PDE analysis and control – Practical lab/ Exercice sessions MISCIT UJF 2016-2017
Modeling and control of an automotive catalyst

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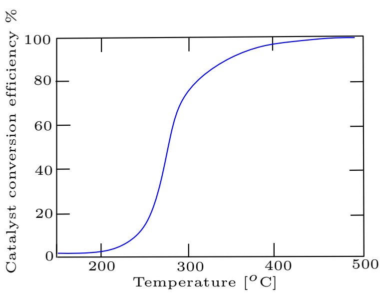


Figure 1: Conversion efficiency depending on a catalyst average temperature. [Heywood, 1998].

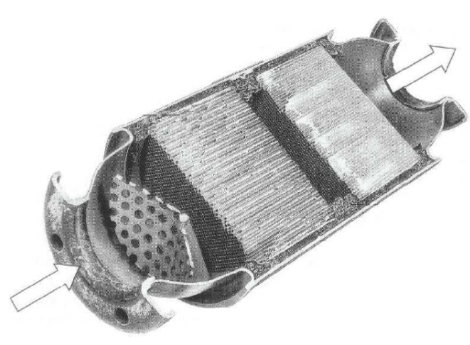
Automotive Gasoline engines are equipped with a Three-Way Catalyst (TWC) located in the exhaust line. This after-treatment device aims at reducing the three major pollutants contained in the gas flowing through it and resulting from the combustion (hydrocarbons HC, carbon monoxide CO and nitrogen oxide NO_x). Yet, conversion efficiency highly depends on the catalyst temperature, as presented in Figure 1. Right after a cold start of the engine, temperatures are too low to activate chemical reactions and the catalyst conversion ratio is poor. Therefore, speed-up of the catalyst warm-up is a point of critical importance to reach high level of pollutant conversion.

We consider here the problem of controlling this warm-up. Various strategies can be performed.

One of them consists in adding an electrical resistance to the catalyst to provide an additive energy supply (see Figure 2). This strategy considerably shortens the duration of the warm-up phase but represents an additional cost and is only chosen by a few automotive companies (Alpina, BMW [not the best example lately...]).

Another standard strategy consists in degrading temporarily the efficiency of the combustion propelling the vehicle, in order to increase the temperature of the exhaust gas flowing through the catalyst.

We want to study here how to operate such strategies, namely, how to choose the resistance characteristics and its operating point or the inlet gas temperature, depending on the considered warm-up solution.



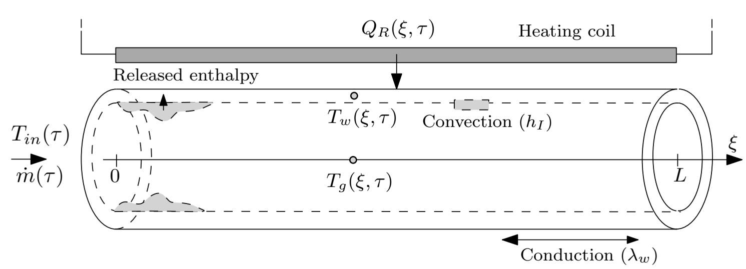


Figure 2: Left: Transversal view of a standard catalyst, composed of two monoliths. Right: Schematic view of a catalyst equipped with a warm-up resistance (represented here externally for the sake of simplicity) and view of the main thermal exchanges.

In the sequel, the questions followed by a $*$ require the use of Matlab/Simulink.

1 Modeling

Assume that:

- the catalyst is cold (no chemical reaction occurs inside it and no enthalpy is released);
- radial and angular dependencies can be neglected;
- the mass flow rate of the gas does not vary along the catalyst and is constant over time.

Then, denoting ξ the longitudinal position and τ the current time, the dynamics of the monolith/gas temperatures can be described by the following equations

$$\begin{cases} \dfrac{\partial T_w}{\partial \tau}(\xi,\tau) = \alpha_w \dfrac{\partial^2 T_w}{\partial \xi^2}(\xi,\tau) + \beta_w (T_g(\xi,\tau) - T_w(\xi,\tau)) + \gamma_w Q_R(\xi,\tau) \\ \dfrac{\partial T_g}{\partial \tau}(\xi,\tau) = -\alpha_g \dfrac{\partial T_g}{\partial x}(\xi,\tau) + \beta_g (T_w(\xi,\tau) - T_g(\xi,\tau)) \\ \dfrac{\partial T_w}{\partial \xi}(0,t) = \dfrac{\partial T_w}{\partial \xi}(L,t) = 0 \\ T_g(0,\tau) = T_{in}(\tau) \end{cases} \tag{1}$$

in which:

- T_w is the distributed temperature of the monolith;
- T_g the one of the gas flowing into it;
- Q_R is the heat released by the external resistance (if present);
- $\dot{m}$ is the gas mass flow rate; and
- $\alpha_w, \alpha_g, \beta_w, \beta_g$ and γ_w are positive variables that are considered constant here.

1. Explain the physical meaning of this model, including its boundary conditions.

2.* Compare this model with the one implemented in your Simulink model. Explain the discretization methodology employed there.

3. Show that (1) can be rewritten as

$$\begin{cases} u_t(x,t) = u_{xx}(x,t) + a(v-u)(x,t) + b f_R(x,t) \\ v_t(x,t) = -c v_x(x,t) + d(u-v)(x,t) \\ u_x(0,t) = u_x(1,t) = 0 \\ v(0,t) = T_{in}(t) \end{cases} \tag{2}$$

In particular, exhibit the transformation allowing this reformulation and the expressions of a, b, c and d depending on $\alpha_w, \alpha_g, \beta_w, \beta_g, \gamma_w$ and L.

2 First control solution: additional resistance

In this context, one can reasonably neglect the dynamics of the gas temperature compared to the heat provided by the resistance. This leads to the following simplified dynamics

$$\begin{cases} u_t(x,t) = u_{xx}(x,t) - au(x,t) + bf(x,t) \\ u_x(0,t) = u_x(1,t) = 0 \end{cases} \tag{3}$$

in which we introduced $f(x,t) = f_R(x,t) + \frac{a}{b}v(x,t) \approx f_R(x,t)$. In this set-up, the catalyst is equipped with an internal sensor located at a position x_0 and which can be modeled as

$$y(t) = \frac{1}{2\varepsilon}\int_{x_0-\varepsilon}^{x_0+\varepsilon} u(x,t)dx \tag{4}$$

where ε is supposed to be small enough.

2.1 Analysis

4. Rewrite (3) as the abstract problem defined on $\mathscr{L}_2([0,1],\mathbb{R})$

$$\begin{cases} \dot{X}(t) = \mathscr{A}\cdot X(t) + \mathscr{B}\cdot U(\cdot,t) \\ y(t) = \mathscr{C}X(t) \end{cases} \tag{5}$$

   Specify the dimension of X, the definition of the operators $\mathscr{A}$, $\mathscr{B}$ and $\mathscr{C}$ and the domain of definition $\mathscr{D}(\mathscr{A})$ of the operator $\mathscr{A}$.

5. What are the eigenvalues λ_n of the operator $\mathscr{A}$? Are they simple? What are corresponding eigenvectors φ_n?

6. Show that $(\varphi_n)_{n\in\mathbb{N}}$ can be chosen s.t. it is an orthonormal basis.

7. How can you then express the solution of (9) (for a given initial condition X_0)?

8.* Implement a truncation of this solution in your Simulink model and compare it with the model output. How does the truncation order influence your results?

9. Justify why a control action is needed for this problem from an applicative point of view.

10. Show that the abstract problem (9) is controllable (specify in which sense).

11. Study if the abstract problem (9) is observable (specify in which sense) according to the values of x_0 and ε.

2.2 Output-feedback control

12. Consider the Lyapunov functional candidate

$$V_1(t) = \int_0^1 u(x)^2 dx \tag{6}$$

   Design a full-state control action f to improve the exponential decay rate.

13.* Implement this control law on your Simulink model. Compare with your theoretical results.

14. As only the output (4) is measured in practice, we consider the following observer

$$\begin{cases} \hat{u}_t(x,t) = \hat{u}_{xx}(x,t) - a\hat{u}(x,t) + bf(x,t) + l\left(y(t) - \frac{1}{2\varepsilon}\int_{x_0-\varepsilon}^{x_0+\varepsilon} \hat{u}(x,t)dx\right) \\ \hat{u}_x(0,t) = \hat{u}_x(1,t) = 0 \end{cases} \tag{7}$$

   in which l is a constant gain. Formulate the dynamics of the observation error $\tilde{u} = u - \hat{u}$ and determine its points spectrum depending on the value of l.

15. Which value of l is thus suitable for observation?

16.* Implement your observer in Simulink.

17.* Implement now the corresponding output-feedback control strategy.

3 Second control solution: warm-up through the inlet gas temperature

We now aim at studying the second and most standard control strategy, which consists in heating the monolith through convection with the gas. In this context, the now control variable is $U(t) = T_{in}(t)$, no internal resistance is present (namely, $f = 0$) and the output gas temperature is measured, i.e.,

$$y(t) = v(1,t) \tag{8}$$

3.1 Analysis

18. Rewrite (3) as the boundary control problem defined on $\mathscr{L}_2([0,1],\mathbb{R})^2$

$$\begin{cases} \dot{X}(t) = \mathscr{A} \cdot X(t) \\ \beta X(t) = U(t) \\ y(t) = \mathscr{C} X(t) \end{cases} \tag{9}$$

Specify the dimension of X, the definition of the operators $\mathscr{A}$, β and $\mathscr{C}$ and the domain of definition $\mathscr{D}(\mathscr{A})$ of the operator $\mathscr{A}$.

19. Express (without entirely solving them) the final equations characterizing the eigenvalues (and corresponding eigenvectors) of the operator $\mathscr{A}$.

20. Propose a procedure to study controllability.

3.2 Output-feedback control

21. Consider the Lyapunov functional candidate

$$V_2(t) = \int_0^1 e^{\mu x} u^2 dx + \int_0^1 e^{\mu x} v^2 dx \tag{10}$$

Design an output-feedback law assuming that $a, b, c, d >> 1$ and for an appropriate constant $\mu > 0$.

22*. Implement your corresponding control law in Simulink.

23. Conclude on the respective efficiency of the two strategies. What are their respective drawbacks and limitations from a practical point of view?