![1; p-1]\!],\ L_j^{n+1} = L_{j+1}^n$$

and

$$L_p^{n+1} = (1-\alpha)L_p^n - \alpha L_{p-r_{n+1}}^n + \alpha L_{p-m_{n+1}}^n$$

Hence, according to (24)

$$L_p^{n+1} = (1-\alpha)L_p^n - \alpha L_p^{n-r_{n+1}} + \alpha L_{p-m_{n+1}}^n \tag{27}$$

To proceed to the induction, we choose to develop the second term of (27). According to (25) at rank n

$$L_p^{n-r_{n+1}+1} = (1-\alpha)L_p^{n-r_{n+1}} - \alpha L_{p-r_{n-r_{n+1}+1}}^{n-r_{n+1}} + \alpha L_{p-m_{n-r_{n+1}+1}}^{n-r_{n+1}}$$

It follows that

$$L_p^{n-r_{n+1}} = \frac{1}{1-\alpha}(L_p^{n-r_{n+1}+1} + \alpha L_{p-r_{n-r_{n+1}+1}}^{n-r_{n+1}} - \alpha L_{p-m_{n-r_{n+1}+1}}^{n-r_{n+1}}) \tag{28}$$

As a consequence, after substitution with (28), (27) gives

$$\begin{aligned} L_p^{n+1} &= (1-\alpha)L_p^n + \alpha L_{p-m_{n+1}}^n \\ &\quad - \frac{\alpha}{1-\alpha}(L_p^{n-r_{n+1}+1} + \alpha L_{p-r_{n-r_{n+1}+1}}^{n-r_{n+1}} - \alpha L_{p-m_{n-r_{n+1}+1}}^{n-r_{n+1}}) \end{aligned}$$

Recursively, from rank $n - r_{n+1} + 1$ to n, we get

$$\begin{aligned} L_p^{n+1} &= \left[(1-\alpha) - \frac{\alpha}{(1-\alpha)^{r_{n+1}}}\right] L_p^n + \alpha L_{p-m_{n+1}}^n \\ &\quad -\alpha^2 \sum_{i=0}^{r_{n+1}-1} \frac{1}{(1-\alpha)^{i+1}}(L_{p-r_{n-r_{n+1}+1+i}}^{n-r_{n+1}+i} - L_{p-m_{n-r_{n+1}+1+i}}^{n-r_{n+1}+i}) \end{aligned}$$

Then, if the following condition holds

$$0 \leq (1-\alpha) - \frac{\alpha}{(1-\alpha)^{r_{n+1}}} \tag{29}$$

Using one cancellation and a succession of terms reorderings, one has

$$\|L_p^{n+1}\|_1 \leq \left(1 - \frac{\alpha}{(1-\alpha)^{r_{n+1}}} + 2\alpha^2 \sum_{i=0}^{r_{n+1}-1} \frac{1}{(1-\alpha)^{i+1}}\right) \max_{\substack{j \in [\![1; p]\!] \\ k \in [\![n - r_{\max}; n]\!]}} \|L_j^k\|$$

And, finally, using the explicit summation of the geometric sequence

$$\|L_p^{n+1}\|_1 \leq \left(1 - \frac{\alpha}{(1-\alpha)^{r_{n+1}}} + \frac{2\alpha}{1-\alpha}(1-(1-\alpha)^{r_{n+1}})\right) \max_{\substack{j \in [\![1; p]\!] \\ k \in [\![n - D_{\max}; n]\!]}} \|L_j^k\|$$

which leads, by induction with (26), to

$$\|L_p^{n+1}\|_1 \leq K\eta^{\lfloor \frac{n-D_{\max}+1-2}{p+D_{\max}} \rfloor + 1}$$

and after a simplification

$$\|L_p^{n+1}\|_1 \leq K\eta^{\lfloor \frac{(n+1)+p-2}{p+D_{\max}} \rfloor}$$

This proves (26) at rank $n+1$. As a consequence, (22) directly follows using the relation between the infinity norm of a matrix and the one norm of its rows

$$\forall n \in \mathbb{N},\ \|\Pi_n\|_\infty = \max_{i \in [\![1; p]\!]} \|L_i^n\|_1$$

5.2. General case

Based on this first result, we introduce a small model error, and formulate an extension by continuity. This last result shows that the proposed controller solves the control problem at stake, in the presence of model mismatch, delayed measurements and dating error.

According to (22) there exists $N_0 \in \mathbb{N}$ such that if there is no model error

$$M_{N_0,\infty} \leq \frac{1}{2}$$

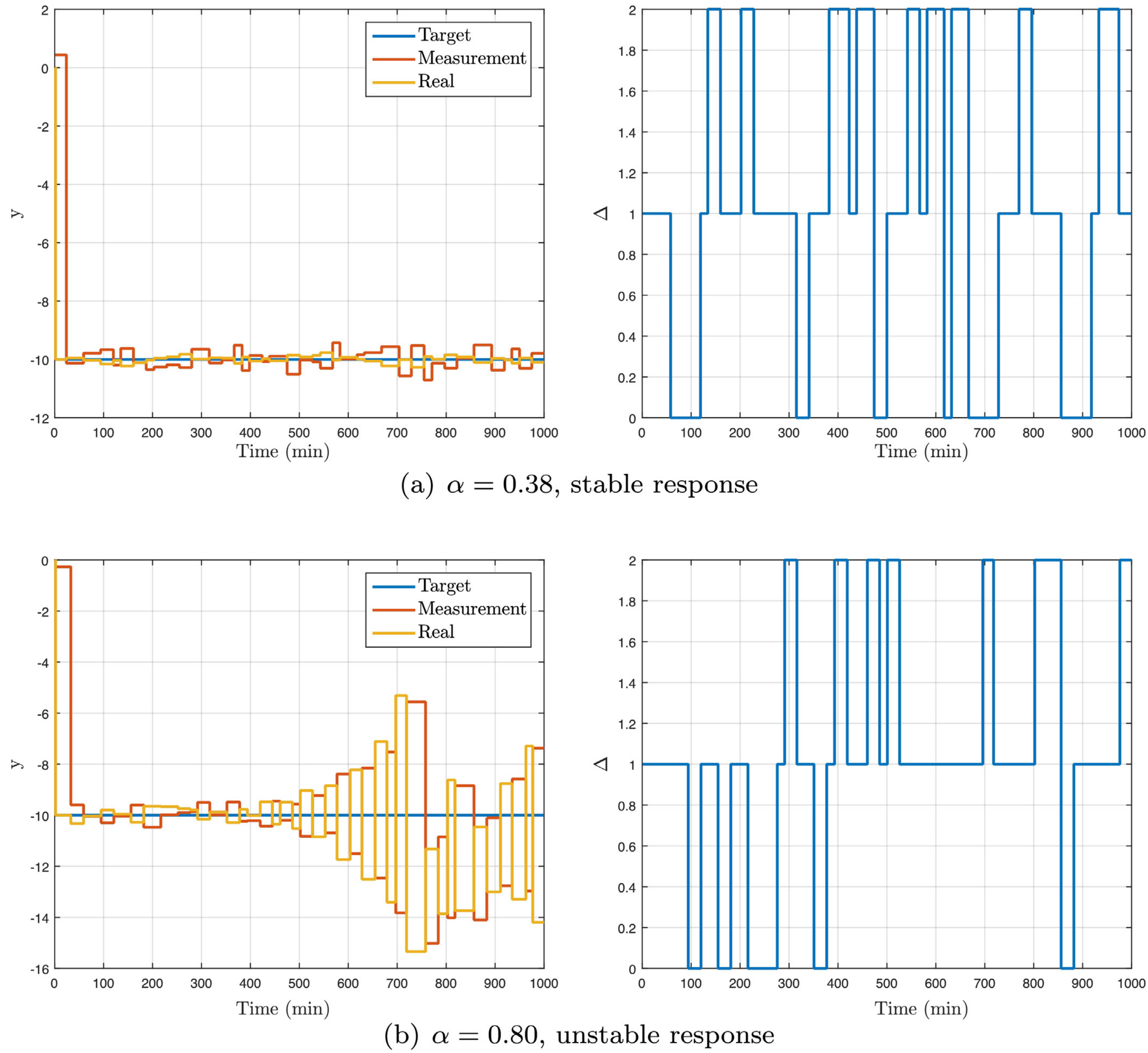


Fig. 4. No model mismatch, measurement dating error, small measurement error.

With model error, any transition matrix of the dynamics A_n can be written under the additive form

$$A_n = A_n^0 + P_n \tag{30}$$

where A_n^0 is a matrix of the set (20)

$$A_n^0 \in \{C + \alpha E_{k_n} - \alpha E_{k'_n},\ (k_n, k'_n) \in [\![1; p-1]\!] \times [\![p-1-D_{\max}; p-1]\!]\} \tag{31}$$

and P_n is a perturbation matrix

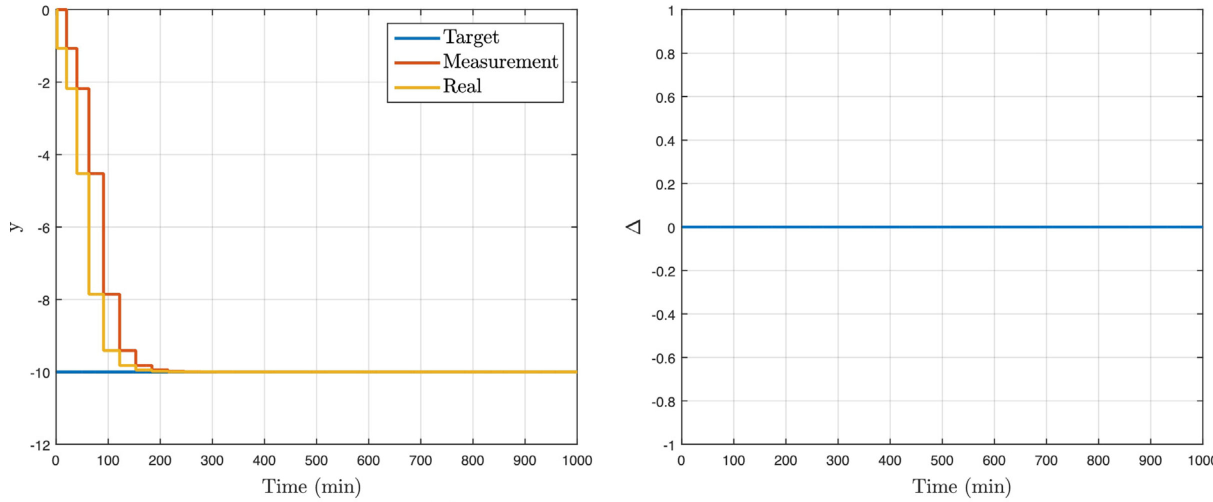
$$P_n = \alpha h(x_n) E_{k'_n} \tag{32}$$

with x_n a given real number. Consider any collection of N_0 such matrices $(A_i)_{i \in [\![0; N_0-1]\!]}$ and assume that there exists $\epsilon > 0$ such that $\| h \|_\infty \le \epsilon$, then

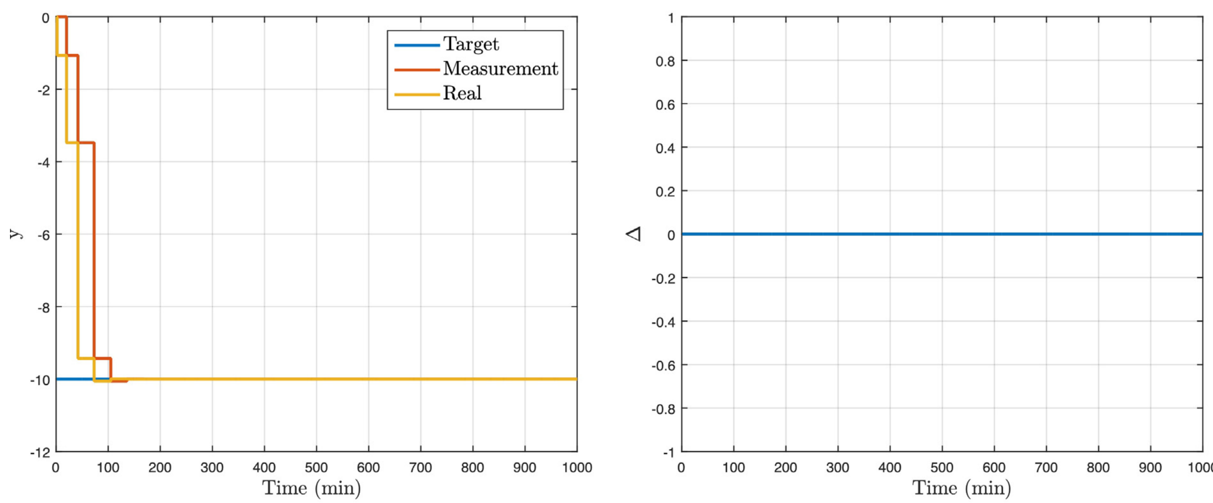
$$\begin{aligned} \| \prod_{i=0}^{N_0-1} A_i \|_\infty &\le \| \prod_{i=0}^{N_0} A_i^0 \|_\infty + \sum_{i=1}^{N_0} C_{N_0}^{N_0-i} (1+\alpha)^{N_0-i} \alpha^i \epsilon^i \\ &\le \frac{1}{2} + \epsilon \sum_{i=1}^{N_0} C_{N_0}^{N_0-i} (1+\alpha)^{N_0-i} \alpha^i \epsilon^{i-1} \end{aligned}$$

By upper-bounding the (finite) sum appearing in the right-hand side, it follows that there exists a sufficiently small value of ϵ such that for any $(A_i)_{i \in [\![0; N_0-1]\!]}$

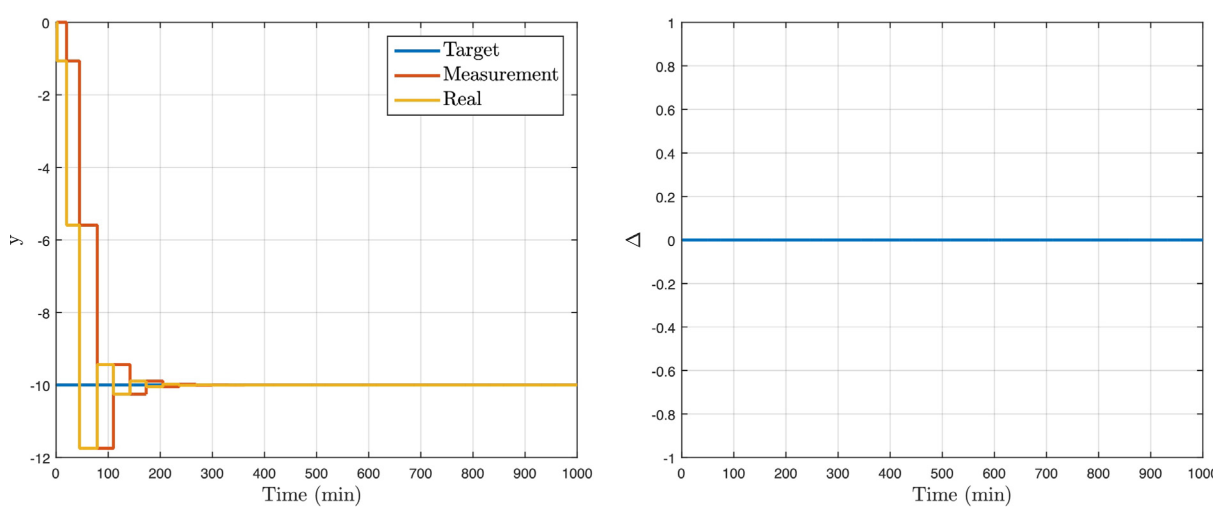
$$\| \prod_{i=0}^{N_0-1} A_i \|_\infty \le \frac{3}{4} < 1 \tag{33}$$



(a) $\alpha = 0.38$, stable response

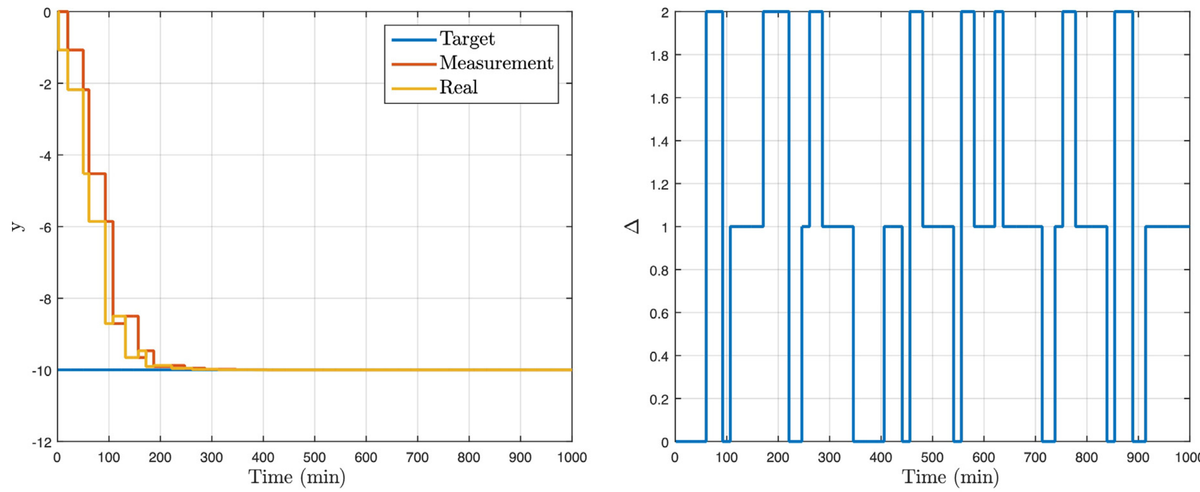


(b) $\alpha = 0.60$, stable response

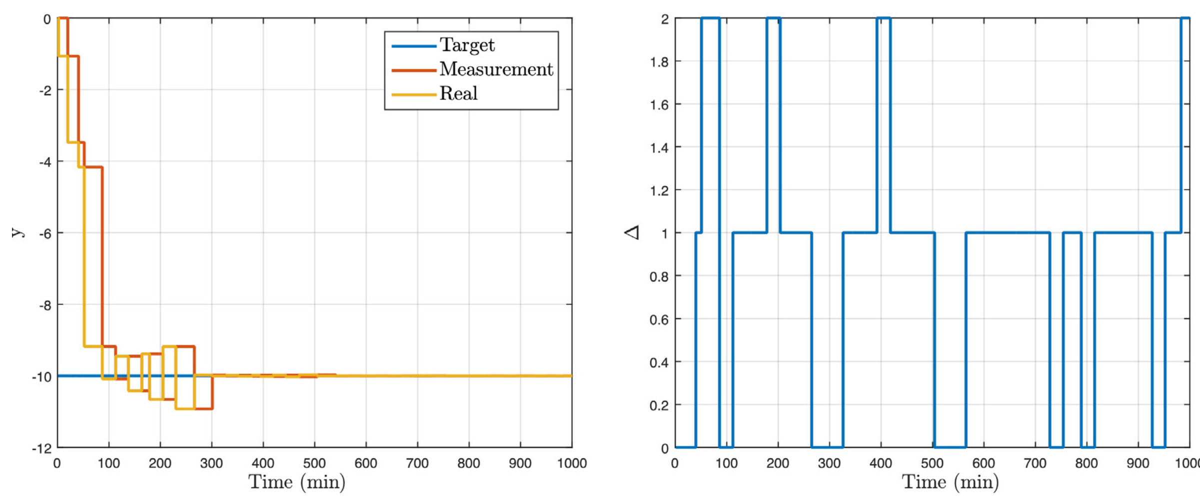


(c) $\alpha = 0.80$, stable response

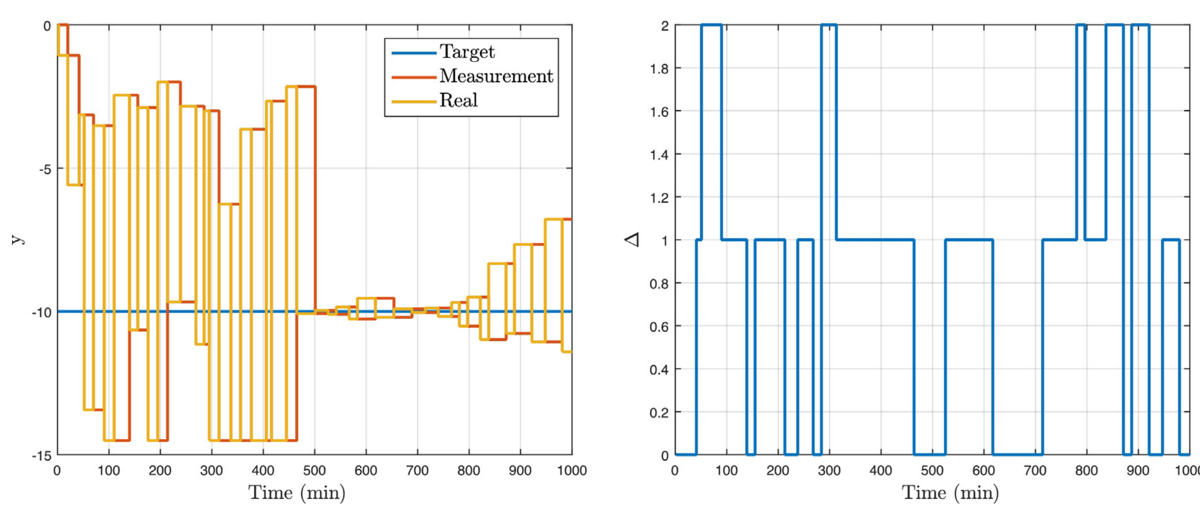
Fig. 5. Model mismatch, no measurement dating error, no measurement noise.



(a) $\alpha = 0.38$, response remains stable with delay estimation error



(b) $\alpha = 0.60$, response starts to exhibit erratic behaviour with delay estimation error



(c) $\alpha = 0.80$, response becomes unstable with delay estimation error

Fig. 6. Model mismatch, measurement dating error, no measurement noise.

Then, Proposition 1 yields the exponential convergence of X_n and leads to the following (main) result

Theorem 2 (Exponential convergence under measurement dating uncertainty and model mismatch). *Let $\Delta \le \Delta_{max}$. Consider any $0 < \alpha \le 1$ such that*

$$0 \le (1-\alpha) - \frac{\alpha}{(1-\alpha)^{D_{\max}+1}} \tag{34}$$

and

$$\max_{r \in [\![1;D_{\max}+1]\!]} 1 - \frac{\alpha}{(1-\alpha)^r} + \frac{2\alpha}{1-\alpha}(1-(1-\alpha)^r) < 1 \tag{35}$$

There exists $\epsilon \in \mathbb{R}_+$ such that, if $\|h\|_\infty \le \epsilon$, then the controller (5) is exponentially stabilizing and guarantees

$$\lim_{n \to +\infty} f_p(u_n) = c$$

Remark 3. In particular, one sees from (22) that the larger D_{max} and $\Delta_{\max}$ is, the slower the guaranteed convergence rate is.

Remark 4. One can easily check that there always exist a neighbourhood of $\alpha = 0$ on which conditions (34) and (35) are verified. Indeed

$$(1-\alpha) - \frac{\alpha}{(1-\alpha)^{D_{\max}+1}} \underset{\alpha \to 0^+}{\longrightarrow} 1$$

and $\forall r \ge 1$

$$1 - \frac{\alpha}{(1-\alpha)^r} + \frac{2\alpha}{1-\alpha}(1-(1-\alpha)^r) = 1 - \alpha + \mathcal{O}(\alpha^2)$$

Remark 5. In terms of controller design's specifications, we can draw the following statements from our analysis:

- the sign of the estimated gain must be correct and its value cannot be too small compared with the reality. On the other hand, taking an estimated gain larger than the true one will slow the controller down but cannot jeopardize its stability
- variable delays cause no specific problem of convergence if we assume that exact measurement dating is available
- if there is dating uncertainty of the measurement, stability can still be retained, provided that the measurements filtering is strong enough (α small enough)

6. Simulation

In this section, we consider a setting where each sample is analysed during a certain lapse of time during which no other sample is taken. New control actions are only implemented when a new measurement result is received. In that way, the time-sampled measurement delay is always zero ($D_n = 0$, $D_{\max} = 0$), i.e. the measurement we receive is always informative of the result of the last control action taken. This is a special case of Theorem 2 which is of practical importance in the implementation of many controllers. In classic run-to-run cases, nonzero $D_{\max}$ could be considered, without loss of generality.

We will assume that the actual physical time required for the measurement to reach the controller, T_p, depends on the measured value according to the following relation

$$T_p(y) = 8.5 - 0.75 y_{mes} + 10(\xi_1 + \xi_2) \tag{36}$$

where $\xi_{1,2}$ are the realizations of two independent random variables taking the values 0 or 1 with respective probabilities of $\frac{1}{2}$. In some simulations, we will assume that timestamps are not available and that the times at which the measurements are taken have to be estimated using an approximate model T of T_p. This estimation may be inexact, thus leading to dating uncertainty of the measurements, i.e. $\Delta_{\max} \ge 0$. In all simulations, the target will be $c = -10$.

6.1. Influence of pure dating uncertainty

In this subsection, the response of the system, f_p, is assumed to be a simple linear function of the control u. We further assume that it is perfectly known, so that

$$f_p(u) = f(u) = -25u \tag{37}$$

Furthermore, we consider a situation where no timestamp is available. As a consequence, T_p must be estimated and we assume that the approximate model available to the controller is

$$T(y) = 17 - 1.5 y_{mes} \tag{38}$$

This mismatch results into nonzero values of Δ which are plotted to provide a graphical view of its statistical distribution. We also corrupt the measurements with a zero-mean, uniform noise of small amplitude to excite the system.

We simulate the response of the system for various values of the gain α. The results show that while the closed-loop system remains stable for α small enough, as α increases, destabilization can arise in absence of any model error on the function f_p, simply because of measurement dating uncertainty. This illustrates the fact that Theorem 2 does not hold after a critical value of α, which is conservatively estimated by conditions (34) and (35) $\left(\alpha = \frac{3-\sqrt{5}}{2} \approx 0.38\right)$. The results of these simulations are shown in Fig. 4.

6.2. Influence of the system's response uncertainty

We further illustrate Theorem 2 by simulating the closed-loop response of the system with a model mismatch (but without any measurement noise). We assume that f_p is given by

$$f_p(u) = -8.4339 - 6\ \arctan(8u - 6) \tag{39}$$

and f by

$$f(u) = -25u \tag{40}$$

We first run the simulations without measurement dating error with three different values of α corresponding to a low ($\alpha = 0.38$ discussed above), medium ($\alpha = 0.60$) and high ($\alpha = 0.80$) filtering. Fig. 5 shows the results of the simulations. Despite model mismatch, all of them display stable responses.

Fig. 6 then shows the results of the simulations where measurement dating error is introduced (the previous expressions for T_p and T are used). One sees that beyond a critical value for the gain α, the response becomes increasingly poor, and eventually becomes unstable for the high α scenario.

These simulations illustrate the merits of the theoretical results established in this article. A tuning of the controller gain following the (conservative) estimate provided by the small-gain condition gives satisfactory closed-loop responses even when the measurement dating error is not negligible. If the gain is chosen above the threshold, some divergence (or strong oscillations) can be observed. The situation is similar with reasonable levels of measurement noise.

7. Conclusions

As a static SISO control problem, the core problem tackled in this paper appears, at first sight, as simple as it could be. However, the variability and uncertainty of the delay makes the problem particularly tricky. We have provided explicit robustness margins in regard of model error and asymptotic analysis on the consequences of uncertain measurement dating.

In the case where an underlying dynamical system should be considered to model the system, the preceding approach should be updated, significantly. Because the measurement will remain sampled by nature, the closed loop system will naturally become a sampled-data ordinary differential equation as considered in e.g. [23]. Also, it is known that the introduction of time-varying gains may improve the exponential convergence, when measurements are subjected to (known) delays. If estimates of the delay are available, such tuning rules could bring some performance improvement. While the problem becomes significantly harder due to the time-varying nature of the discretized system transition matrices, it would be interesting to investigate whether, in a more general context of multi-input multi-output (MIMO) dynamical systems, an event-triggered discretization approach such as the one developed in this paper could be used to obtain results on the influence of measurement dating uncertainty.

Appendix A. Proof of Proposition 1

The proof is relatively straightforward

$$\forall n \in \mathbb{N},\ X_n = \prod_{i=1}^{n} A_{n-i} X_0$$

Hence, grouping terms in N_0-size bundles starting from the right

$$\begin{aligned}\|X_n\|_* \le \ & \| \prod_{i=1}^{n-\lfloor \frac{n}{N_0} \rfloor N_0} A_{n-i} \|_* \\ & \times \prod_{i=1}^{\lfloor \frac{n}{N_0} \rfloor} \| \prod_{j=1}^{N_0} A_{\lfloor \frac{n}{N_0} \rfloor N_0 - (i-1)N_0 - j} \|_* \|X_0\|_*\end{aligned}$$

and

$$\|X_n\|_* \le M_{n-\lfloor \frac{n}{N_0} \rfloor N_0, *} M_{N_0,*}^{\lfloor \frac{n}{N_0} \rfloor} \|X_0\|_*$$

Besides,

$$\forall n \in \mathbb{N},\ 0 \le n - \lfloor \frac{n}{N_0} \rfloor \ < N_0$$

Hence, we get the desired result by defining

$$K \triangleq \max_{k \in [\![0;N_0-1]\!]} M_{k,*}$$

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