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Brief paper

Output-feedback adaptive control of a wave PDE with boundary anti-damping☆

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ABSTRACT

We develop an adaptive output-feedback controller for a wave PDE in one dimension with actuation on one boundary and with an unknown anti-damping term on the opposite boundary. This model is representative of a torsional stick–slip instability in drillstrings in deep oil drilling, as well as of various acoustic instabilities. The key feature of the proposed controller is that it requires only the measurements of boundary values and not of the entire distributed state of the system. Our approach is based on employing Riemann variables to convert the wave PDE into a cascade of two delay elements, with the first of the two delay elements being fed by control and the same element in turn feeding into a scalar ODE. This enables us to employ a prediction-based design for systems with input delays, suitably converted to the adaptive output-feedback setting. The result's relevance is illustrated with simulation example.

1. Introduction

Adaptive control of distributed parameters systems is a challenging topic due to the infinite dimension of the state and sometimes also of the parameter. As underlined in Bohm, Demetriou, Reich, and Rosen (1998), one of the most important drawbacks of most of the existing adaptive schemes for partial differential equations (PDEs) (see Bentsman & Orlov, 2001 and Duncan, Pasik-Duncan, & Maslowski, 1992 for example) is that they require measurement of the full distributed state, which is seldom the case in applications. The same observation can be made for analogous finite-dimensional approaches, often considered in practice (Dochain, Babary, & Tali-Maamar, 1992; Evesque, Dowling, & Annaswamy, 2001).

Over the past decade there has been a steady increase of interest in adaptive boundary control. Design of adaptive boundary regulation for Burgers' equations has been reported in Kobayashi (2001) and extended to the similar but higher order Kuramoto–Sivashinsky equation in Kobayashi (2002). Output-feedback adaptive designs for parabolic PDEs have been developed in Krstic and Smyshlyaev (2008a) and Smyshlyaev and Krstic (2007a,b) via the backstepping approach.

In this paper, we consider an unstable wave equation controlled from a boundary, and where the source of instability arises from an anti-damping boundary condition which is not collocated with control. This PDE has all of its infinitely many eigenvalues in the right-half plane, with arbitrary positive real parts, which is a reason why we refer to this PDE as "anti-stable".

Such a wave model with anti-damping phenomenon can be used, for example, to model duct combustion dynamics. For such a process, depicted in Fig. 1, the pressure field is subject to an acoustic dynamics (De Queiroz & Rahn, 2002), disturbed by a varying heat release produced by a flame anchored at a specific location. This heat release varies according to the pressure rate, which leads to combustion instabilities (Annaswamy & Ghoniem, 1995). These instabilities can be controlled thanks to a loudspeaker which is placed as far as possible from the flame front for thermal protection.[2] A second application fitting into this framework is the stick–slip phenomenon for drilling (Sagert, Di Meglio, Krstic, & Rouchon, 2013; Saldivar, Mondié, Loiseau, & Rasvan, 2011), which

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[2] Nevertheless, note that the pressure at $x = 0$ is not measured in this set-up. Therefore, the control law proposed in the sequel of this paper cannot be immediately applied.

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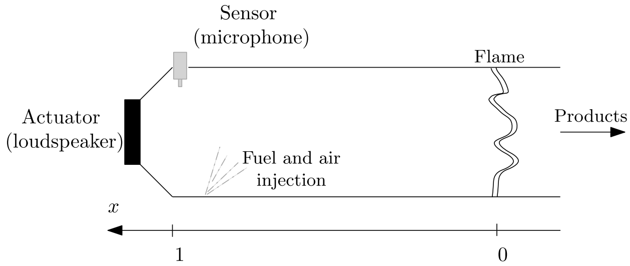


Fig. 1. A schematic of a premixed combustor with a sensor (microphone) and an actuator (loudspeaker). The distributed pressure inside the duct is disturbed by the heat release due to the combustion flame front at $x = 0$ and controlled with a loudspeaker located at $x = 1$.

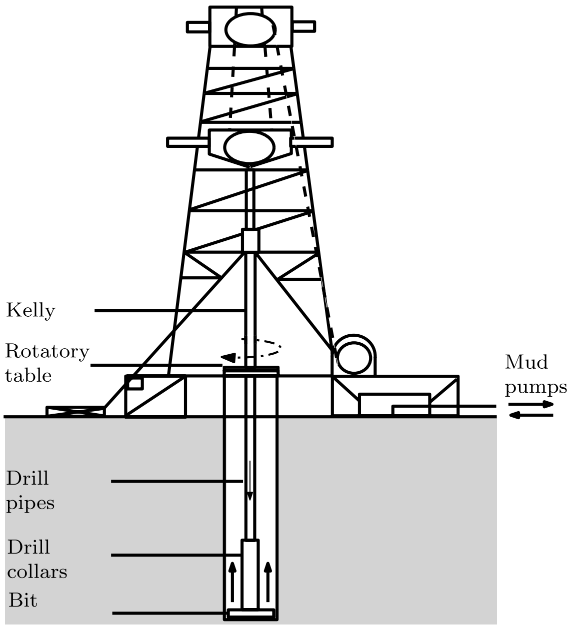


Fig. 2. Schematic view of a drilling system. A well is drilled with a rotating rock-crushing device, called a bit, driven by a rotatory table at the surface, equipped with an electric motor (actuator). The top rotatory table position is measured, along with the top and bottom bit velocities. The latter measurement can be transmitted via the mud system flowing back to the surface or via dedicated acoustic waves.

is an undesirable limit cycle of the drillstring velocity yielding potentially significant damages on oil production facilities[3] (see Fig. 2). In this context, adaptive control of the (distributed) bit velocity is of considerable interest, as the "drill bit on rock" friction term involved in modeling is poorly known. This results into an experimental set-up captured by the PDE dynamics addressed in this paper.

Even in the non-adaptive case, the underlying anti-damping wave PDE has been an open problem until the recent contribution (Smyshlyaev & Krstic, 2009), in which a backstepping transformation has been proposed to design boundary control. This approach has been pursued in Krstic (2010) to provide an adaptive controller in the case of unknown anti-damping coefficient. Nevertheless, the obtained controller suffers from the requirement of the measurement of the entire distributed state of the system.

In this paper, we revisit this problem via the introduction of Riemann variables, reformulating the plant in the form of an input-delay model cascaded with a stable transport equation occurring in the direction opposite from the propagation delay. This formulation allows one to use infinite-dimensional time-delay control strategy, namely a prediction-based controller which has recently been reinterpreted in the light of the PDE backstepping technique (Krstic & Smyshlyaev, 2008b). Exploiting the transport equation structure of the dynamics under consideration, we present a global output-feedback adaptive controller.

Both the controller and the parameter estimator that we design employ only boundary measurements. This is our paper's main achievement. While Krstic (2010) was the first result reporting adaptive control of an unstable wave PDE with unmatched parametric uncertainty, the present paper is the first result on *output-feedback* adaptive control for the same class of systems.

Our paper's other significant achievement is that, unlike (Krstic, 2010), where stability was achieved only in an energy-type norm and regulation was achieved only in the sense of an essential supremum (in time) of the system's energy norm (in space) going to zero, in this paper we prove both boundedness and regulation pointwise in the spatial variable for both the displacement and velocity components of the wave PDE's state.

Contrary to previous prediction-based adaptive controllers proposed in Bresch-Pietri, Chauvin, and Petit (2012) and Bresch-Pietri and Krstic (2010) where the delay itself is considered as uncertain and adaptation was designed to handle this lack of knowledge, here we only consider a linear parametric uncertainty. While delay adaptive output feedback cannot be solved globally because of the non-linear parametrization of the delay, the unknown parameter considered here appears in a linear manner in the plant. Therefore, the stability result proposed in this paper is global, in the sense that the initial parameter error can be arbitrarily large without jeopardizing stabilization (see Bresch-Pietri & Krstic, 2009 in which global prediction-based stabilization is also obtained with parametric uncertainties).

The paper is organized as follows. In Section 2, we present and reformulate the problem under consideration, before providing the proposed adaptive control law in Section 3, along with global stability statements. We illustrate these theorems through numerical simulations in Section 4 before providing their proof in Sections 5–7 and concluding with directions of future work.

Notations. In this paper, $|\cdot|$ is the Euclidean norm and $\|u(\cdot)\|$ is the spatial $\mathscr{L}_2$-norm of a signal $u(x,\cdot)$, $x \in [0, 1]$, which is denoted as

$$\|u(t)\|_2 = \sqrt{\int_0^1 u(x,t)^2 dx}. \tag{1}$$

For a scalar x, we write the sign function as $\text{sgn}(x)$ with $\text{sgn}(x) = 1$ if $x > 0$ and $\text{sgn}(x) = -1$ if $x < 0$. For $(a, b) \in \mathbb{R}^2$ such that $a < b$, we define the standard projector operator on the interval $[a, b]$ as a function of two scalar arguments f (denoting the parameter being update) and g (denoting the nominal update law) in the following manner:

$$\text{Proj}_{[a,b]}(f, g) = g \begin{cases} 0 & \text{if } f = a \text{ and } g < 0 \\ 0 & \text{if } f = b \text{ and } g > 0 \\ 1 & \text{otherwise.} \end{cases}$$

2. Problem statement

Consider the system

$$u_{xx} = u_{tt} \tag{2}$$

$$u_x(0, t) = -q u_t(0, t) \tag{3}$$

$$u_x(1, t) = U(t) \tag{4}$$

in which $U(t)$ is the input, appearing in the form of Neumann actuation, and (u, u_t) is the system state, with $(u(\cdot, 0), u_t(\cdot, 0))$

[3] However, the anti-damping wave equation is only an approximation of the model commonly used to account for this phenomenon (Sagert et al., 2013), in which a friction ODE is used as the boundary condition instead. The extension of the proposed adaptive technique to this framework is a direction of future work.

$\in H_1([0,1]) \times (L_2([0,1]) \cap L_\infty([0,1]))$ and $(u_x(\cdot,0), u_{xx}(\cdot,0), u_{xt}(\cdot,0)) \in L_\infty([0,1])^3$.

Our objective is to provide a feedback law stabilizing the anti-stable wave equation, despite large uncertainty in the anti-damping coefficient $q \geq 0$. This point is dealt with by employing an adaptive controller, fed by an estimate $\hat{q}(t)$ which is updated based on the system's real-time measurements to guarantee closed-loop stability.

As always in indirect adaptive control, certain a priori assumptions on the parameter values are needed in order to ensure stabilizability under parameter estimates. For our system, this gives rise to the following assumption.

Assumption 1. There exist known constants $\underline{q}$ and $\overline{q}$ such that $\underline{q} < \overline{q}$ and $q \in [\underline{q}, \overline{q}]$, with either $\overline{q} < 1$ or $\underline{q} > 1$.

As discussed in Krstic (2010), when $q = 1$, the real part of the plant (infinite) eigenvalues is $+\infty$ while, for $q \neq 1$ and $q \geq 0$, the real part is positive but finite.

Our second objective is to design a feedback law which does not employ the distributed state, but only boundary values measurements. We assume that the signals $u(0,\cdot)$, $u(1,\cdot)$ and $u_t(1,\cdot)$ are measured for all time.

As a first step in our development, we reformulate plant (2)–(4) by introducing the following intermediate Riemann variables[4] and transformed control variable

$$\zeta = u_t + u_x \tag{5}$$

$$\omega = \frac{1-q}{1+q}(u_t - u_x) \tag{6}$$

$$W(t) = U(t) + u_t(1,t) \tag{7}$$

which lead to the following new dynamics

$$u_t(0,t) = \frac{1}{1-q}\zeta(0,t) \tag{8}$$

$$\zeta_t = \zeta_x \tag{9}$$

$$\zeta(1,t) = W(t) \tag{10}$$

$$\omega_t = -\omega_x \tag{11}$$

$$\omega(0,t) = \zeta(0,t). \tag{12}$$

In this new framework, the wave phenomenon is represented as the cascade of two transport PDEs, with one ODE (simple integrator) being driven by the first of the two PDEs. The ODE (8) with state $u(0,\cdot)$ plays a central role and it has to be made asymptotically stable by feedback, which is applied through the transport equation (9) controlled at the boundary $x = 1$. A second transport phenomenon (11) is also present, in the opposite direction, accounting for the reflection of the wave at $x = 0$.

Remark 1. From the transport equations (9) and (11), we have that $\zeta(x,t) = \zeta(y, t+x-y)$ and $\omega(x,t) = \omega(y, t-x+y) = \zeta(x, t-2x)$ for any $0 \leq y \leq x \leq 1$ and any $t \geq 0$. In particular, $\zeta(x,t) = W(t-1+x)$ and $\omega(x,t) = W(t-1-x)$ for any $x \in [0,1]$ and $t \geq 0$.

Following Krstic (2008), (8)–(10) can also be interpreted as an input-delay ordinary differential equation, delayed by 1 unit of time, followed by a stable transport phenomenon (11)–(12). This motivates the control design.

[4] The choice of these asymmetric variables (ζ unweighted and ω weighted) is made so that they can be directly computed from past values of the input, as explained below in Remark 1.

3. Control design

Consider the following control law

$$U(t) = -u_t(1,t) - c_0\bigg((1-\hat{q})u(0,t) + \int_{t-1}^{t} U(\tau)d\tau + u(1,t) - u(1,t-1)\bigg) \tag{13}$$

in which $c_0 > 0$ is a constant and $\hat{q}$ is an estimate of the unknown parameter q. We choose the parameter estimate update law as

$$\dot{\hat{q}}(t) = \frac{\gamma c_0}{1+N(t)}\mathrm{Proj}_{[\underline{q},\overline{q}]}\bigg\{\hat{q}(t), \mathrm{sgn}(1-\overline{q})\bigg(b_1(U(t-1) + u_t(1,t-1))\int_{t-1}^{t} e^{1+\tau-t}w(\tau,t)d\tau - u(0,t)^2\bigg)\bigg\} \tag{14}$$

$$N(t) = u(0,t)^2 + b_1\int_{t-1}^{t} e^{1+\tau-t}w(\tau,t)^2 d\tau + b_2\int_{t-2}^{t-1} e^{2+\tau-t}(U(\tau)+u_t(1,\tau))^2 d\tau \tag{15}$$

in which the bounds $\underline{q}, \overline{q}$ are defined in Assumption 1, Proj is the standard projection operator, the normalization constants $b_1, b_2 > 0$ and the update gain $\gamma > 0$ are tuning parameters and, for $t \geq 0$ and $t-1 \leq \tau \leq t$,

$$w(\tau,t) = U(\tau) + u_t(1,\tau) + c_0\bigg((1-\hat{q})u(0,t) + u(1,\tau) - u(1,t-1) + \int_{t-1}^{\tau} U(\sigma)d\sigma\bigg). \tag{16}$$

In order to properly interpret this adaptive control law, we provide several comments next.

The choice of the control law (13) originates from the interpretation of (8)–(10) as an input delay system. Indeed, if q was known, the following predictor-based control law (Artstein, 1982; Kwon & Pearson, 1980; Manitius & Olbrot, 1979) would compensate exactly the delay

$$W(t) = -c_0\left((1-q)u(0,t) + \int_0^1 \zeta(x,t)dx\right) \tag{17}$$

i.e., after 1 unit of time, it would result into the closed-loop dynamics $u_t(0,t) = -c_0 u(0,t)$ which is exponentially stable for any $c_0 > 0$. Noticing that $\zeta(x,t) = U(t+x-1) + u_t(t+x-1)$ and employing a simple change of variable and integration, the control law (13) follows applying the certainty equivalent principle.

The choice of the update law is based on Lyapunov design, as detailed in the following section. As common in adaptive control (Ioannou & Fidan, 2006; Ioannou & Sun, 1996; Krstic, Kanellakopoulos, & Kokotovic, 1995), a projector operator is used in (14). In addition, normalization (15) is employed in order to limit the rate of change of the parameter estimate, which could otherwise act as a destabilizing disturbance. The role of projection is subtle—keeping the parameter estimate within a priori known bounds enables us to chose the normalization coefficients so that the update rate, in its role as a disturbance, is dominated by the stabilizing terms in the analysis.

As a final remark, we would like to stress the fact that the proposed controller (13)–(16) is entirely computable with only the measurement of the boundary values $u(0,\cdot)$, $u(1,\cdot)$ and $u_t(1,\cdot)$. This is the main advantage of this control law compared to ones previously obtained, like e.g. in Krstic (2010) which requires the measurement of the entire distributed state.

Theorem 1. *Consider the closed-loop system consisting of the plant (2)–(4), the control law (13) and the parameter update law (14)–(15). Define the functional*

$$\Gamma(t) = u(0,t)^2 + \int_0^1 u_x(x,t)^2 dx + \int_0^1 u_t(x,t)^2 dx + (q-\hat{q}(t))^2. \quad (18)$$

For any $c_0 > 0$ and any

$$b_2 < b_2^* = \frac{1}{2ec_0 \max\left\{(1-\bar{q})^2, (1-\underline{q})^2\right\}} \quad (19)$$

there exists $\gamma^*(b_2, c_0) > 0$ such that for any $\gamma \in (0, \gamma^*)$, the following hold:

(1) *there exist $R, \rho > 0$ such that*

$$\Gamma(t) \le R(e^{\rho\Gamma(0)} - 1) \quad (20)$$

(2) *$(u(\cdot,t), u_t(\cdot,t)) \in L_\infty([0,1])^2$ for all $t \ge 0$;*

(3) *and the regulation in maximum norm follows, i.e.,*

$$\lim_{t\to\infty} \max_{x\in[0,1]} |u(x,t)| = \lim_{t\to\infty} \max_{x\in[0,1]} |u_x(x,t)| = \lim_{t\to\infty} \max_{x\in[0,1]} |u_t(x,t)| = 0. \quad (21)$$

Remark 2. It is possible to obtain explicit bound on u and u_t. Namely, under the conditions of Theorem 1, the following pointwise bounds on the state (u, u_t) hold for all $x \in [0,1]$ and $t \ge 0$:

$$u(x,t)^2 \le e^{V_0} - 1 + 2\left(\left(2 + 4\left(\frac{1}{b_2}\left(\frac{1+\bar{q}}{\min\{1-\underline{q}, 1-\bar{q}\}}\right)^2 + r_1 + \frac{r_2}{b_1}\right)\right)(e^{V_0}-1)\right)^{\frac{1}{2}} \left(\left(\frac{1}{b_2}\left(\frac{1+\bar{q}}{\min\left\{1-\underline{q}, 1-\bar{q}\right\}}\right)^2 + r_1 + \frac{r_2}{b_1}\right)(e^{V_0}-1)\right)^{\frac{1}{2}} \quad (22)$$

$$\begin{aligned} u_t(x,t)^2 \le\ & \frac{1}{2}\Bigg(3\Bigg(2\sqrt{\frac{e^{V_0}-1}{b_1}}\left(2e\left(c_0^2\left(r_1+\frac{r_2}{b_1}\right)(e^{V_0}-1) + \|u_{xt}(0)\|^2 + \|u_{xx}(0)\|^2\right)\right) + \frac{3ec_0^2}{\eta}e^{V_0}V_0 \\ & \times \left(\frac{\gamma V_0}{\min\{1-\underline{q}, 1-\bar{q}\}} + c_0^2 \max\{1-\underline{q}, 1-\bar{q}\}^2 + \gamma c_0 \frac{b_1 c_0^2 \max\left\{1-\underline{q}, 1-\bar{q}\right\}^2 b_1 + 1}{2}\right)\Bigg)^{\frac{1}{2}} \\ & + c_0^2\left(\max\{1-\underline{q}, 1-\bar{q}\}^2 + \frac{1}{b_1}\right)\left(e^{V_0}-1\right)\Bigg) + \left(\frac{1+\bar{q}}{\min\{1-\underline{q}, 1-\bar{q}\}}\right)^2 R_0\Bigg) \end{aligned} \quad (23)$$

where

$$\begin{aligned}\eta = \min\Big\{& c_0 - 2eb_2c_0^2 \max\left\{(1-\underline{q})^2, (1-\bar{q})^2\right\}, \\ & b_1 - c_0\gamma - 2eb_2 - \frac{1}{c_0 \min\left\{(1-\underline{q})^2, (1-\bar{q})^2\right\}}, \\ & 1 - \gamma c_0^2 b_1^2 e^2 \left(1 + c_0^2\left(1 - \max\left\{(1-\underline{q})^2, (1-\bar{q})^2\right\}\right)\right)\Big\}\end{aligned}$$

$$V_0 = \log\left(1 + (1+b_1r_1e)u(0,0)^2 + b_1er_2\int_0^1 (U(x-1) + u_t(x-1))dx + b_2\int_0^1 (U(-x-1) + u_t(-x-1))dx\right) + \frac{(q-\hat{q}(0))^2}{\gamma \min\left\{|1-\underline{q}|, |1-\bar{q}|\right\}} \quad (24)$$

$$\begin{aligned} R_0 =\ & \max_{x\in[0,1]} \frac{u_t(x,0)^2 + u_x(x,0)^2}{2} + c_0^2\left(\max\{1-\underline{q}, 1-\bar{q}\}^2 + r_1 + \frac{r_2}{b_1}\right)(e^{V_0}-1) + 2\sqrt{\frac{e^{V_0}-1}{b_2}} \\ & \times \left(\frac{\|u_{xt}(0)\|^2 + \|u_{xx}(0)\|^2}{2} + \max_{x\in[0,1]} \frac{u_{xx}(x,0)^2 + u_{xt}(x,0)^2}{2}\right) \\ & + 3c_0^2\left(\frac{\gamma V_0}{\min\{1-\underline{q}, 1-\bar{q}\}} + c_0^2 \max\{1-\underline{q}, 1-\bar{q}\}^2 + \gamma c_0 \frac{b_1c_0^2\max\{1-\underline{q}, 1-\bar{q}\}^2 + b_1 + 1}{2}\right)\frac{1}{\eta}e^{V_0}V_0 \\ & + 3\Bigg(c_0^2\max\{1-\underline{q}, 1-\bar{q}\}^2(e^{V_0}-1) + 2\sqrt{\frac{e^{V_0}-1}{b_1}} \\ & \times \Bigg(2ec_0^2\left(r_1+\frac{r_2}{b_1}\right)(e^{V_0}-1) + \|u_{xt}(0)\|^2 + \|u_{xx}(0)\|^2 \\ & + 3ec_0^2\left(\frac{\gamma V_0}{\min\{1-\bar{q}, 1-\underline{q}\}} + c_0^2\max\{1-\underline{q}, 1-\bar{q}\}^2 + \gamma c_0 \right. \\ & \left. \times \frac{b_1c_0^2\max\{1-\underline{q}, 1-\bar{q}\}^2 + b_1 + 1}{2}\right)\frac{e^{V_0}V_0}{\eta}\Bigg)^{\frac{1}{2}}\Bigg)^{\frac{1}{2}} \end{aligned} \quad (25)$$

$$b_1 = 1 + c_0\gamma + 2eb_2 + \frac{1}{c_0 \min\left\{(1-\bar{q})^2, (1-\underline{q})^2\right\}} \quad (26)$$

and in which the positive constants r_1 and r_2 can be chosen as

$$r_1 = 3c_0^2 \max\left\{1-\underline{q}, 1-\bar{q}\right\}^2, \qquad r_2 = 3(1+c_0^2). \quad (27)$$

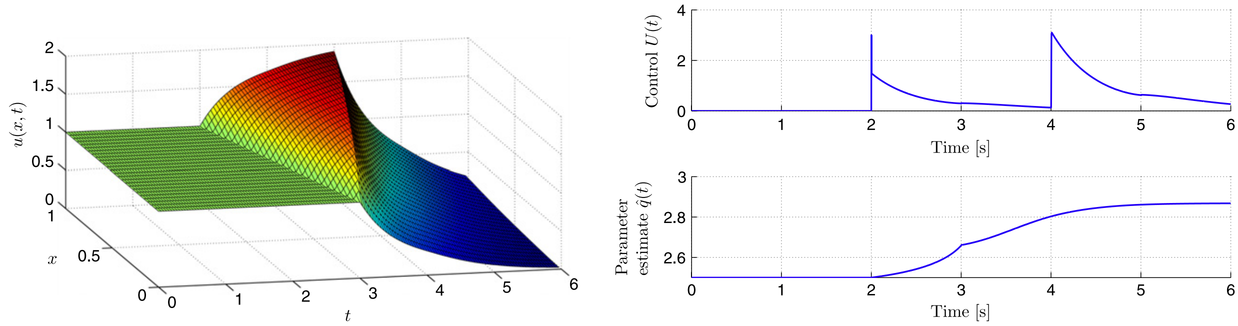
Theorem 1 states a stability result (20) jointly with pointwise boundedness and asymptotic convergence properties (21). Possible expressions of the pointwise bounds are provided in Remark 2. These conservative bounds cannot be made arbitrarily small with the choice of the adaptive controller's parameters but it is remarkable that explicit bounds on the system's states can be derived.

Before providing the proofs of this theorem, we illustrate it with a simulation example.

4. Illustrative example

We now present numerical simulation on a toy example illustrating the merits of the controller (13)–(15). The unknown anti-damping coefficient is chosen as $q = 3$ (with $\underline{q} = 2$ and $\bar{q} = 4$) and two sets of initial conditions are considered, respectively $u(\cdot, 0) = 1$ which is an unstable equilibrium point and $u(x,0) = 0.1\sin(x)$. Corresponding simulation results are pictured respectively in Figs. 3 and 4. For both cases, the controller is turned on after 2 s and the controller parameters are chosen as $c_0 = 2$, $\gamma = 5\mathrm{e}-4$ and $b_2 = 1\mathrm{e}-2$.

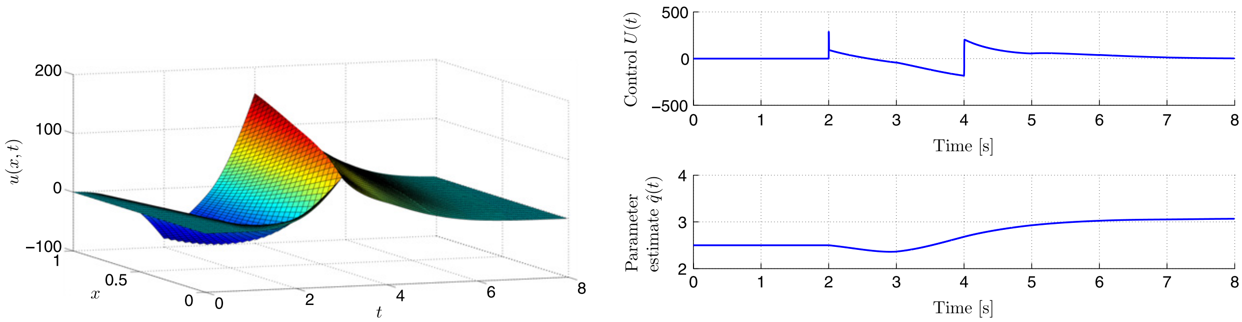
The internal behavior of the proposed controller is particularly clear in Fig. 3(a). At $t = 2$ s, the control law starts acting and



(a) System state evolution. The adaptive controller is turned on after 2 s.

(b) Control input and parameter estimate evolutions. The adaptive controller is turned on after 2 s.

Fig. 3. Stabilization of plant (2)–(4) starting from the (unstable) equilibrium state $u(\cdot, 0) = 1$.



(a) System state evolution. The adaptive controller is turned on after 2 s.

(b) Control input and parameter estimate evolutions. The adaptive controller is turned on after 2 s.

Fig. 4. Stabilization of plant (2)–(4) initialized at time $t = 0$ with $u(x, 0) = 0.1 \sin(x)$.

kicks in the dynamics of $u(0, \cdot)$ at $t = 3$ s, consistently with the interpretation of the wave equation as delay systems with 1 unit of time delay. From there, the boundary value $u(0, \cdot)$ exponentially converges to zero and, while propagating back to $x = 1$, the control acts now in a stabilizing manner on the system state: $u(x, t)$ starts to converge exponentially at $t = 3+x$, corresponding to the stable transport dynamics (11). Note that this exponential behavior is only observed here and that only asymptotic stability is stated in Theorem 1. This behavior is consistent with the evolution of the control law pictured in Fig. 3(b), which is turned on at $t = 2$ and is finally "updated" after a wave round-trip, in the sense that the control peak observed at $t = 4$ s is due to the change of sign and of scale of $u_t(1, t)$ at this instant.

A similar behavior can be observed in Fig. 4 (the control starts acting at $t = 2$ s corresponding to the minimal value of $u(1, \cdot)$, kicks in at $t = 3$ s corresponding to the maximum value of $u(0, \cdot)$ and propagates backward from there) for which the initial condition is not an equilibrium point and the beginning of the simulation consequently exhibits diverging performance.

Finally, in both cases, the parameter update law does not provide the convergence of the parameter estimate to the unknown parameter value, even if stabilization is achieved, as it is usually the case in adaptive control (Ioannou & Fidan, 2006; Ioannou & Sun, 1996).

5. Stability—proof of (20)

5.1. Backstepping transformation and target system

Consider the backstepping transformation of the distributed variable ζ,

$$z(x,t) = \zeta(x,t) + c_0\left((1-\hat{q})u(0,t) + \int_0^x \zeta(y,t)dy\right). \tag{28}$$

Following Remark 1 with a suitable change of variable and direct integration, this backstepping transformation is closely related to (16) with

$$z(x,t) = w(t-1+x,t), \quad x \in [0,1],\ t \geq 0. \tag{29}$$

Further, using the exact same steps, one can rewrite (13) as

$$W(t) = -c_0\left((1-\hat{q})u(0,t) + \int_0^1 \zeta(x,t)dx\right) \tag{30}$$

and, jointly with (28), the plant (8)–(12) can then be reformulated as the following target system

$$u_t(0,t) = -c_0 u(0,t) + \frac{1}{1-q}\left(z(0,t) - \tilde{q}c_0 u(0,t)\right) \tag{31}$$

$$z_t = z_x + c_0\left(\frac{\tilde{q}}{1-q}(z(0,t) - c_0(1-\hat{q})u(0,t)) - \dot{\hat{q}}u(0,t)\right) \tag{32}$$

$$z(1,t) = 0 \tag{33}$$

$$\omega_t = -\omega_x \tag{34}$$

$$\omega(0,t) = z(0,t) - c_0(1-\hat{q})u(0,t) \tag{35}$$

in which $\tilde{q}(t) = q - \hat{q}(t)$ is the parameter estimation error. This target system is the one which is exploited in the Lyapunov analysis, as it presents the advantage of having a boundary condition $z(1, t) = 0$.

5.2. Lyapunov analysis

We are now ready to start the Lyapunov analysis. Define the Lyapunov–Krasovskii functional candidate

$$V(t) = \log(1 + N(t)) + \frac{\tilde{q}(t)^2}{\gamma|1-q|} \tag{36}$$

in which, using (29), suitable change of variable and direct integration jointly with Remark 1, the normalization factor originally defined in (15) can be expressed as

$$N(t) = u(0,t)^2 + b_1 \int_0^1 e^x z(x,t)^2 dx + b_2 \int_0^1 e^{1-x} \omega(x,t)^2 dx. \quad (37)$$

Note that, similarly, (14) can be reformulated as

$$\dot{\hat{q}}(t) = \frac{\gamma c_0}{1+N(t)} \mathrm{Proj}_{[\underline{q},\bar{q}]} \Bigg\{ \hat{q}(t), \mathrm{sgn}(1-q) \Bigg(b_1 (z(0,t) - c_0(1-\hat{q})u(0,t)) \int_0^1 e^x z(x,t)dx - u(0,t)^2 \Bigg) \Bigg\} \quad (38)$$

observing that $\mathrm{sgn}(1-q) = \mathrm{sgn}(1-\bar{q})$ is known according to Assumption 1. Taking a time-derivative of (36), one gets

$$\dot{V}(t) \le \frac{1}{1+N(t)} \Bigg(-2c_0 u(0,t)^2 + \frac{2u(0,t)}{1-q}(z(0,t) - c_0 u(0,t) \times \tilde{q}(t)) - b_1 z(0,t)^2 - b_1 \|z(t)\|^2 + 2b_1 c_0 \int_0^1 e^x z(x,t)dx \times \left(\frac{\tilde{q}}{1-q}(z(0,t) - c_0(1-\hat{q})u(0,t)) - \dot{\hat{q}}(t)u(0,t) \right) + b_2 \left(e\omega(0,t)^2 - \|\omega(t)\|^2 \right) \Bigg) - \frac{2\tilde{q}(t)\dot{\hat{q}}(t)}{\gamma|1-q|}. \quad (39)$$

Using projection operator properties and (14)–(15) (or equivalently but with a more suitable set of variables (38)–(37)), one obtains

$$\dot{V}(t) = \frac{1}{1+N(t)} \Bigg(-2c_0 u(0,t)^2 + \frac{2u(0,t)}{1-q} z(0,t) - b_1 z(0,t)^2 - b_1 \|z(t)\|^2 + b_2 \left(e\omega(0,t)^2 - \|\omega(t)\|^2 \right) - 2b_1 c_0 \dot{\hat{q}}(t) u(0,t) \int_0^1 e^x z(x,t)dx \Bigg). \quad (40)$$

Observing that, if $b_1 > 1$,

$$\left| \frac{2u(0,t)}{1+N(t)} \int_0^1 e^x z(x,t)dx \right| \le 1$$

it follows with (38) that

$$\left| 2\dot{\hat{q}}(t)u(0,t) \int_0^1 e^x z(x,t)dx \right| \le \gamma c_0 (b_1^2 e^2 (1 + c_0^2(1-\hat{q})^2) \|z(t)\|^2 + z(0,t)^2) \quad (41)$$

and therefore

$$\dot{V}(t) \le \frac{1}{1+N(t)} \Bigg(-2c_0 u(0,t)^2 + \frac{2u(0,t)}{1-q} z(0,t) - (b_1 - c_0\gamma) z(0,t)^2 - b_1(1 - \gamma c_0^2 b_1^2 e^2 (1 + c_0^2 \times (1-\hat{q})^2)) \|z(t)\|^2 + b_2 \left(e\omega(0,t)^2 - \|\omega(t)\|^2 \right) \Bigg). \quad (42)$$

Finally, one gets

$$\omega(0,t)^2 \le 2(z(0,t)^2 + c_0^2 (1-\hat{q})^2 u(0,t)^2) \quad (43)$$

and therefore, applying Young's inequality,

$$\dot{V}(t) = \frac{1}{1+N(t)} \Bigg(-(c_0 - 2eb_2 c_0^2 (1-\hat{q})^2) u(0,t)^2 - \left(b_1 - c_0\gamma - 2eb_2 - \frac{1}{c_0(1-q)^2} \right) z(0,t)^2 - b_1 (1 - \gamma c_0^2 b_1^2 e^2 (1 + c_0^2(1-\hat{q})^2)) \|z(t)\|^2 - b_2 \|\omega(t)\|^2 \Bigg). \quad (44)$$

Consequently, by choosing b_2 according to (19), b_1 as (26) and the update gain $\gamma \in (0, \gamma^*)$ with γ^* such that

$$\gamma^* \left(1 + c_0\gamma^* + 2eb_2 + \frac{1}{c_0 \min\left\{(1-\bar{q})^2, (1-\underline{q})^2\right\}} \right)^2 \times c_0^2 e^2 (1 + c_0^2 \max\{(1-\underline{q})^2, (1-\bar{q})^2\}) < 1 \quad (45)$$

one obtains that there exists a constant $\eta > 0$ such that

$$\dot{V}(t) \le -\frac{\eta}{1+N(t)} \left(u(0,t)^2 + z(0,t)^2 + \|z(t)\|^2 + \|\omega(t)\|^2 \right) \quad (46)$$

and finally that

$$V(t) \le V(0), \quad \forall t \ge 0. \quad (47)$$

5.3. Stability in terms of the functional Γ

Finally, we need to establish the stability in terms of Γ. First, from the definition of the Riemann variables (5)–(6), one gets

$$u_t(x,t) = \frac{1}{2} \left(\zeta(x,t) + \frac{1+q}{1-q}\omega(t) \right) \quad (48)$$

$$u_x(x,t) = \frac{1}{2} \left(\zeta(x,t) - \frac{1+q}{1-q}\omega(t) \right). \quad (49)$$

Second, from the backstepping transformation (28) and its inverse

$$\zeta(x,t) = z(x,t) - c_0 \left((1-\hat{q}) e^{-c_0 x} u(0,t) + \int_0^x e^{-c_0(x-y)} z(y,t)dy \right) \quad (50)$$

using Cauchy–Schwarz's and Young's inequalities, one obtains the existence of strictly positive constants r_1 and r_2 such that

$$\|\zeta(t)\|^2 \le r_1 u(0,t)^2 + r_2 \|z(t)\|^2 \quad (51)$$

$$\|z(t)\|^2 \le r_1 u(0,t)^2 + r_2 \|\zeta(t)\|^2. \quad (52)$$

A constructive choice of such constants is given in (27) for example. With these inequalities and Young's inequality, it follows that

$$\|u_t(t)\|^2 + \|u_x(t)\|^2 \le \|\zeta(t)\|^2 + \left(\frac{1+q}{1-q}\right)^2 \|\omega(t)\|^2 \le \left(\frac{1+q}{1-q}\right)^2 \|\omega(t)\|^2 + r_1 u(0,t)^2 + r_2 \|z(t)\|^2 \le \left(\frac{1}{b_2} \left(\frac{1+\bar{q}}{\min\{1-\underline{q}, 1-\bar{q}\}} \right)^2 + r_1 + \frac{r_2}{b_1} \right) (e^{V(t)} - 1) \quad (53)$$

and also that

$$u(0,t)^2 \le V(t) \le e^{V(t)} - 1 \quad (54)$$
$$\tilde{q}(t) \le \gamma |1-q| V(t). \quad (55)$$

Consequently, we have that

$$\Gamma(t) \le \left(1 + \frac{1}{b_2}\left(\frac{1+\bar{q}}{\min\{1-\underline{q}, 1-\bar{q}\}}\right)^2 + r_1 + \frac{r_2}{b_1} + \gamma \max\{|1-\underline{q}|, |1-\bar{q}|\}\right)(e^{V(t)} - 1). \quad (56)$$

Finally, with (52) one gets

$$V(t) \le \left(1 + b_1 e(r_1 + 2r_2) + 2eb_2 + \frac{1}{\gamma|1-q|}\right)\Gamma(t). \quad (57)$$

Matching the previous inequalities gives the stability result stated in the theorem with

$$R = 1 + \frac{1}{b_2}\left(\frac{1+\bar{q}}{\min\{1-\underline{q}, 1-\bar{q}\}}\right)^2 + r_1 + \frac{r_2}{b_1} + \gamma \max\{|1-\underline{q}|, |1-\bar{q}|\} \quad (58)$$

$$\rho = 1 + b_1 e(r_1 + 2r_2) + 2eb_2 + \frac{1}{\gamma \min\{|1-\underline{q}|, |1-\bar{q}|\}}. \quad (59)$$

6. Proof of the convergence in the L_2-norm

Lemma 1. *Consider the closed-loop system consisting of the plant (2)–(4), the control law (13) and the parameter update law (14)–(15). Consider also the corresponding Riemann variables (5)–(6) and the backstepping transformation (28). Then, under the conditions stated in Theorem 1,*

$$\lim_{t\to\infty} u(0,t) = \lim_{t\to\infty} \|z(t)\| = \lim_{t\to\infty} \|\omega(t)\| = 0. \quad (60)$$

Proof. From (47), one easily gets that $\tilde{q}$ and $N(t)$ are uniformly bounded for $t \ge 0$, and therefore $u(0,t)$, $\|z(t)\|$ and $\|\omega(t)\|$ are also uniformly bounded for $t \ge 0$. Consequently, from (51), $\|\zeta(t)\|$ is uniformly bounded for $t \ge 0$.

From there, applying Cauchy–Schwarz's inequality to (30), one obtains that $W(t) = \zeta(1,t)$ is uniformly bounded for $t \ge 0$. Further, as $\zeta(x,t) = \zeta(1, t-1+x)$, $\zeta(x,t)$ is also uniformly bounded for $t \ge 1-x$ and in particular $\zeta(x,t)$ is for $t \ge 1$. From (28),

$$z(0,t) = \zeta(0,t) + c_0(1-\hat{q})u(0,t) \quad (61)$$

and, consequently, $z(0,t)$ is also uniformly bounded for $t \ge 1$. From there, applying Young's inequality to (38), one obtains that $\dot{\hat{q}}$ is uniformly bounded for $t \ge 1$. Further, from (31)–(35),

$$\frac{d}{dt}u(0,t)^2 = 2u(0,t)\left(-c_0 u(0,t) + \frac{1}{1-q} \times (z(0,t) - \tilde{q}c_0 u(0,t))\right) \quad (62)$$

$$\frac{d}{dt}\|z(t)\|^2 = 2\|z(t)\|\left(-\frac{1}{2}z(0,t)^2 + \int_0^1 z(x,t)dx \times \left(c_0\left(\frac{\tilde{q}}{1-q}(z(0,t) - c_0(1-\hat{q})u(0,t)) - \dot{\hat{q}}u(0,t)\right)\right)\right) \quad (63)$$

$$\frac{d}{dt}\|\omega(t)\|^2 = \|\omega(t)\|(\omega(0,t)^2 - \omega(0,t-1)^2) \quad (64)$$

in which $\omega(0,t) = \zeta(0,t)$. Applying Cauchy–Schwarz's inequality and the previous considerations, it is straightforward that the right-hand terms in the previous equations are all uniformly bounded for $t \ge 2$.

Finally, integrating (46) from 0 to ∞, it follows that $u(0,t)$, $\|z(t)\|$ and $\|\omega(t)\|$ are square integrable. Following Barbalat's Lemma, $u(0,t)$, $\|z(t)\|$ and $\|\omega(t)\|$ tend to zero as t tends to ∞. ■

7. Proof of pointwise bounds (22)–(25) and convergence

7.1. Pointwise boundedness

In this section, we prove Remark 2. Following Agmon's inequality, one obtains for $x \in [0,1]$ and $t \ge 0$

$$u(x,t)^2 \le u(0,t)^2 + 2\|u(t)\|\,\|u_x(t)\| \quad (65)$$

or, using now Poincare's inequality,

$$u(x,t)^2 \le u(0,t)^2 + 2\sqrt{2u(0,t)^2 + 4\|u_x(t)\|^2}\,\|u_x(t)\|. \quad (66)$$

From (47) and (53), it follows that

$$u(0,t)^2 \le e^{V(0)} - 1 \quad (67)$$

$$\|u_x(t)\|^2 \le \left(\frac{1}{b_2}\left(\frac{1+\bar{q}}{\min\{1-\underline{q}, 1-\bar{q}\}}\right)^2 + r_1 + \frac{r_2}{b_1}\right)(e^{V(0)} - 1). \quad (68)$$

Observing, from (52), that

$$V(t) \le \log(1 + (1 + b_1 r_1 e)u(0,t)^2 + b_1 e r_2\|\zeta(t)\|^2 + b_2\|\omega(t)\|^2) + \frac{\tilde{q}(t)^2}{\gamma|1-q|} \quad (69)$$

the bound (22) follows, matching the previous inequalities.

Now, with (48) and (51) and applying Cauchy–Schwarz's and Young's inequalities, one can observe that

$$\begin{aligned} u_t(x,t)^2 &\le \frac{1}{2}\left(\zeta(x,t)^2 + \left(\frac{1+q}{1-q}\right)^2\omega(x,t)^2\right) \\ &\le \frac{1}{2}\left(3(z(x,t)^2 + c_0^2(1-\hat{q})^2u(0,t)^2 + c_0^2\|z(t)\|^2) + \left(\frac{1+q}{1-q}\right)^2\omega(x,t)^2\right) \\ &\le \frac{1}{2}\left(3\left(z(x,t)^2 + \left(c_0^2(1-\hat{q})^2 + \frac{1}{b_1}\right)(e^{V(0)} - 1)\right) + \left(\frac{1+q}{1-q}\right)^2\omega(x,t)^2\right) \end{aligned} \quad (70)$$

in which we have used (67) and a similar inequality for $\|z(t)\|^2$. Now, to bound $z(x,t)^2$ and $\omega(x,t)^2$ appearing in this expression, we employ Agmon's inequality. Taking into account the fact that $z(1,t) = 0$, one gets

$$\max_{x\in[0,1]} |z(x,t)|^2 \le 2\|z(t)\|\,\|z_x(t)\| \quad (71)$$

in which, from (47),

$$\|z(t)\| \le \sqrt{\frac{e^{V(0)} - 1}{b_1}}. \quad (72)$$

Further, from (32)–(33), the spatial-derivative of the backstepping transformation z_x satisfies the following equations

$$z_{xt} = z_{xx} \tag{73}$$

$$z_x(1,t) = -c_0\left(\frac{\tilde{q}}{1-q}(z(0,t) - c_0(1-\hat{q})u(0,t)) - \dot{\hat{q}}u(0,t)\right). \tag{74}$$

Consequently, one deduces that

$$\frac{d}{dt}\left[\int_0^1 e^x z_x(x,t)^2 dx\right] = e z_x(1,t)^2 - z_x(0,t)^2 - \int_0^1 e^x z_x(x,t)^2 dx \tag{75}$$

and, solving this equation,

$$\|z_x(t)\|^2 \le \int_0^1 e^x z_x(x,t)^2 dx \le e^{1-t}\left\|z_x(0)^2\right\| + \int_0^t e^{-(t-s)} e z_x(1,s)^2 ds. \tag{76}$$

Following (74), one further obtains that

$$\|z_x(t)\|^2 \le e^{1-t}\|z_x(0)\|^2 + 3ec_0^2\int_0^t \left(\left(\frac{\tilde{q}}{1-q}\right)^2 (z(0,s)^2 + c_0^2(1-\hat{q})^2 u(0,s)^2) + \dot{\hat{q}}(s)^2 u(0,s)^2\right) ds \tag{77}$$

in which, integrating (46) and as $1+N(t) \le e^{V(t)},\ t \ge 0$,

$$\int_0^t \frac{z(0,s)^2}{1+N(t)} ds \le \frac{V(0)}{\eta} \tag{78}$$

$$\int_0^t (z(0,s)^2 + u(0,s)^2) ds \le \frac{1}{\eta} e^{V(0)} V(0) \tag{79}$$

and, from (38) and taking a spatial-derivative of (28) and using Young's inequality,

$$|\dot{\hat{q}}(t)| \le \gamma c_0\left(\frac{b_1 z(0,t)^2}{2(1+N(t))} + \frac{b_1 c_0^2(1-\hat{q})^2 + 1}{2}\right) \tag{80}$$

$$\|z_x(0)\|^2 \le 2(\|\zeta_x(0)\|^2 + c_0^2\|\zeta(0)\|^2) \le 2\left(c_0^2\left(r_1 + \frac{r_2}{b_1}\right)(e^{V(0)} - 1) + \|u_{xt}(0)\|^2 + \|u_{xx}(0)\|^2\right) \tag{81}$$

in which we have used (51), (36) and a spatial derivative of (5) to obtain the last inequality. Matching (80) and (81) into (77) and using (78)–(79), one finally obtains

$$\|z_x(t)\|^2 \le 2e\left(c_0^2\left(r_1 + \frac{r_2}{b_1}\right)(e^{V(0)} - 1) + \|u_{xt}(0)\|^2 + \|u_{xx}(0)\|^2\right) + 3ec_0^2\left(\frac{\gamma V(0)}{1-q} + c_0^2(1-\hat{q})^2 + \gamma c_0 \frac{b_1 c_0^2(1-\hat{q})^2 + b_1 + 1}{2}\right)\frac{1}{\eta} e^{V(0)} V(0). \tag{82}$$

Consequently, matching (72) and this last inequality into (71), one obtains

$$z(x,t)^2 \le 2\sqrt{\frac{e^{V(0)} - 1}{b_1}}\left(2e\left(c_0^2\left(r_1 + \frac{r_2}{b_1}\right)(e^{V(0)} - 1) + \|u_{xt}(0)\|^2 + \|u_{xx}(0)\|^2\right) + 3ec_0^2\left(\frac{\gamma V(0)}{1-q} + c_0^2(1-\hat{q})^2 + \gamma c_0 \frac{b_1 c_0^2(1-\hat{q})^2 + b_1 + 1}{2}\right)\frac{1}{\eta} e^{V(0)} V(0)\right)^{1/2}. \tag{83}$$

Now, applying Agmon's inequality and using (12), one gets

$$\omega(x,t)^2 \le \zeta(0,t)^2 + 2\|\omega(t)\|\,\|\omega_x(t)\| \tag{84}$$

where, from (47),

$$\|\omega(t)\|^2 \le \frac{e^{V(0)} - 1}{b_2}. \tag{85}$$

With arguments similar to those used to obtain (76), one also gets

$$\|\omega_x(t)\|^2 \le e^{-t}\|\omega_x(0)\|^2 + \int_0^t e^{-(t-s)}\omega_x(0,s)^2 ds \tag{86}$$

in which, using a spatial derivative of (50) with Young's inequality and the fact that both z_x and ζ_x satisfy the transport PDE (73),

$$\omega_x(0,t)^2 = \zeta_x(0,t)^2 \le \begin{cases} \max_{x\in[0,1]} \zeta_x(x,0)^2 & \text{if } t \le 1 \\ 3(z_x(1,t-1)^2 + c_0^2((1-\hat{q})^2 u(0,t)^2 + z(0,t)^2)) & \text{otherwise.} \end{cases} \tag{87}$$

Consequently, using (74) jointly with (36) and (46) to bound the second expression in (87) and following (86), one obtains

$$\|\omega_x(t)\|^2 \le \|\omega_x(0)\|^2 + \max_{x\in[0,1]}\zeta_x(x,0)^2 + 3c_0^2\left(\frac{\gamma V(0)}{1-q} + c_0^2(1-\hat{q})^2 + \gamma c_0\frac{b_1 c_0^2(1-\hat{q})^2 + b_1 + 1}{2}\right) \times \frac{e^{V(0)}V(0)}{\eta} + 3(c_0^2(1-\hat{q})^2 \times (e^{V(0)} - 1) + z(0,t)^2) \tag{88}$$

in which $z(0,t)^2$ can be bounded following (83) which holds for any $x \in [0,1]$. Finally,

$$\zeta(0,t)^2 \le \begin{cases} \max_{x\in[0,1]}\zeta(x,0)^2 & \text{if } t \ge 1 \\ \zeta(1,t-1)^2 & \text{otherwise} \end{cases} \le \begin{cases} \max_{x\in[0,1]}\frac{u_t(x,0)^2 + u_x(x,0)^2}{2} & \text{if } t \ge 1 \\ c_0^2\left((1-\hat{q})^2 u(0,t)^2 + \|\zeta(t)\|^2\right) & \text{otherwise} \end{cases} \le \begin{cases} \max_{x\in[0,1]}\frac{u_t(x,0)^2 + u_x(x,0)^2}{2} & \text{if } t \ge 1 \\ c_0^2\left((1-\hat{q})^2 + r_1 + \frac{r_2}{b_1}\right)(e^{V(0)} - 1) & \text{otherwise.} \end{cases} \tag{89}$$

Plugging (85), (88) with (83) and (89) into (84), one finally obtains (23), using (70) with (83).

7.2. Pointwise convergence property (21)

Consider Agmon's inequality (71). From (82), $\|z_x(t)\|$ is uniformly bounded for $(u_{xx}(\cdot,0), u_{xt}(\cdot,0)) \in L_\infty([0,1])^2$ and from Lemma 1 $\|z(t)\|$ tends to zero as t tends to ∞. Therefore, it follows from (71) that $\max_{x\in[0,1]}|z(x,t)|$ tends to 0 as t tends to ∞. Therefore, from (50) and as $u(0,t)$ asymptotically converges from Lemma 1, $\max_{x\in[0,1]}|\zeta(x,t)|$ also tends to 0 as t tends to ∞.

Besides, consider (84). From (88) and (89) respectively, $\|\omega_x(t)\|$ and $\zeta(0,t)$ are uniformly bounded for $(u(\cdot,0), u_t(\cdot,0)) \in H_1([0,1]) \times L_2([0,1])$ and $(u_{xx}(\cdot,0), u_{xt}(\cdot,0)) \in L_\infty([0,1])^2$. Further, from Lemma 1, $\|\omega(t)\|$ tends to zero as t tends to ∞. Consequently, $\max_{x\in[0,1]} |\omega(x,t)|$ tends to zero as t tends to ∞.

Finally, from the inverse transformations (48)–(49), applying the triangle inequality, one obtains that $\max_{x\in[0,1]} |u_x(x,t)|$ and $\max_{x\in[0,1]} |u_t(x,t)|$ tend to zero asymptotically. The convergence to zero of $\max_{x\in[0,1]} |u(x,t)|$ follows from (65) and the fact that both $u(0,t)$ and $\|u_x(t)\|$ tend to zero while t tends to ∞ and $\|u(t)\|$ remains bounded.

8. Conclusion

In this paper, we considered a wave PDE subject to anti-damping with unknown coefficient and have proposed an output-feedback adaptive controller. The main advantage of this new control strategy is that it does not require the measurements of the entire system state but only of boundary values. The extension of this technique to other types of boundary conditions, such as an ODE of order two or higher for $u(0,\cdot)$ which is usually employed in drillstring modeling (Saldivar et al., 2011), is a topic of future work.

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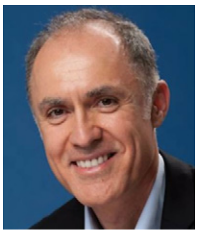

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