Robust control of variable time-delay systems
Theoretical contributions and applications to engine control

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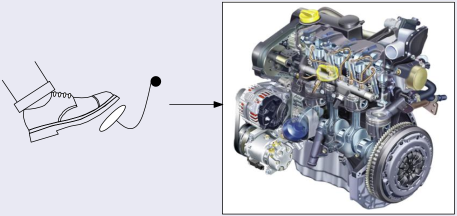
Context

Variable delays **ubiquitous** in internal combustion engines

- communication lags (ECU)
- spatially distributed after treatment devices
- physical flow transportation (transport delay)
- few embedded sensors
- measurement dead times



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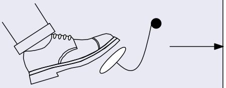

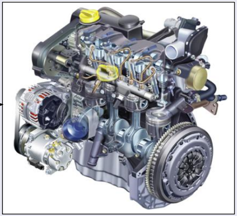

Delays are **prejudicial** to closed-loop stability and transient performances

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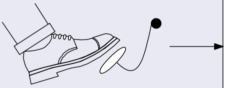

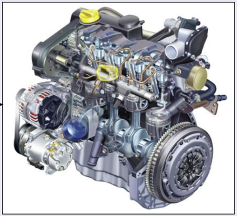

Delays are **prejudicial** to closed-loop stability and transient performances

Thesis objective

Design real-time control strategies to compensate uncertain and time-varying delays using a unified methodology.

Outline

1. Introduction : state prediction
2. Adaptive control scheme for uncertain systems with constant input delay
   - Delay-adaptive methodology principle
   - Control strategy with delay update law
   - Case study for a SI engine : FAR regulation
3. Robust compensation of a class of time- and input-dependent input delays
   - Integral transport delay class and application to EGR estimation
   - Sufficient conditions for transport delay robust compensation
4. Conclusion

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Input delay compensation

Dynamics under consideration

$$\dot{X}(t) = AX(t) + BU(t - D)$$

with $X \in \mathbb{R}^n$, U scalar and $D > 0$ **constant**.
(A, B) controllable and $K \in \mathbb{R}^{1\times n}$ s.t. $A + BK$ Hurwitz.

Prediction-based control law (Smith,1959)

$$U(t) = KX(t + D) = K\left[e^{AD}X(t) + \int_{t-D}^{t} e^{A(t-s)}BU(s)ds\right]$$

achieves **exact compensation** of the input delay

- finite spectrum assignment (FSA) (Manitius and Olbrot, 1978)
- reduction method (Artstein, 1979)

Closed-loop delay compensation

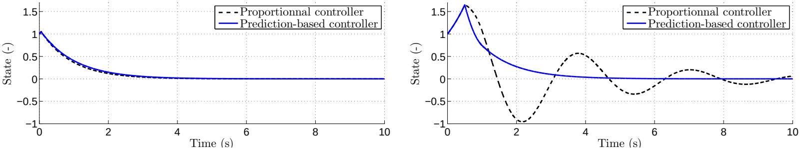
$\dot{X} = (A + BK)X(t)$ delay-free exponential convergence after D units of time

Input delay compensation

Illustrative scalar unstable example

$$\dot{x} = x + U(t - D)$$

with initial conditions $x(0) = 1$ and $u(s) = 1$, $s \in [-D, 0]$
Feedback gain chosen as $K = -2$



(a) $D = 0.05$ s (b) $D = 0.5$ s

Prediction-based control laws

- Improvement of transient performances
- Delay-independent behavior as $\dot{X} = (A + BK)X(t)$

Time-varying input delay compensation

$$\dot{X}(t) = AX + BU(r(t)) \quad \text{with} \quad r(t) = t - D(t)$$

Time-varying state prediction horizon

Exact delay compensation is achieved with

$$U(t) = KX(r^{-1}(t))$$

provided that

1. D time-differentiable and uniformly bounded
2. r is invertible (i.e. $|\dot{D}| < 1$)

- for a constant delay, $D > 0$, $r^{-1}(t) = t + D$
- $r^{-1}(t) \neq t + D(t)$

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- for a constant delay, $D > 0$, $r^{-1}(t) = t + D$
- $r^{-1}(t) \neq t + D(t)$: for the control law $U(t) = KX(t + D(t))$,

$$\dot{X}(t) = AX(t) + BU(t - D(t)) = AX(t) + BKX(t \underbrace{- D(t) + D(t - D(t))}_{\neq 0 \text{ a priori}})$$

- requires to predict future values of the delay

Input delay compensation : open questions

Constant input delay

$$U(t) = K\left[e^{AD}X(t) + \int_{t-D}^{t} e^{A(t-s)}BU(s)ds\right]$$

Time-varying input delay

$$U(t) = KX(r^{-1}(t)) \quad \text{with} \quad r(t) = t - D(t)$$

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Constant input delay

$$U(t) = K\left[e^{AD}X(t) + \int_{t-D}^{t} e^{A(t-s)}BU(s)ds\right]$$

- robustness to delay mismatch ?

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Input delay compensation : open questions

Constant input delay

$$U(t) = K\left[e^{AD}X(t) + \int_{t-D}^{t} e^{A(t-s)}BU(s)ds\right]$$

- robustness to delay mismatch ?
- robustness to plant uncertainties ?
- disturbance rejection ?
- output feedback ?

Time-varying input delay

$$U(t) = KX(r^{-1}(t)) \quad \text{with} \quad r(t) = t - D(t)$$

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Question

How to design delay-adaptive prediction-based control tackling these issues ?

Input delay compensation : open questions

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$$U(t) = KX(r^{-1}(t)) \quad \text{with} \quad r(t) = t - D(t)$$

- the case of unknown delay variations remains to be addressed
- extension to input-dependent delay

Input delay compensation : open questions

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How to ensure causal robust compensation for time- and input-varying delays ?

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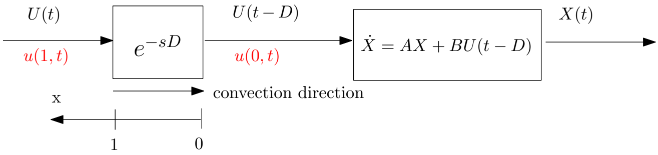
Transport PDE and backstepping transformation [Krstic, 2008]

$$\dot{X}(t) = AX(t) + BU(t - D)$$

- **Hyperbolic PDE delay representation**

consider the distributed actuator
$u(x,t) = U(t + D(x-1)),\ x \in [0;1]$

$$\begin{cases} \dot{X}(t) & = & AX(t) + Bu(0,t) \\ Du_t(x,t) & = & u_x(x,t) \\ u(1,t) & = & U(t) \end{cases}$$



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- **Backstepping transformation** :

  change of variables $(X,u) \to (X,w)$ to obtain the target system

$$\begin{cases} \dot{X}(t) &= (A+BK)X(t) + Kw(0,t) \\ Dw_t(x,t) &= w_x(x,t) \\ w(1,t) &= 0 \end{cases}$$

sol. : $w(x,t) = u(x,t) - Ke^{ADx}X(t) - DK\int_0^x e^{AD(x-y)}Bu(y,t)dy$

$= u(x,t) - KX(t+xD) \quad \Rightarrow u(1,t) = U(t) = KX(t+D)$

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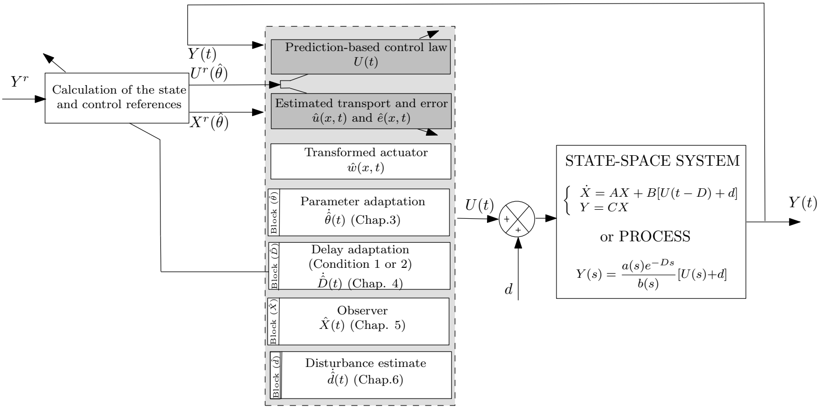
(1)$\Rightarrow$ linear parameterization
(2)$\Rightarrow$ Lyapunov-Krasovskii analysis

General prediction-based delay-adaptive scheme

We investigate robust compensation for different versions of the general linear system

$$\begin{cases} \dot{X}(t) = A(\theta)X(t) + B(\theta)[U(t-D)+d] \\ Y(t) = CX(t) \end{cases}$$



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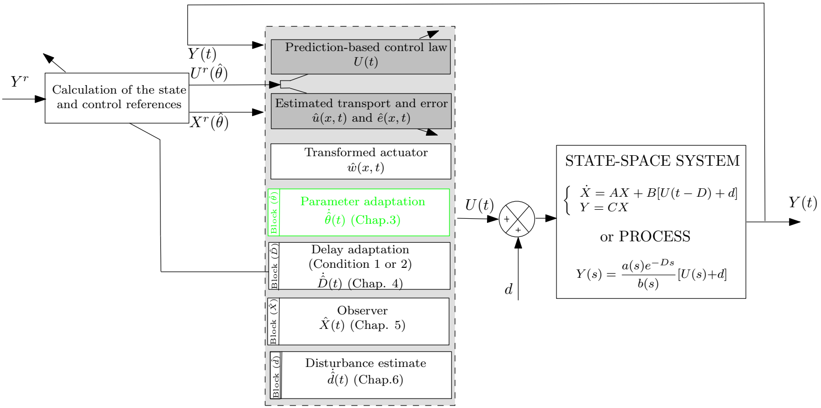

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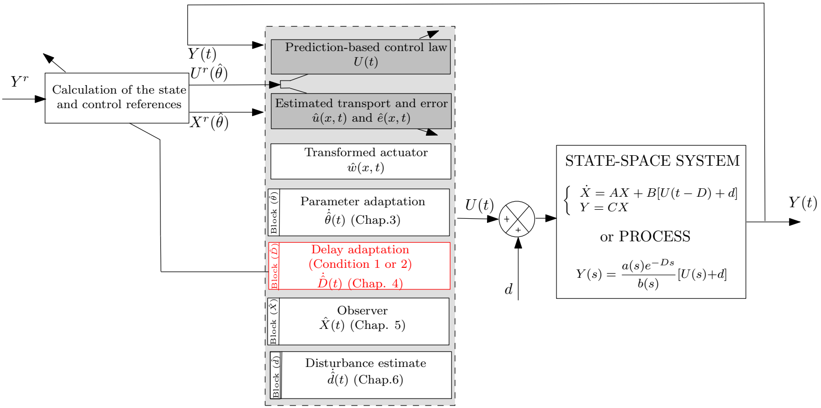

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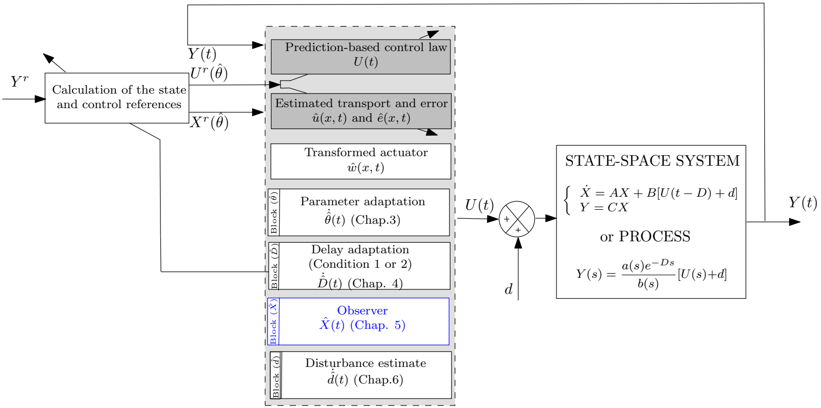

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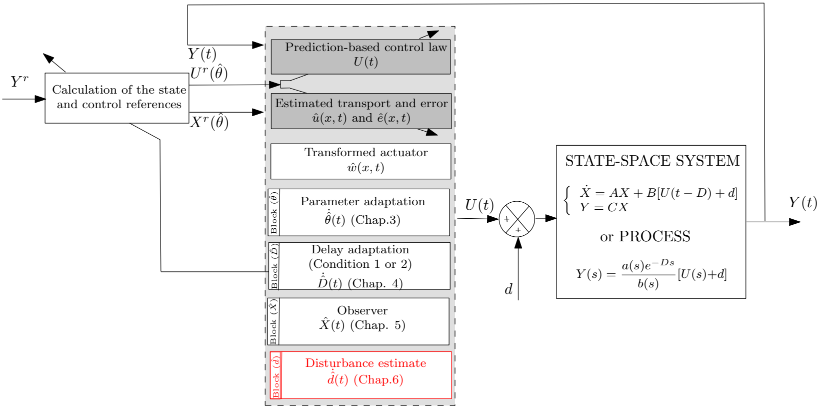

General methodology for Lyapunov analysis

D **uncertain** $\rightarrow$ delay estimate $\hat{D}$

Implementable control law :

$$U(t) = KX_P(t+\hat{D}) \neq KX(t+D)$$

with X_P predicted using estimates

Question

What is the influence of the prediction error on closed-loop stability in case of

1. a time-varying delay update law ?

$$X_P(t+\hat{D}) = X(t+\hat{D}(t))$$

2. plant parameter uncertainties ?

$$X_P(t+\hat{D}) = X_{\hat{\theta}}(t+\hat{D})$$

3. an output feedback strategy ?

$$X_P(t+\hat{D}) = X_{\hat{X}}(t+\hat{D})$$

4. disturbance rejection ?

$$X_P(t+\hat{D}) = X_{\hat{d}}(t+\hat{D})$$

General methodology for Lyapunov analysis

Robustness of $U(t) = KX_P(t+\hat{D}) \neq KX(t+D)$?

Estimation of distributed variables

$$\hat{u}(x,t) = U(t+\hat{D}(t)(x-1)) \quad \rightarrow \text{ control synthesis}$$

$$\hat{w}(x,t) = \hat{u}(x,t) - KX_P(t+x\hat{D}(t)) \quad \rightarrow \text{ Lyapunov analysis}$$

with X_P predicted state using erroneous estimates

Error variable PDEs

$$\begin{cases} \hat{D}(t)\hat{u}_t(x,t) = \hat{u}_x(x,t) + \dot{\hat{D}}(t)(x-1)\hat{u}_x(x,t) \\ \hat{u}(1,t) = U(t) \end{cases}$$

$$\begin{cases} \hat{D}(t)\hat{w}_t(x,t) = \hat{w}_x(x,t) + \dot{\hat{D}}(t)(x-1)\hat{w}_x(x,t) + \psi(x,t) \\ \hat{w}(1,t) = 0 \end{cases}$$

ψ source term aggregating quantities arising from the estimation errors

Lyapunov-Krasovskii analysis for system state stability analysis

study of delay update law and robustness to other mismatches.

Local robust compensation result

Result

Consider the closed-loop system obtained using the control law

$$U(t) = KX(t+\hat{D}(t)) = K\left[e^{A\hat{D}(t)}X(t) + \int_{t-\hat{D}(t)}^{t} e^{A(t-s)}BU(s)ds\right]$$

with a delay estimate $\hat{D}$ satisfying one of the following Growth Conditions. Define

$$\Gamma(t) = |X(t)|^2 + \|u(t)\|^2 + \|\hat{u}(t)\|^2 + \|\hat{u}_x(t)\|^2 + (D-\hat{D}(t))^2$$

There exist $\gamma^* > 0$, $R > 0$ et $\rho > 0$ such that, provided $\Gamma(0) < \rho$ and $\gamma_D < \gamma^*$, then

$$\forall t \geq 0 \quad \Gamma(t) \leq R\Gamma(0)$$

$$X(t) \underset{t\to\infty}{\to} 0 \quad \text{and} \quad U(t) \underset{t\to\infty}{\to} 0$$

- stability result, which can be generalized to tracking

(Lyapunov analysis giving conservative expressions of the bounds γ^* and ρ)

Growth conditions for the delay update law

Growth Condition 1 [Estimation improvement]

There exist $M > 0$ and $\tau_D \in \mathcal{C}^0([0, +\infty[)$ s.t.

$$\dot{\hat{D}}(t) = \gamma_D \text{Proj}_{[\underline{D}, \bar{D}]} \{\tau_D(t)\}$$
$$\forall t \geq 0, \quad \tau_D(t)(D - \hat{D}(t)) \geq 0 \quad \text{and} \quad |\tau_D(t)| < M$$

or

Growth Condition 2 [Rate of change compliant with the overall dynamics]

There exist $M > 0$ and $\tau_D \in \mathcal{C}^0([0, +\infty[)$ s.t.

$$|\dot{\hat{D}}(t)| = \gamma_D \text{Proj}_{[\underline{D}, \bar{D}]} \{\tau_D(t)\}$$
$$|\tau_D(t)| \leq M(|X(t)|^2 + \|u(t)\|^2 + \|\hat{u}_x(t)\|^2)$$

Proof : backstepping transformation and error equations

Backstepping transformation

$$\hat{w}(x,t) = \hat{u}(x,t) - K\left[e^{A\hat{D}(t)x}X(t) + \hat{D}(t)\int_{t-x\hat{D}(t)}^{x} e^{A(t-s)}U(s)ds\right]$$

with $\hat{u}$ satisfying $\hat{D}(t)\hat{u}_t(x,t) = \hat{u}_x(x,t) + \dot{\hat{D}}(t)(x-1)\hat{u}_x(x,t)$

Cascade ODE-PDE

$$\begin{aligned}
\dot{X}(t) =& (A+BK)X(t) + B\hat{w}(0,t) + B[u(0,t) - \hat{u}(0,t)] \\
\hat{D}(t)\hat{w}_t =& \hat{w}_x + \dot{\hat{D}}(t)g(x,t) - \hat{D}(t)Ke^{A\hat{D}(t)x}B[u(0,t) - \hat{u}(0,t)] \\
\hat{w}(1,t) =& 0
\end{aligned}$$

Asymptotically stable dynamics with additive terms :

- one coupling term
- two estimation error terms $u(0,t) - \hat{u}(0,t)$ ($\dot{\hat{D}}$ at first order)
- one source term in $\dot{\hat{D}}$ with g function of $X(t), \hat{w}(\xi,t), \hat{w}_x(\xi,t)$ ($\xi \in [0,x]$)

Proof : Lyapunov-Krasovskii analysis

- Consider the candidate functional

$$V(t) = X(t)^T P X(t) + b_1 D \int_0^1 (1+x)(u(x,t) - \hat{u}(x,t))^2 dx + b_2 \hat{D}(t) \int_0^1 (1+x)\hat{w}(x,t)^2 dx + b_2 \hat{D}(t) \int_0^1 (1+x)\hat{w}_x(x,t)^2 dx + \tilde{D}(t)^2$$

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- Taking a time-derivative of J and using the error equation gives

$$dJ/dt = 2\int_0^1 (1+x) \underbrace{\hat{D}(t)\hat{w}_t(x,t)}_{=\hat{w}_x + \ldots} \hat{w}(x,t) dx + \dot{\hat{D}}(t) \int_0^1 (1+x)\hat{w}(x,t)^2 dx$$

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$$\overset{IBP}{=} [(1+x)\hat{w}(x,t)^2]_0^1 - \|\hat{w}(t)\|^2 - 2\dot{\hat{D}}(t) \int_0^1 (1+x)g(x,t)\hat{w}(x,t)dx$$
$$-2\hat{D}(t) \int_0^1 (1+x)Ke^{A\hat{D}(t)x} B[u(0,t)-\hat{u}(0,t)]\hat{w}(x,t)dx$$
$$+\dot{\hat{D}}(t) \int_0^1 (1+x)\hat{w}(x,t)^2 dx$$

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$$\overset{IBP}{=} -\hat{w}(0,t)^2 - \|\hat{w}(t)\|^2 - 2\dot{\hat{D}}(t)\int_0^1 (1+x)g(x,t)\hat{w}(x,t)dx$$
$$-2\hat{D}(t)\int_0^1 (1+x)Ke^{A\hat{D}(t)x}B[u(0,t)-\hat{u}(0,t)]\hat{w}(x,t)dx$$
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- With Young's inequality and the expression of g, one obtains

$$dJ/dt \leq -\hat{w}(0,t)^2 - \frac{1}{2}\|\hat{w}(t)\|^2 + M_1|\dot{\hat{D}}(t)|V_0(t) + M_2|u(0,t) - \hat{u}(0,t)|^2$$

with $V_0(t) = |X(t)|^2 + \|\hat{w}(t)\|^2 + \|\hat{w}_x(t)\|^2$

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$$+ b_2 \hat{D}(t) \int_0^1 (1+x)\hat{w}(x,t)^2 dx + b_2 \hat{D}(t) \int_0^1 (1+x)\hat{w}_x(x,t)^2 dx + \tilde{D}(t)^2$$

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$$dJ/dt \leq -\hat{w}(0,t)^2 - \frac{1}{2}\|\hat{w}(t)\|^2 + M_1 |\dot{\hat{D}}(t)| V_0(t) + M_2 |u(0,t) - \hat{u}(0,t)|^2$$

with $V_0(t) = |X(t)|^2 + \|\hat{w}(t)\|^2 + \|\hat{w}_x(t)\|^2$

- Acting similarly with other modified $\mathcal{L}_2$-norms and judiciously choosing the intermediate constants b_1 and b_2 yields either

$$\dot{V}(t) \leq -(\eta_1 - \gamma_D M_2 V(t))\, V_0(t) \quad \text{for Growth Condition 1}$$

or $\dot{V}(t) \leq -(\eta_2 - \gamma_D M_3 V(t))\, V_0(t) - (\eta_3 - \gamma_D M_4)\, V_0(t)^2$ for Growth Condition 2

Proof : Lyapunov-Krasovskii analysis

- Consider the candidate functional

$$V(t) = X(t)^T P X(t) + b_1 D \int_0^1 (1+x)(u(x,t)-\hat{u}(x,t))^2 dx$$
$$+ b_2 \hat{D}(t) \int_0^1 (1+x)\hat{w}(x,t)^2 dx + b_2 \hat{D}(t) \int_0^1 (1+x)\hat{w}_x(x,t)^2 dx + \tilde{D}(t)^2$$

- With Young's inequality and the expression of g, one obtains

$$dJ/dt \leq -\hat{w}(0,t)^2 - \frac{1}{2}\|\hat{w}(t)\|^2 + M_1|\dot{\hat{D}}(t)|V_0(t) + M_2|u(0,t)-\hat{u}(0,t)|^2$$

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- Stability follows by choosing $V(0)$ and γ_D sufficiently small. Convergence is obtained by applying Barbalat's Lemma to $|X(t)|^2$ and $U(t)^2$

General prediction-based adaptive control strategy

D **uncertain** $\rightarrow$ delay estimate $\hat{D}$ and $U(t) = KX_P(t+\hat{D}(t)) \neq KX(t+D)$

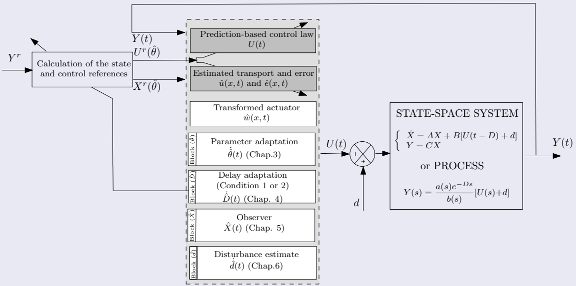
Question

What is the influence of the prediction error on closed-loop stability in case of

1. a time-varying delay update law ?
2. plant parameter uncertainties ?
3. an output feedback strategy ?
4. disturbance rejection ?

Answer

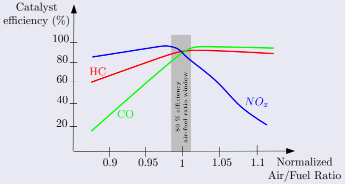
Robust compensation can be achieved for various combinations of the blocks below



Application to Fuel-to-Air Ratio (FAR) for SI engines

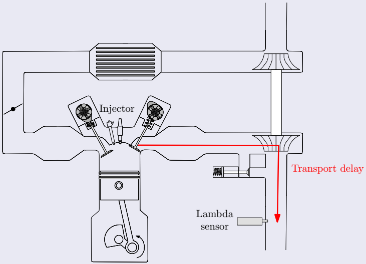
Stoichiometric blend for SI engines

- Catalyst conversion efficiency optimal at stoichiometry



Classical FAR control architecture

- air path dedicated to the driver torque request
- fuel path adjusted (manipulated variable = injected fuel mass)
- feedback loop using an oxygen sensor (Lambda sensor)

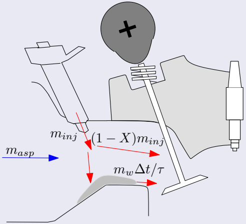


Application to Fuel-to-Air Ratio (FAR) for SI engines

Define the normalized Fuel-to-Air Ratio as follows

$$\phi = \frac{1}{FAR_S} \frac{m_f}{m_{asp}} \quad \text{and} \quad \phi_m \quad \text{the exhaust FAR measurement}$$

Modeling of the composition from the intake chamber to Lambda sensor



- Dilution and transport delay
- Sensor dynamics
- Wall-wetting dynamics (indirect injection)
- input = m_{inj}, output = ϕ_m

Application to Fuel-to-Air Ratio (FAR) for SI engines

Modeling (cont.)

Defining the control variable as $U(t) = \frac{1}{FAR_S} \frac{m_{inj}^{sp}}{m_{asp}^{est}(t)}$, one obtains

$$\begin{cases} \tau_\phi \tau \ddot{\phi}_m(t) + (\tau_\phi + \tau)\dot{\phi}_m + \phi_m = \theta(t)\left[\tau(1-X)\dot{U}(t-D) + U(t-D)\right] \\ y(t) = \phi_m \\ \theta = \theta(N_e, F_{air}) \\ D = D(N_e, F_{air}) \end{cases}$$

with F_{air} unreliable

- an unknown gain $\theta \in [0.75, 1.25]$, that varies with the operating point (and extremely slowly over time)
- an uncertain input time delay, estimated by $\hat{D} = \hat{D}(N_e)$ in $[\underline{D}, \bar{D}] = [100, 600]$ ms
- only one available measurement, ϕ_m

$\Rightarrow$ combination of block $(\hat{X})$ and block $(\hat{\theta})$

Application to Fuel-to-Air Ratio (FAR) for SI engines

Modeling (cont.)

Defining the control variable as $U(t) = \frac{1}{FAR_S} \frac{m_{inj}^{sp}}{m_{asp}^{est}(t)}$, one obtains

$$\begin{cases} \dot{X}(t) = AX(t) + B(\theta)U(t-D) \\ Y(t) = CX(t) \\ A = \begin{pmatrix} 0 & 1 \\ -\frac{1}{\tau_\phi \tau} & -\frac{1}{\tau_\phi} - \frac{1}{\tau} \end{pmatrix}, \quad B(\theta) = \begin{pmatrix} 0 \\ \frac{\theta}{\tau \tau_\phi} \end{pmatrix} \quad \text{and} \quad C = (1 \quad \tau(1-X)) \end{cases}$$

The reference trajectories are $(X^r, U^r(\hat{\theta})) = ([\phi^r \quad 0]^T, \phi^r/\hat{\theta})$, with

- an unknown gain $\theta \in [0.75, 1.25]$, that varies with the operating point (and extremely slowly over time)
- an uncertain input time delay, estimated by $\hat{D} = \hat{D}(N_e)$ in $[\underline{D}, \bar{D}] = [100, 600]$ ms
- only one available measurement, ϕ_m

$\Rightarrow$ combination of block $(\hat{X})$ and block $(\hat{\theta})$

Application to Fuel-to-Air Ratio (FAR) for SI engines

Controller

- **Control law**

$$U(t) = U^r(\hat{\theta}) - KX^r + K\left[e^{A\hat{D}}\hat{X}(t) + \int_{t-\hat{D}}^{t} e^{A(t-s)}B(\hat{\theta})U(s)ds\right]$$

- **Observer**

$$\dot{\hat{X}}(t) = A\hat{X}(t) + B(\hat{\theta})\hat{u}(0,t) - L(Y(t) - C\hat{X}(t))$$

- **Update law (Lyapunov design)**

$$\dot{\hat{\theta}}(t) = \gamma\left[\frac{(\hat{X}(t) - X^r)^T P(\hat{\theta})}{b} - \hat{D}K(\hat{\theta})\int_0^1 (1+x)[\hat{w}(x,t) + A\hat{D}\hat{w}_x(x,t)]e^{A\hat{D}x}dx\right]\begin{pmatrix} 0 \\ \frac{\phi^r}{\hat{\theta}\tau\tau_\phi} \end{pmatrix}$$

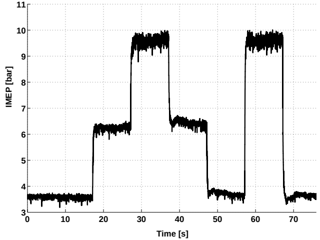
- **Backstepping transformation of** $\hat{e}(x,t) = \hat{u}(x,t) - U^r(\hat{\theta})$

$$\forall x \in [0,1], \quad \hat{w}(x,t) = \hat{e}(x,t) - \hat{D}\int_0^x Ke^{A\hat{D}(x-y)}B(\hat{\theta})\hat{e}(y,t)dy - Ke^{A\hat{D}x}\left[\hat{X}(t) - X^r\right]$$

FAR, experimental results

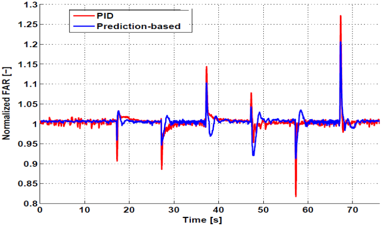
Test-bench experiments

- Experimental setup
  - 1.4L four-cylinder SI engine
  - indirect injection
- Comparison between
  - the proposed prediction-based strategy
  - a PID controller
- Constant feedback gains K, L and update gain γ_D
- Torque variations at constant engine speed

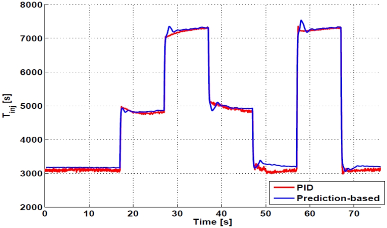


(c) IMEP [bar]

FAR, experimental results

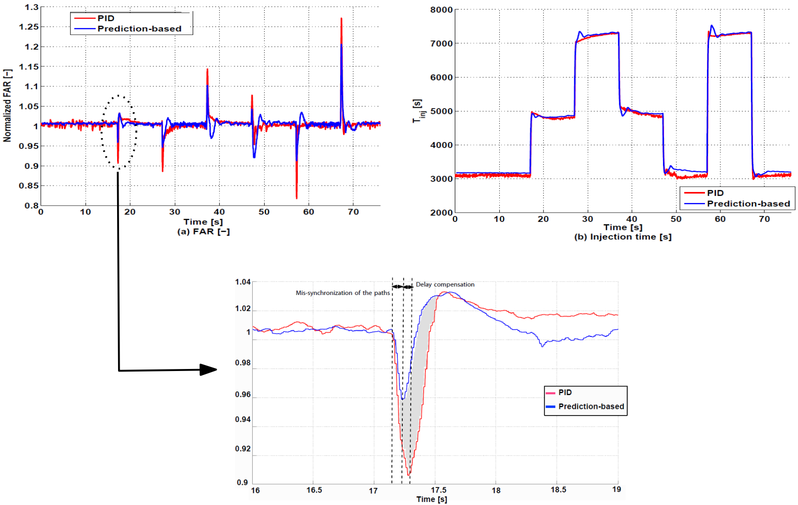


(a) FAR [−]



(b) Injection time [s]

FAR, experimental results



Outline

1. Introduction : state prediction
2. Adaptive control scheme for uncertain systems with constant input delay
   - Delay-adaptive methodology principle
   - Control strategy with delay update law
   - Case study for a SI engine : FAR regulation
3. Robust compensation of a class of time- and input-dependent input delays
   - Integral transport delay class and application to EGR estimation
   - Sufficient conditions for transport delay robust compensation
4. Conclusion

Transport delay integral model

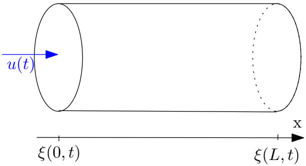
$$\int_{t-D(t)}^{t} \varphi(s, u(s))ds = 1$$

with φ a **strictly positive** function of time and of the input

Transport between $x = 0$ and L with a speed $u(t)$ of a variable ξ

The solution of the equation $\xi_t = u(t)\xi_x$ satisfies $\xi(L,t) = \xi(0, t - D(t))$ with $D(t)$ defined by

$$\int_{t-D(t)}^{t} \underbrace{\frac{u(s)}{L}}_{=\varphi(u(s))} ds = 1$$



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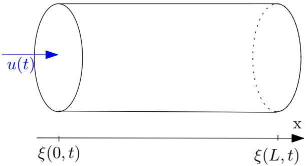
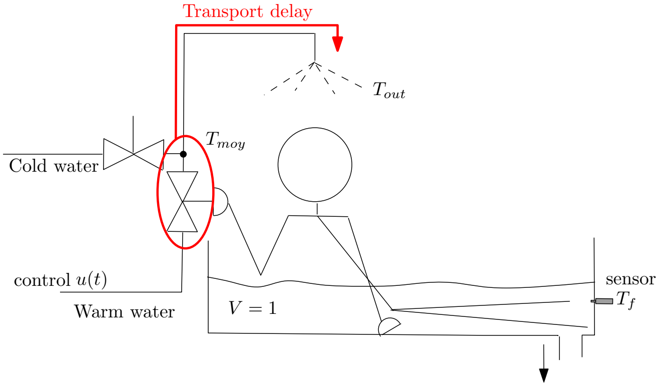


- $D > 0$, as φ positive
- $\dot{D} \leq 1$ (causal), as φ positive and

$$\dot{D}(t) = 1 - \frac{\varphi(t, u(t))}{\varphi(t - D(t), u(t - D(t)))} \leq 1$$

- $D(t)$ can be calculated numerically : $D \mapsto \int_{t-D}^{t} \varphi(.)ds$ is a class $\mathcal{K}$ function

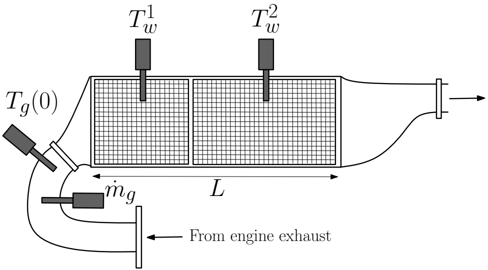
Example 1 : the bath/shower



$T_{out} = T_{moy}(t - D(t))$ with

$$\int_{t-D(t)}^{t} \underbrace{\frac{(1+u(s))}{V_P}}_{=\varphi(u(s))} ds = 1 \qquad \Rightarrow \text{input-dependent delay}$$

Example 2 : catalyst internal temperature



- control = inlet gas temperature (and the mass flow rate for hybrid vehicle)
- state = (distributed) temperature
- residence time

From a low-frequencies analysis of the PDE thermal model, one can obtain

$$T_w(L,t) = \frac{1}{1+\tau s} T_g(0, t - D(t))$$

with ($k_1, k_2 > 0$ given constants)

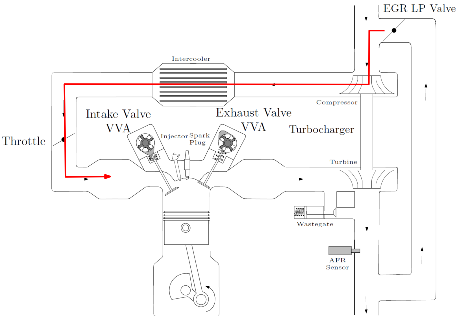
$$\int_{t-D(t)}^{t} \underbrace{\frac{k_1}{k_2} \frac{\dot{m}_g(s)}{L}}_{=\varphi(s,u(s))} ds = 1 \qquad \Rightarrow \text{ time- and input-dependent delay}$$

Example 3 : exhaust gas recirculation (EGR) for SI engine

- state = intake burned gas rate
- control = reintroduced burned gas mass flow rate

Input delay defined as

$$\int_{t-D(t)}^{t} \underbrace{\frac{v_{gas}(s)}{L}}_{=\varphi(s,u(s))} ds = 1 \qquad \Rightarrow \text{ time- and input-dependent delay}$$



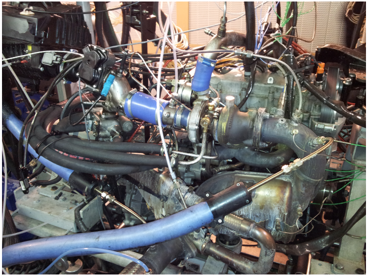


FIG.: IFPEn test-bench, 1.8L RSA F5Rt

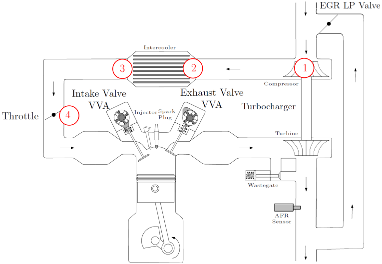
Example 4 : EGR for SI engine, practical implementation

v_{gas} infinite dimensional and not measured

Practical delay calculation by perfect gas law fed by measurements

$$\int_{t-D(t)}^{t} v_{gas}(s)ds = L_P \quad \Rightarrow \quad \int_{t-D(t)}^{t} \frac{rT(s)}{P(s)}[F_{air}(s) + F_{egr}(s)]ds = V_P$$

- $1 \longrightarrow 2$: homogeneous temperature and pressure (measured)
- $2 \longrightarrow 3$: homogeneous pressure and linear temperature profile
- $3 \longrightarrow 4$: intake manifold temperature (measured)



three delays calculated sequentially by direct iterations

Example 4 : EGR for SI engine, practical implementation

Intake burned gas rate model

Define x_{lp} the low-pressure burned gas rate

$$\begin{aligned} \dot{x}_{lp} =& \alpha \left[-(F_{egr}(t) + F_{air}(t)) x_{lp}(t) + F_{egr}(t) \right] \\ x(t) =& x_{lp}(t - D(t)) \end{aligned}$$

with $D(t)$ the transport delay defined earlier and α a known constant (ambiant conditions)

Open-loop burned gas rate estimate $\hat{x}$

aspirated mass flow rate model
measurement of the low-pressure fresh air mass flow rate $\Big\}$ $\Rightarrow \hat{x}_{lp}$

delay calculated by the previous methodology $\Rightarrow \hat{x}$

Example 4 : EGR for SI engine, practical implementation

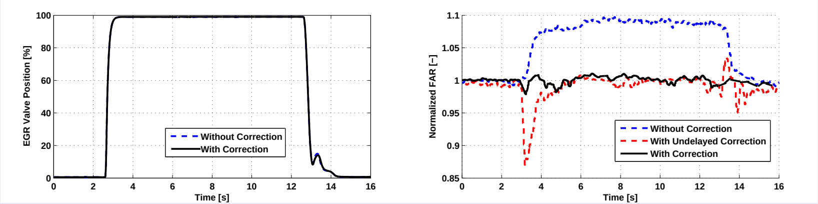
Experimental set-up

- 1.8L four cylinder turbocharged SI engine
- no real-time intake burned gas rate measurement → use of Lambda sensor
- *indirect* validation methodology through estimation-based feedforward strategy on the FAR

$$m_{inj} = FAR_S m_{air}^{est} = FAR_S (1 - \hat{x}) m_{asp}$$

FAR staying close to 1 ⟷ accurate estimate

Experimental results for a given operating point ($N_e = 2000$ rpm and $IMEP = 8$ bar)



A general problem statement

Problem

To design a prediction-based control law for the (potentially unstable) plant

$$\begin{cases} x^{(n)} + a_{n-1}x^{(n-1)} + \ldots + a_0 x = b_0 u(t - D(t)) \\ \displaystyle\int_{t-D(t)}^{t} u(s)ds = 1 \quad \text{with} \quad u \geq \underline{u} > 0 \end{cases}$$

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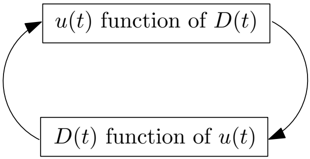
$$\begin{cases} x^{(n)} + a_{n-1}x^{(n-1)} + \ldots + a_0 x = b_0 u(t - D(t)) \\ \int_{t-D(t)}^{t} u(s)ds = 1 \quad \text{with} \quad u \geq \underline{u} > 0 \end{cases}$$

The control law

$$u(t) = KX(r^{-1}(t)) \quad \text{with} \quad r(t) = t - D(t)$$

achieves exact delay compensation but results into an implicit loop.



A general problem statement

Problem

To design a prediction-based control law for the (potentially unstable) plant

$$\begin{cases} x^{(n)} + a_{n-1}x^{(n-1)} + \ldots + a_0 x = b_0 u(t - D(t)) \\ \int_{t-D(t)}^{t} u(s)ds = 1 \quad \text{with} \quad u \geq \underline{u} > 0 \end{cases}$$

Control : $u(t) = KX(t + D(t)) \neq KX(r^{-1}(t))$

State-space representation : **assume that X is fully known**

$$A = \begin{pmatrix} 0 & 1 & & 0 \\ \vdots & & \ddots & \\ 0 & 0 & & 1 \\ -a_0 & -a_1 & \ldots & -a_{n-1} \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ b_0 \end{pmatrix}$$

Question

Under which conditions is the control law $u(t) = KX(t + D(t))$ stabilizing ?
(without exact delay compensation)

Methodology

Question

Under which conditions is the control law $u(t) = KX(t + D(t))$ stabilizing ?

1st step : find a condition on the delay variations $D(t)$

Lyapunov-Krasovskii analysis (backstepping)

2nd step : relate delay variations to the input and study the input dynamics

DDE stability results (Halanay)

1st step : find a condition on the delay variations $D(t)$

Robust compensation of a time-varying delay

Consider the closed-loop system

$$\begin{aligned} \dot{X}(t) &= AX(t) + BU(t - D(t)) \\ U(t) &= U^r - KX^r + K\left[e^{AD(t)}X(t) + \int_{t-D(t)}^{t} e^{A(t-s)}BU(s)ds\right] \end{aligned} \quad (1)$$

with U scalar, K s.t. $A + BK$ Hurwitz and (X^r, U^r) an equilibrium point.
$D : \mathbb{R}_+ \mapsto [0, \overline{D}]$ is assumed to be time-differentiable.

There exists $\delta^* \in]0,1[$ such that, provided $\forall t \geq 0\,, |\dot{D}(t)| \leq \delta^*$, then (1) exponentially converges to X^r.

1st step : find a condition on the delay variations $D(t)$

Robust compensation of a time-varying delay

Consider the closed-loop system

$$\begin{aligned} \dot{X}(t) &= AX(t) + BU(t-D(t)) \\ U(t) &= U^r - KX^r + K\left[e^{AD(t)}X(t) + \int_{t-D(t)}^{t} e^{A(t-s)}BU(s)ds\right] \end{aligned} \quad (1)$$

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$D : \mathbb{R}_+ \mapsto [0, \overline{D}]$ is assumed to be time-differentiable.

There exists $\delta^* \in]0,1[$ such that, provided $\forall t \geq 0, |\dot{D}(t)| \leq \delta^*$, then (1) exponentially converges to X^r.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t-D(t)) \\ &= Ax(t) + BKx(t - \underbrace{D(t) + D(t-D(t))}_{\approx 0 \text{ if } |\dot{D}| \text{ small}}) \end{aligned}$$

Methodology

Question

Under which conditions is the control law $u(t) = KX(t + D(t))$ stabilizing ?

1st step : find a condition on the delay variations $D(t)$

Lyapunov-Krasovskii analysis (backstepping)

Solved : get $|\dot{D}(t)| < \delta^*$, $t \geq 0$

2nd step : relate delay variations to the input and study the input dynamics

DDE stability results (Halanay)

2nd step : relate delay variations to the input and study the input dynamics

$$\int_{t-D(t)}^{t} u(s)ds = 1 \,, \quad u \geq \underline{u} > 0$$

implies

$$\dot{D}(t) = 1 - \frac{u(t-D(t))}{u(t)} = \frac{\varepsilon(t) - \varepsilon(t-D(t))}{u(t)}$$

Sufficient condition for $|\dot{D}(t)| < \delta^*$

$$\max |\varepsilon_t| \leq \frac{\underline{u}\delta^*}{2}$$

with $\varepsilon(t) = u(t) - u^r$ the error variable and $\varepsilon_t : s \in [-\overline{D}, 0] \mapsto \varepsilon(t+s)$

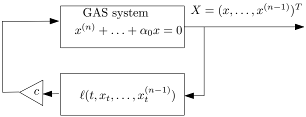
Problem reformulation

To guarantee the condition

$$\max |\varepsilon_t| \leq \delta = \frac{\underline{u}\delta^*}{2}$$

2nd step : relate delay variations to the input and study the input dynamics

We use the following stability result.



Halanay (local)

Let x be a solution of the n^{th} order DDE

$$\begin{cases} x^{(n)} + \alpha_{n-1} x^{(n-1)} + \ldots + \alpha_0 x = c\ell(t, x_t, \ldots x_t^{(n-1)}), \, t \geq t_0 \\ x_{t_0} = \psi \in \mathcal{C}^0([-\overline{D}, 0], \Omega) \end{cases}$$

where the left-hand side of the differential equation defines a polynomial which roots have only strictly negative real parts, $c > 0$, ℓ is a continuous functional and Ω is a neighborhood of the origin in which ℓ satisfies the sup-norm relation

$$\forall t \geq t_0 \,, \quad |\ell(t, x_t, \ldots, x_t^{(n-1)})| \leq \max |X_t| \tag{2}$$

with $X = [x \quad \dot{x} \ldots x^{(n-1)}]^T$. Then, there exists $c_{max} > 0$ such that, for any $0 \leq c \leq c_{max}$, there exists $\gamma \geq 0$ and $r > 0$ such that

$$\forall t \geq 0 \,, \quad |X(t)| \leq r \max |X_{t_0}| e^{-\gamma(t-t_0)}$$

2nd step : relate delay variations to the input and study the input dynamics

ε dynamics

The error variable $\varepsilon = u - u^r$ with u defined through a prediction on the horizon $D(t)$ satisfies the differential equation

$$\varepsilon^{(n)} + (a_{n-1} + b_0 k_{n-1})\varepsilon^{(n-1)} + \ldots + (a_0 + b_0 k_0)\varepsilon = \pi_0(\varepsilon_t, \ldots, \varepsilon_t^{(n-1)}) + \pi_1(\varepsilon_t, \ldots, \varepsilon_t^{(n-1)})$$

where $[-k_0 \ldots - k_{n-1}] \triangleq K$ and π_0 and π_1 are polynomial functions s.t.

- there exists a class $\mathcal{K}_\infty$ function β such that $|\pi_0(\varepsilon_t, \ldots, \varepsilon_t^{(n-1)})| \leq \beta(|K|) \max |E_t|$ with $E(t) = [\varepsilon(t) \quad \dot{\varepsilon}(t) \ldots \varepsilon^{(n-1)}(t)]^T$
- π_1 is *at least quadratic* in $\varepsilon_t, \ldots, \varepsilon_t^{(n-1)}$

We apply the previous result $X = E$, $\ell = \pi_0 + \pi_1$. To guarantee the property (2),

- we decrease the gain magnitude $|K| < k^*$ such that $|\pi_0| < (1-\varepsilon)c_{max} \max |E_t|$
- we decrease the open set Ω such that $|\pi_1| < \varepsilon c_{max} \max |E_t|$

then $\ell < c_{max} \max |E_t|$ and one can apply Halanay, to guarantee the exponential convergence of ε

Transport delay robust compensation

Local small gain condition

Consider the closed-loop system

$$\begin{cases} x^{(n)} + a_{n-1}x^{(n-1)} + \ldots + a_0 x = b_0 u(t - D(t)) \quad \text{with} \quad \int_{t-D(t)}^{t} u(s)ds = 1 \\ u(t) = u^r + K\left[e^{AD(t)}X(t) + \int_{t-D(t)}^{t} e^{A(t-s)}B\phi(s)ds - X^r\right] \end{cases}$$

Consider the functional $\Theta(t) = |X(t) - X^r| + \max_{s\in[t-\overline{D},t]} |U(s) - U^r|$ and Q a symmetric positive definite matrix. Assume that, for a given $\varepsilon \in (0,1)$, there exists $k^* > 0$ s. t.

$$\beta(|K_0|) < (1-\varepsilon)\frac{\underline{\lambda}(P)\underline{\lambda}(Q)}{2\overline{\lambda}(P)^2} \quad \text{with} \quad P(A+BK_0)+(A+BK_0)^T P = -Q$$

for any $K_0 \in \mathbb{R}^{1\times n}$ such that $|K_0| < k^*$, with β defined in terms of by A and B. Then, there exists $\theta : \mathbb{R}_+ \mapsto \mathbb{R}_+$ such that for any $K \in \mathbb{R}^{1\times n}$ such that $|K| < k^*$ and $\Theta(0) < \theta(|K|)$ the considered condition is fulfilled and the plant exponentially converges to X^r.

Methodology

Question

Under which conditions is the control law $u(t) = KX(t + D(t))$ stabilizing ?

1st step : find a condition on the delay variations $D(t)$

Lyapunov-Krasovskii analysis (backstepping)

Solved : get $|\dot{D}(t)| < \delta^*$, $t \geq 0$

2nd step : relate delay variations to the input and study the input dynamics

DDE stability results (Halanay)

Solved : independent n^{th} order scalar delay equation

Answer

Small gain condition

Outline

1. Introduction : state prediction
2. Adaptive control scheme for uncertain systems with constant input delay
   - Delay-adaptive methodology principle
   - Control strategy with delay update law
   - Case study for a SI engine : FAR regulation
3. Robust compensation of a class of time- and input-dependent input delays
   - Integral transport delay class and application to EGR estimation
   - Sufficient conditions for transport delay robust compensation
4. Conclusion

Conclusion

Scope of the thesis

Focus on the design of robust compensation for uncertain and time-varying delays

Constant input delay

How to design delay-adaptive prediction-based control ?

Proposed general adaptive scheme

- based on a delay transport PDE representation and a backstepping transformation
- versatile control strategy
- illustrated experimentally on FAR regulation

Time-varying input-delay

How to ensure causal robust compensation for time- and input-varying delays ?

Proposed robust compensation method

- prediction over a time horizon equal to the delay
- two-steps methodology for input-dependent delay, based on Halanay DDE stability results
- focus on integral transport delay model (experimentally tested for EGR)
- small gain condition (detuning, robustness filter)