2015 American Control Conference
Palmer House Hilton
July 1-3, 2015. Chicago, IL, USA

A distributed parameters model for electric hot water tanks

Nathanael Beeker[1,2], Paul Malisani[2] and Nicolas Petit[1]

***Abstract*— To quantify the potential of electric hot water tanks (EHWT) in load shifting and other demand response programs, there is a need for a simple model allowing to reproduce their experimentally observed behavior. The input output response of this system is in fact relatively complex. This paper presents a model of an EHWT in the form of two simple one-dimensional partial differential equations, followed by an experimental validation. The model permits to quantify the effective energy available for consumption which is an important variable for control and optimization purposes.**

I. INTRODUCTION

Recent trends in energy policy have led to a rise of production levels of intermittent energy sources [16] such as wind or photovoltaic energy. In turn, this emergence has raised new questions for electric production management. Among these, the reduction of the induced fluctuations and the determination of new ways of storing energy are central. These topics are of importance in the questions of demand response and load shifting [15]. In this context, the high ratio of equipment of French households with Electric Hot Water Tanks (EHWT) (representing 32,6% of the final energy consumption for personal heated water in 2011 [2]), appears as a promising field because of its large overall energy capacity and its potential flexibility.

Individual EHWT are heated in a static or semi-static way over long periods of time. To develop dynamical piloting strategies for domestic uses (e.g. in response to fluctuating prices of electricity), dynamical models are needed. An EHWT can be seen as a two inputs, single output dynamical system. The two inputs are the heating power and water outflow or drain (which is equal in volume to the inflow). The definition of the output is less straightforward. A natural choice, for later optimization purposes, is to define a variable representing a quality of service. We propose here to consider an "available energy", representing the total energy contained in the water whose temperature is above some prescribed threshold (e.g. 40^{o}C). The rationale is that hot water above 40^{o}C can be used, by blending with cold water, for all domestic uses. By contrast, water below 40^{o}C is useless. To control and optimize the EHWT one needs to model the input-output behavior of these systems. This is the subject of this article.

[1]N. Beeker (PhD candidate) and N. Petit are with MINES ParisTech, PSL Research University, CAS, 60 bd Saint-Michel, 75272 Paris Cedex FRANCE nathanael.beeker-adda@mines-paristech.fr

[2] P. Malisani is with EDF Lab, EnerBat, Avenue des Renardières - Ecuelles, 77818 Moret-sur-Loing Cedex FRANCE, which fund the PhD of N. Beeker

More generally, in the literature [1], [10], [13], [18], hot water storages are described as vertical cylindrical columns driven by thermo-hydraulic phenomena: heat diffusion, buoyancy effects and induced convection and mixing, forced convection induced by draining and associated mixing, and heat losses at the walls. Most existing models are either i) one-dimensional superposition of layers (see e.g. [1], [13]), or ii) three or two-dimensional (using rotational symmetry) models, often using a discretization for numerical simulation purposes such as CFD [1], [6], [8] or iii) so-called zonal models mainly based on the software TRNSYS [8], [9]. These types of model, although accurate, are numerically intensive and do not fit with our piloting goals which require simpler models. On the other hand, when overly simplified, layers models fail to reproduce some physical phenomena whose effects are observed in practice. This is particularly true when one wishes to introduce heating in the dynamics. Unfortunately, this is a strong limitation in view of later control and optimization scheduling tasks.

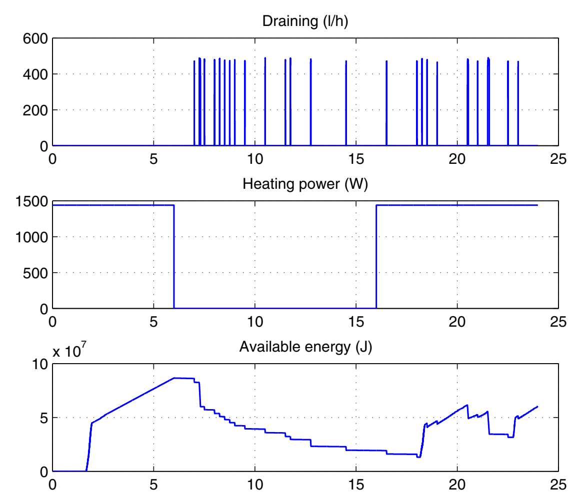


Fig. 1. Available hot water as a function of heating and draining on a 24h run

A careful study of the physical principles at stake in the system suggests some simplifications. The buoyancy effects lead to the so-called stratification phenomenon [6], causing horizontal homogeneity of the temperature with an increasing profile of the temperature with height. This effect is dominant and allows us to consider only one-dimensional models. The model we develop here extends an existing one-dimensional convection-diffusion linear equation modeling the draining

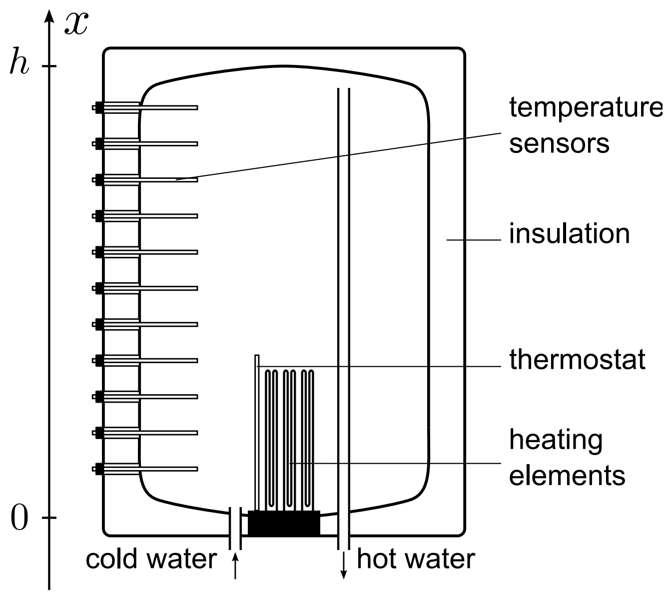


Fig. 2. Simplified scheme of an EHWT

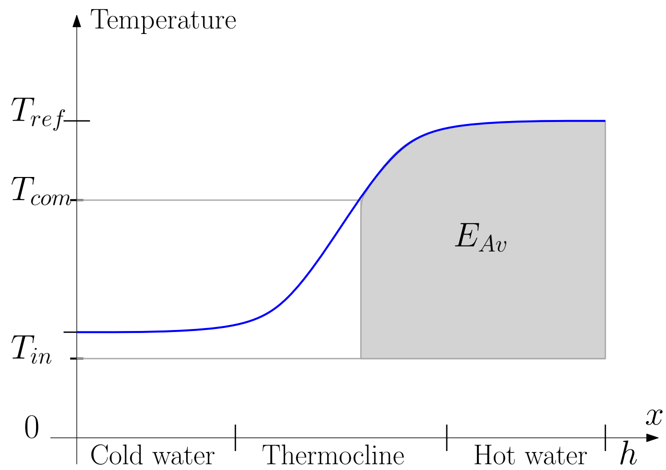


Fig. 3. Example of the temperature profile of a stratified water tank

convection and its mixing developed in the 80s [17], [18]. In details, to the classic governing equation, we add a nonlinear velocity term given by empirical laws representing other phenomena (turbulent natural convection due to heating in particular) and we include heating power as a source term. Experimental data illustrate the relevance of this modeling.

This model, which reproduces experimental data presented in this article, permits to compute the dynamics of the "available energy", defined earlier, in response to the water drain and the power injected in the tank. A typical scenario is reported in Fig.1. As it is visible, the input-output behavior of the system in somewhat complex although not counterintuitive. Heating water takes time. Starting from a uniformly cold tank, the output of the system remains identically equal to zero for hours. Then it jumps, and steadily increases. Draining causes step down on the output, and also causes some internal mixing which is non negligible. As will appear, the simple distributed parameters model proposed in this article is sufficient to reproduce such experimental recordings.

The paper is organised as follows. After having described the proposed model in Section II, we illustrate it by means of simulations and compare it against experimental data in Section III. In this study, a typical 200L tank (equipped with spatially distributed internal sensors) with realistic scenarios of draining and heating is employed.

II. PDE MODEL FOR HEATING AND DRAINING

A. General considerations on water tanks and stratification

Here, we briefly introduce the geometry and functioning of an EHWT, along with the assumptions induced on the model. A typical EHWT is a vertical cylindrical tank of water in which a heating element is plunged at the bottom end (see Fig.2). The heating element is pole-shaped, and its length may be up to one third of the tank. Cold water is injected at the bottom of the tank while hot water is drained from the top at exactly the same flow rate (under the assumption of pressure equilibrium in the water distribution system). Therefore, the tank is always full. In the tank, layers of water with various temperature (increasing with height) can coexist (see Fig.3). At rest, these layers are mixed only by heat diffusion which effects are relatively slow [6]. Existence of a non uniform quasi-equilibrium temperature profile in the tank is called stratification [3], [6], [11]. It practice, this effect is beneficial as hot water available for consumption is naturally stored near the outlet of the EHWT, while the rest of the tank is heated (see Fig.3).

B. Draining model as a PDE

To take advantage of the stratification effect, we can limit our model design to one-dimensional partial differential equations (PDEs). Following the works of Zurigat on draining effects in stratified thermal storage tanks [17], [18], a first model can be considered as the geometry of the storage considered in those works is easily extrapolated to the EHWT under consideration here, at the exclusion of the heating system. This first model focuses only on draining and its induced turbulent mixing effects. It is composed of a simple one-dimensional energy balance where the turbulence is lumped into a diffusion term

$$\frac{\partial T}{\partial t} + \frac{\partial v_d T}{\partial x} = (\alpha + \alpha_d)\frac{\partial^2 T}{\partial x^2}.$$

In this equation, $T(x,t)$ is the temperature at time t and height x, $v_d \geq 0$ is the velocity induced by the draining (assumed spatially uniform but time-varying), α is the thermal diffusivity and α_d is an additional turbulent diffusivity term representing the mixing effects. Moreover, Zurigat [17], [18] introduces the ratio $\epsilon_d = \frac{\alpha+\alpha_d}{\alpha}$ and presents an experimental correlation with the Reynolds number and the Richardson number Ri in the tank. The Richardson number written below is a dimensionless number representing the relative importance of natural convection compared to forced convection [18].

$$Ri = \frac{g\beta(T_{in} - T_a)L_c}{v_d^2}$$

where g is the gravitational acceleration, β is the volumetric coefficient of thermal expansion of the fluid (here water), T_{in} is the temperature of the inlet water, T_a is the ambient temperature and L_c is a characteristic vertical length. This correlation is influenced by the geometry of the inlet nozzle [7], [18].

Then, a heat loss term (to the exterior of the tank assumed to be at temperature T_a) can be added to this equation to obtain

$$\frac{\partial T}{\partial t} + \frac{\partial v_d T}{\partial x} = \epsilon_d \alpha \frac{\partial^2 T}{\partial x^2} - k(T - T_a) \qquad \text{(II.1)}$$

where the factor k is defined by

$$k = \frac{U_h P}{S \rho c_p} \qquad \text{(II.2)}$$

where ρ is the density of water, c_p its specific heat capacity, U_h is the overall heat transfer coefficient based on the tank internal surface area, P is the tank internal perimeter and S its effective cross-section.

Note h the vertical length of the tank internal volume. Equation (II.1) is assumed to hold over $\Omega \times I$, where $I =]t_0, t_1]$ is a time interval and $\Omega =]0, h[$. Classically, we consider boundary conditions of the Dirichlet form $T(0,t) = T_{in}$ at $x = 0$, and of the Neumann form $\frac{\partial T}{\partial x}(h,t) = 0$ in $x = h$, meaning that energy is allowed to leave the system with the outlet flow but not with diffusion.

C. Including heating and buoyancy forces

Equation (II.1) integrates three of the four main phenomena in EHWT modeling. The buoyancy effects can be added to this equation at the expense of its linearity. When the heating system is on, temperature of water around the heating element starts to rise. By buoyancy, hot water replaces colder water above by a phenomenon called Rayleigh-Bénard convection [14], [4], [5]. This convection can take various forms depending on the characteristics of the system, represented by the Rayleigh number

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu\alpha} \qquad \text{(II.3)}$$

where ν is the kinematic viscosity of the fluid and α its thermal diffusivity, T_s is the temperature of the surface (here the heating element), and T_∞ is the temperature in the tank far from it. This adimensional number scales the effects of buoyancy and conduction: if it is low, the conduction will be the main heat transfer factor, if it is high, the natural convection will predominate. Over a critical value ($Ra = 1108$ for [14], $Ra = 3,5 \cdot 10^4$ for [4]), turbulent natural convection appears under the form of a pattern of plumes forming convection cells called Bénard cells which can take various forms and sizes.

For any EHWT found in households, even with a small $T_s - T_\infty$ difference, the Rayleigh number is far over the critical value, and plumes of turbulent water appears over the heating system (see Fig.4 reproduced from [1]). Therefore, convection dominates conduction. To include this effect into our one-dimensional model, we can simply consider that, at each given height, there can exist two distinct temperatures in the convection cells. Then, we append to our equation on T an interacting equation bearing on a new physical quantity $\Delta T(x,t)$ representing the temperature spread at each height x over T. This gives the following system

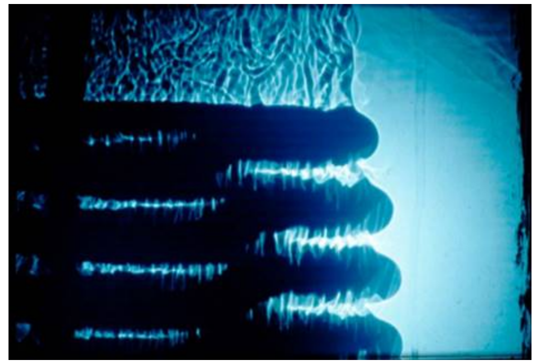


Fig. 4. Plumes of turbulent natural convection over an exchanger

$$\frac{\partial T}{\partial t} + \frac{\partial v_d T}{\partial x} = \epsilon_d \alpha \frac{\partial^2 T}{\partial x^2} + \Phi \Delta T - k(T - T_a) \qquad \text{(II.4)}$$

$$\frac{\partial \Delta T}{\partial t} + \frac{\partial (v_d + v_{nc}) \Delta T}{\partial x} = \epsilon_d \alpha \frac{\partial^2 \Delta T}{\partial x^2} - \Phi \Delta T + P_W \qquad \text{(II.5)}$$

In the two equations above, three terms have been added: a velocity term v_{nc} of natural convection, which is responsible for transport of energy in the system, a heat exchange term $\Phi(x,t)\Delta T(x,t)$ (representing at each height the mixing induced by natural convection being proportional to the temperature spread), and the spatially distributed source term P_W (representing the power injected in the tank via the element), which drives the dynamic of ΔT. The boundary conditions for (II.4) remain unchanged, while the boundary conditions for (II.5), as a temperature spread, are $\Delta T(0,t) = 0$ at $x = 0$ and $\frac{\partial \Delta T}{\partial x}(h,t) = 0$ at $x = h$.

D. Model for speed and exchange flows

We now introduce a model for the transport velocity. This velocity $v_{nc}(x,t)$ is non-constant. It is non-zero at a given altitude x only if there exists colder water over the height x (i.e. downstream). We give to v_{nc} the following integral form

$$v_{nc}(x,t) = v \int_x^h [T(x,t) + \Delta T(y,t) - T(y,t)]_+ dy$$

where $[z]_+$ is the positive part of z and v is a positive factor. In fact, to bring some flexibility into the identification procedure, we use the form

$$v_{nc}(x,t) = v\left(\int_x^h [T(x,t) + \Delta T(y,t) - T(y,t)]_+^\beta dy\right)^\gamma \qquad \text{(II.6)}$$

where β and γ are positive parameters smaller than 1 the value of which will be chosen to fit experimental data. These parameters reduce the impact of the temperatures differences above x and smooth the velocity when it is nonzero. The exchange coefficient Φ between the two equations is also non-constant and we model it as

$$\Phi(x,t) = \phi[v_{max} - v_{nc}(x,t)]_+ \qquad \text{(II.7)}$$

where ϕ and v_{max} are two parameters. The rationale behind this expression is that the horizontal mixing is stronger when the natural convection flow reaches the upper part of the tank and has a lower speed.

E. Summary of the model

According to the previous discussion, the EHWT can be represented by two distributed state variables, T and ΔT, governed by (II.4) and (II.5). In those governing equations two velocities appear: v_d which is spatially uniform and is equal to the output flowrate (drain) of the system, and v_{nc} which is defined in (II.6) to model the effects of natural convection. Heat is injected into the system through a distributed source term P_W and the heat exchange between the two equations is proportional to ΔT with a coefficient (variable in space) defined in (II.7). Finally, α, ϵ_d, k, v, β, γ, ϕ and v_{max} are constant parameters depending on physical constants and the geometry of the tank. Typical values for the EHWT reported in Table I are given in Table II. They result from an identification procedure.

III. MODEL VALIDATION

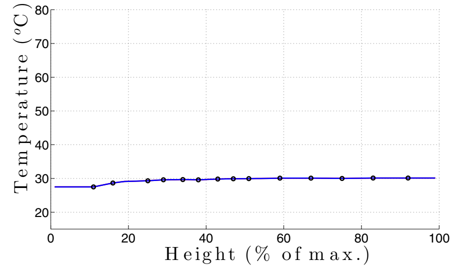
To validate the model, experiments have been conducted in the facilities of EDF Lab Research Center, on an Atlantis ATLANTIC VMRSEL 200L water tank. The power is injected via three nearby elements permitting a power injection up to 2200W. The dimensions of the water tank are specified in Table I.

TABLE I
SPECIFICATIONS OF THE EHWT USED IN EXPERIMENTS

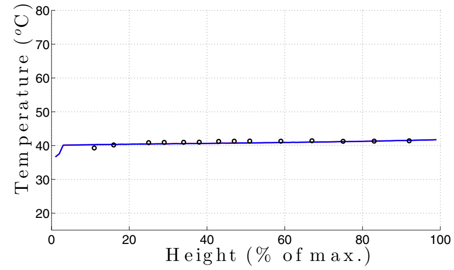
Volume (L)	200
Length (m)	1.37
Maximal power (W)	2200
Heat loss coefficient ($WK^{-1}m^{-2}$)	0.66

TABLE II
PARAMETER OF THE MODEL FOR THE ASSOCIATED EHWT

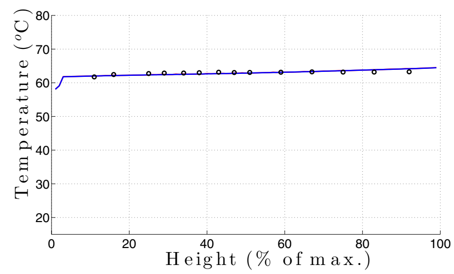
α (m^2s^{-1})	$1.43 \cdot 10^{-7}$
ϵ_d	13
k (s^{-1})	$1.43 \cdot 10 - 6$
v ($m^{1-\gamma}s^{-1}K^{-\gamma\beta}$)	10^{-3}
β	0.2
γ	0.5
ϕ (m^{-1})	0.03
v_{max} (ms^{-1})	0.35



(a)



(b)



(c)

Fig. 5. Evolution of the temperature profile during a heating period (blue: model prediction, black: experimental values)

The water tank has been equipped with internal temperature sensors recording temperature at 15 locations of different heights, 15cm deep inside the water tank (see Fig.2). This depth is sufficient to bypass the insulation of the tank. It is assumed that the sensors have no effect on the flows (e.g. that they do not induce significant drag). Besides, the following quantities have been recorded with external sensors: injected power, water flow at the inlet, water temperature at the inlet. These three quantities feed the model, the output of which can be compared with the temperature measured by the sensors. The comparisons are directed into an optimization procedure identifying the coefficients given in Table II. Conducted experiments took the form of fourteen 24h runs with a sampling rate of 1Hz. Histories for drain are taken from the normative sheets emitted by the French norm organism [12] for a tank of such capacity, associated with a classical night-time heating policy until total load. Subsequent experiments consider similar total consumption but with different drain/heat combination and overlaps to test the model under various situations.

For sake of illustration, several operating conditions are reported next. Fig.5 shows the evolution of the temperature during a heating period for a tank completely cold

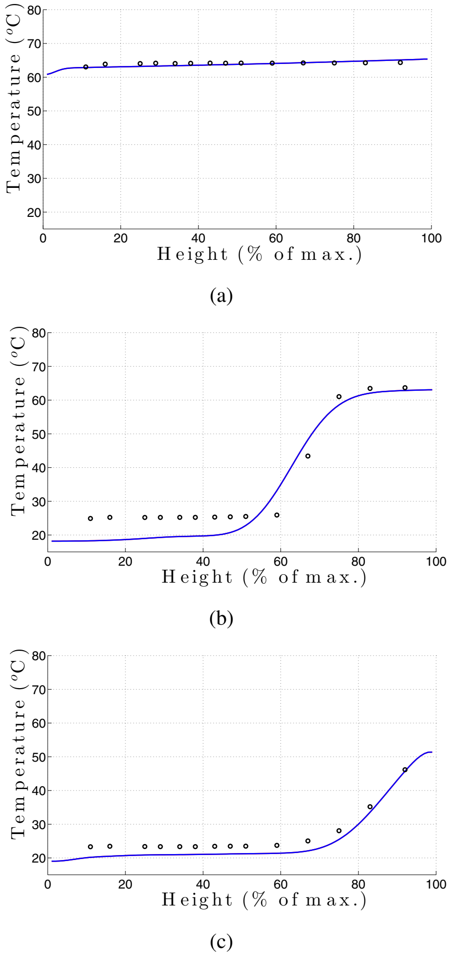


Fig. 6. Evolution of the temperature profile during a draining period (blue: model prediction, black: experimental values)

initially. Fig.6 shows the response to draining of a heated tank. Some mismatch appear in the lower and upper parts of the tank, probably due the one-dimensional approximation, since the neglected effects of radial inhomogeneity may be stronger in the ends of the tank. However, the results show that the model is quite accurate, even in 24h open-loop runs. To support this statement, the distribution of the absolute difference between experimental value and model prediction is given in Table III (produced over the whole set of data).

TABLE III

COMPARISON OF ABSOLUTE DIFFERENCE BETWEEN EXPERIMENTAL RESULTS AND MODEL PREDICTIONS. PERCENTAGE OF SAMPLE FOR EACH ERROR INTERVAL.

Err.	0-2^{o}C	2-4^{o}C	4-6^{o}C	6-8^{o}C	8^{o}C+	Time
Distr.	53.9%	22.9%	10.7%	5.1%	7.4%	2435.6s

IV. TOWARDS OPTIMIZATION AND CONTROL

A. *Practical considerations*

This model is to be used in future works with the goal of planning the heating. For example, given some price variations over a given time horizon, one wishes to determine an optimal heating policy, minimizing end-user cost, while ensuring a certain level of comfort. For this purpose, we need to define, at every instant, how much hot water is available for the user. As discussed earlier, this is a function of the internal distributed state of the system. Estimating this state is an interesting topic, which can be achieved in several ways using this model and sensors information. Given an initial profile, the model is able to deduce the current profile if heating and draining histories are known. This requires a flow sensor (either at the input or output of the tank), and recording of the power injected via the element. To estimate the initial condition, we will need an internal temperature sensor. In the EHWT installed in the French market, such a sensor is already installed at the bottom of the tank and is coupled with the heating system (see Fig.2): heating goes on until this sensor detects a temperature of T_{ref}. With this information, we know that the temperature is uniform and equal to T_{ref} in the tank (no water has been heated over T_{ref} in the past, and the profile is increasing). Finally, the model and the sensor information should be combined in a state observer for this distributed parameter system. This topic is currently under study.

B. *Simulating the energy available for the consumer*

Given the temperature profile in the EHWT, we are able to construct variables of interest for the consumer. As discussed earlier, a common indicator is the available volume of water at 40^{o}C (after potential dilution with cold water) or, more generally, the energy E_{Av} contained in the water which temperature is above a certain comfort level T_{com}. As a way to confirm the accuracy of the model, a comparison of the experimental E_{Av} (in red) and the model prediction (in blue) is depicted in Fig.7. As a reference, the value of E_{Av} obtained with a single-zone (homogeneous) model is shown (in black) in the figure. During a heating period, the available energy can be subject to large variations in a small amount of time. These strong variations are due to the appearance of a a temperature plateau during the heating. This plateau starts at the bottom of the tank and has increasing temperature and length, but leaves higher temperature (at greater heights) untouched and progressively covers the whole of the tank. This effect causes a threshold effect when the temperature of the plateau reach the comfort temperature T_{com} which explains the variations in E_{Av}.

The scenario employed for comparisons here starts with a partially heated tank, which is a favourable case for the simplest model. However some important errors can be observed. In particular the single-zone model fails to reproduce the slow decrease of the output at the end of the scenario. Instead it simulates a very early step down (time 18). This error would reveal very troublesome in optimization tasks, inducing additional cost for the user, and numerical issues such as convergence problems.

V. CONCLUSION

This paper presents a model of behavior of an EHWT with consumed water and injected power as inputs. This

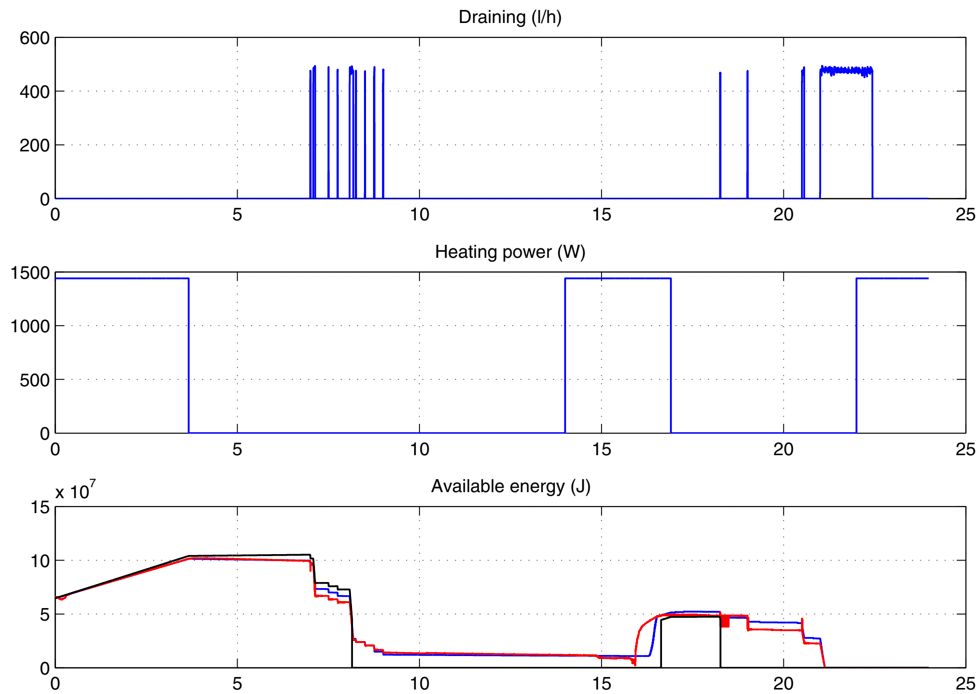


Fig. 7. Evolution of drainig, heating and available energy over a 24h run. Red: experimental data, blue:model predictions, black: Single-zone model

model presents the advantage of being computable with few ressources while being quite accurate. It could be embedded into the control system of the tanks, at a relatively low-cost since no internal sensor is needed. It would allow the real-time computation of available hot water (see Fig.7) or any other comfort indicator which can be used for minimizing the cost of the electricity consumed with comfort constraints fixed by the consumer. It opens the way, after the definition of some variable of interest, to interesting problems of control for large sets of EHWTs.

Interestingly, further simplifications seem possible. Both numerical resolution of the proposed model and experimental results highlight the formation of a spatially uniform temperature distribution which gradually extends itself upwards to the top of the tank. Buoyancy induced forces, generating a local natural convection phenomenon are the root cause. This homogeneous zone is followed by an increasing profile of temperature in the upper part of the tank, staying untouched due to stratification if heat diffusion is neglected. This property will certainly allow us to simplify the system in future works, e.g. by cascading an ordinary differential equation to a mobile interface distributed parameters system.

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