Delay-Adaptive Control for Nonlinear Systems

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***Abstract*—We present a systematic delay-adaptive prediction-based control design for nonlinear systems with unknown long actuator delay. Our approach is based on the representation of the constant actuator delay as a transport Partial Differential Equation (PDE) in which the convective speed is inversely proportional to the unknown delay. We study two different frameworks, assuming first that the actuator state is measured and relaxing afterward. For the full-state feedback case, we prove global asymptotic convergence of the proposed adaptive controller while, in the second case, replacing the actuator state by its adaptive estimate, we prove local regulation. The relevance of the obtained results are illustrated by simulations of a biological activator/repressor system.**

***Index Terms*— Partial differential equation (PDE).**

I. Introduction

THIS paper addresses the general problem of stabilization of (potentially) unstable nonlinear systems subject to *unknown* constant input delay. Such systems are ubiquitous in applications, from automotive engines [17], [32] to process industry [10], [12], [39]. The delay involved in such applications is typically long, slowly time-varying (thus, often considered as constant) and therefore highly uncertain.

To prevent transient performance degradation [29], predictor-based approaches [2], [24], [25] are often considered. Indeed, by using a system state prediction in lieu of the usual current system state, this class of control laws provides input delay compensation. Ideally, the closed-loop system is therefore delay-free and transient performance is notably improved. Due to this substantial advantage, this technique has received considerable attention in the past few years, from alternative implementations design [18], [26], [28] to extension to nonlinear plants [21], [27] or time-varying delays [3], [4], [30].

However, this technique is well-known to suffer from being sensitive to delay mismatch [13], [33], [36]. To the best of the authors' knowledge, the only previous attempt to derive an adaptive design for unknown constant delay was carried out in [22] by applying Padé approximation to approximate the input delay. Nevertheless, this technique is only successful for relatively short delay. The main difficulty in addressing the general problem of unknown long actuator delay comes from the inherent nonlinear parametrization.

In recent papers [6], [8], [9], we tackled this difficulty by representing the constant actuator delay as an Ordinary Differential Equation (ODE) cascaded with a transport Partial Differential Equation (PDE) in which the convective speed is inversely proportional to the (unknown) delay. Indeed, this representation introduces the delay parameter in a linear manner and is therefore in accordance with adaptive design. Here, we pursue this approach, which was developed only for linear systems and extend it to nonlinear dynamics subject to constant input delay. This is the main contribution of the paper.

We consider here two distinct frameworks. First, in Section II, we investigate the case of full-state feedback design,[1] specifically assuming that the distributed input, i.e. the history of the input on a time window of length equal to the delay, is measured. This corresponds to a dynamical system where delay arises from a physical transport process, as opposed to computational or communication-based delay, and in which the distributed input is measured but not the speed of propagation.[2] An example of systems fitting this description is provided as illustration in Section III-D. In this framework, we establish a global delay-adaptive prediction-based stability result. Then, as the distributed input is seldom measured in application, we consider the more realistic case in which both delay and actuator state are unknown (Section IV). The assumption of the actuator state being measured is then only alleviated at the expense of the global validity of the results. Indeed, in this second framework, we establish a local delay-adaptive prediction-based stability result for which design of consistent delay update laws is not Lyapunov-based.

The paper is organized as follows. We begin in Section II by formulating the general problem under consideration. Then, in Section III, we design a delay-adaptive prediction-based control in the case of full-state feedback and, in Section IV, in the case of unmeasured actuator state. In both sections, we formulate a stability result which is then proved by a Lyapunov analysis inspired from the elements proposed in [20] and illustrated by simulation results which enhance the interest of the proposed approach.

Notations: In the following, we use the common definitions of class $\mathcal{K}$ and $\mathcal{K}_\infty$ given in [19]. $|\cdot|$ refers to the Euclidean norm,

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[1] *Full-state feedback* refers here to a control law in terms of (X, U_t) in which $U_t : s \in [-D, 0] \mapsto U(t+s)$ is the actuator state, which is part of the system state.

[2] In practice, only finite samples of this infinite-dimensional variable are known. Therefore, one cannot estimate without uncertainties the delay value from the time- and space-derivatives of the distributed input.

the matrix norm is defined accordingly, for $M \in \mathcal{M}_l(\mathbb{R})(l \in \mathbb{N}^\star)$, as $|M| = \sup_{|x|\leq 1} |Mx|$ and the spatial $\mathcal{L}_1$ and $\mathcal{L}_2$ norms are defined as follows:

$$\|u(t)\|_1 = \int_0^1 |u(x,t)|\, dx,\ \|u(t)\|_2 = \sqrt{\int_0^1 u(x,t)^2 dx}. \quad (1)$$

For $(a,b) \in \mathbb{R}^2$ such that $a < b$, we define the standard projector operator on the interval $[a,b]$ as a function of two scalar arguments f (denoting the parameter being update) and g (denoting the nominal update law) in the following manner:

$$\mathrm{Proj}_{[a,b]}(f,g) = g \begin{cases} 0 & \text{if } f = a \text{ and } g < 0 \\ 0 & \text{if } f = b \text{ and } g > 0 \\ 1 & \text{otherwise.} \end{cases}$$

II. Problem Statement

Consider the following nonlinear plant:

$$\dot{X}(t) = f\left(X, U(t-D)\right) \quad (2)$$

in which $X \in \mathbb{R}^n$, f is a nonlinear function of class $\mathcal{C}^2$ such that $f(0,0) = 0$, U is scalar and D is an unknown delay belonging to a known interval $[\underline{D}, \overline{D}]$ (with $\underline{D} > 0$). The control objective is to stabilize plant (2) following a prediction-based approach, despite delay uncertainties. To handle this last point, we employ an adaptive controller. Before presenting it, we first further characterize the plant under consideration.

Assumption 1: The plant $\dot{X} = f(X, \Omega)$ with Ω scalar is strongly forward complete.

Assumption 2: There exists a feedback law $U(t) = \kappa(X(t))$ such that the nominal delay-free plant is globally exponentially stable and such that κ is a class $\mathcal{C}^2$ function, i.e. there exist (see resp. Theorem 4.14 and Theorem 2.207 in [19], [35]) $\lambda > 0$ and a class $\mathcal{C}^\infty$ radially unbounded positive definite function V such that for $X \in \mathbb{R}^n$

$$\frac{dV}{dX}(X) f\left(X, \kappa(X)\right) \leq -\lambda V(X) \quad (3)$$

$$|X|^2 \leq V(X) \leq c_1 |X|^2 \quad (4)$$

$$\left|\frac{dV}{dX}(X)\right| \leq c_2 |X| \quad (5)$$

for given $c_1, c_2 > 0$.

Assumption 1 guarantees that (2) does not escape in finite time and, in particular, before the input reaches the system at $t = D$. This is a reasonable assumption to enable stabilization. The difference from the standard notion of forward completeness [1] comes from the fact that we assume that $f(0,0) = 0$. *Global* exponential stabilizability required by Assumption 2 is necessary to obtain global adaptive results in presence of delay (Section III) in the following. However, the local result presented in Section IV can be obtained with only a relaxed local version of Assumption 2. For the sake of clarity, we do not pursue this point in the sequel.

To analyze the closed-loop stability despite delay uncertainties, we use the systematic Lyapunov tools introduced in [21] and first reformulate plant (2) in the form

$$\begin{cases} \dot{X}(t) = f\left(X(t), u(0,t)\right) \\ Du_t(x,t) = u_x(x,t) \\ u(1,t) = U(t) \end{cases} \quad (6)$$

by introducing the following distributed input:

$$u(x,t) = U\left(t + D(x-1)\right), \quad x \in [0,1]. \quad (7)$$

In details, the input delay is now represented as a coupling with a transport PDE driven by the input and with unknown convection speed $1/D$.

In the remainder of the paper, we study two distinct problems. In Section III, we investigate delay-adaptive prediction-based control design under the assumption that this distributed actuator $u(\cdot,t)$ is measured. In Section IV, we alleviate this assumption, at the expense of global validity of the results. In both cases, we consider that the system state X is fully measured.

III. Adaptive Control in the Case of Full-State Feedback

In this section, we consider that the actuator state $u(\cdot,t)$ is measured. In this framework, plant (6) introduces a linear delay parametrization which enables delay adaptation via Lyapunov design. Before stating the main result of this section, we formulate additional assumptions.

Assumption 3: The function f is globally Lipschitz with respect to its second arguments, uniformly in the first argument, i.e. there exists $M_L \in \mathbb{R}_+^*$ such that

$$\forall (u_1,u_2) \in \mathbb{R}^2\ \forall X \in \mathbb{R}^n\ \ |f(X,u_1) - f(X,u_2)| \leq M_L |u_1 - u_2|. \quad (8)$$

This Lipschitz assumption bearing on the unknown parameter is usual in nonlinear adaptive control to obtain global results [38].

Assumption 4: The function f, the Jacobian matrix $\partial f / \partial X$, the control law κ and its derivative satisfy the following growth conditions, for given positive constants M_1, M_2, M_3 and M_4

$$|f(X,U)| \leq M_1 \left(|X| + |U|\right), \quad \left|\frac{\partial f}{\partial X}(X,U)\right| \leq M_2 \quad (9)$$

$$|\kappa(X)| \leq M_3 |X|, \quad \left|\frac{d\kappa}{dX}(X)\right| \leq M_4. \quad (10)$$

Assumption 4 actually encompasses Assumption 1. Assumption 3 and 4 can be found quite restrictive but are necessary to state a global delay adaptive stability result. An alternative result which does not require these two assumptions is discussed later in Section III-C.

A. Control Design

Define, for $x \in [0,1]$, the distributed state prediction as

$$p(x,t) = X(t + DX) = X(t) + D \int_0^x f\left(p(y,t), u(y,t)\right) dy \quad (11)$$

which provides a prediction of the system state on the time interval $[t, t+D]$. This expression can be obtained in a straightforward manner with a change of variable and from the definition of the distributed input given in (7). When the delay is known, it can be exactly compensated with the control law $U(t) = \kappa(p(1,t))$. Therefore, following the certainty equivalence principle, the control law is chosen here as:

$$U(t) = \kappa\left(\hat{p}(1,t)\right) \tag{12}$$

in which the distributed predictor estimate is defined as

$$\hat{p}(x,t) = X(t) + \hat{D}(t)\int_0^x f\left(\hat{p}(y,t), u(y,t)\right) dy. \tag{13}$$

Motivated by Lyapunov analysis (presented in the proof of Theorem 1), we choose the delay estimator as[3]

$$\dot{\hat{D}}(t) = \gamma \mathrm{Proj}_{[\underline{D},\bar{D}]}\left\{\hat{D}(t), \tau_D(t)\right\} \tag{14}$$

$$\tau_D(t) = -\frac{\int_0^1 (1+x) q_1(x,t) w(x,t) dx}{1 + V(X) + b\int_0^1 (1+x) w(x,t)^2 dx} \tag{15}$$

where $\mathrm{Proj}_{[\underline{D},\overline{D}]}$ is the standard projector operator on the interval $[\underline{D}, \overline{D}]$ and the transformed state of the actuator is given by

$$w(x,t) = u(x,t) - \kappa\left(\hat{p}(x,t)\right) \tag{16}$$

in which the scalar function q_1 is defined as

$$q_1(x,t) = \frac{d\kappa}{d\hat{p}}\left(\hat{p}(x,t)\right)\Phi(x,0,t) f\left(\hat{p}(0,t), u(0,t)\right) \tag{17}$$

with Φ the transition matrix associated with the space-varying time-parametrized equation $(dr/dx)(x) = \hat{D}(t)(\partial f/\partial \hat{p})(\hat{p}(x,t), u(x,t)) r(x)$ and b a positive parameter.

Theorem 1: Consider the closed-loop system consisting of the plant (2), the control law (12), (13) and the delay update law defined through (14)–(17), satisfying Assumption 1–4. Define the functional

$$\Gamma(t) = |X(t)|^2 + \int_{t-D}^{t} U(s)^2 ds + \tilde{D}^2 \tag{18}$$

in which $\tilde{D}(t) = D - \hat{D}(t)$ is the delay estimation error. If the normalization parameter b is chosen as $b > c_2^2 \bar{D} M_L^2 / 2\lambda$, then there exist $\gamma^* > 0$ and $\rho, R > 0$ (independent of initial conditions) such that, if $\gamma < \gamma^*$

$$\forall t \geq 0, \quad \Gamma(t) \leq R\left(e^{\rho\Gamma(0)} - 1\right) \tag{19}$$

$$X(t) \underset{t\to\infty}{\rightarrow} 0 \quad \text{and} \quad U(t) \underset{t\to\infty}{\rightarrow} .0 \tag{20}$$

[3]Consider the particular case of a linear dynamics $f(X,U) = AX + BU$. In this case, the transition matrix can be explicitly expressed as $\Phi(x,0,t) = e^{A\hat{D}(t)x}$ and one has $\kappa(X) = KX(t)$ with the matrix K such that $A + BK$ is Hurwitz. Therefore, $q_1(x,t) = Ke^{A\hat{D}(t)x}(AX(t) + Bu(0,t))$ and one recovers the delay update law proposed in [23] or in [8] which also deals with plant parameters uncertainties.

B. Proof of Theorem 1—Lyapunov Analysis

We exploit the backstepping transformation (16) to provide a Lyapunov design of the delay update law and to analyze the stability of the closed-loop system. This transformation allows to reformulate plant (6) as stated in the following lemma.

Lemma 1: The infinite-dimensional backstepping transformation (16) together with the control law (12) transform the plant (6) into

$$\dot{X}(t) = f\left(X(t), \kappa\left(X(t)\right) + w(0,t)\right) \tag{21}$$

$$D w_t = w_x - \tilde{D}(t) q_1(x,t) - D\dot{\hat{D}}(t) q_2(x,t) \tag{22}$$

$$w(1,t) = 0 \tag{23}$$

where $\tilde{D}(t) = D - \hat{D}(t)$ is the delay estimation error, the scalar function q_1 is defined in (17) and q_2 is given by

$$q_2(x,t) = \frac{d\kappa}{d\hat{p}}\left(\hat{p}(x,t)\right)\int_0^x \Phi(x,y,t) f\left(\hat{p}(y,t), \kappa\left(\hat{p}(y,t)\right) + w(y,t)\right) dy \tag{24}$$

in which Φ is the transition matrix associated with the space-varying time-parametrized equation $(dr/dx)(x) = \hat{D}(t)(\partial f/\partial \hat{p})(\hat{p}(x,t), w(x,t) + \kappa(\hat{p}(x,t))) r(x)$.

Proof: First, (21) can be directly obtained from (6) using (16) and (13). Then, before studying the governing equation of the transformed distributed actuator, we focus on the dynamics of the distributed predictor. The temporal and spatial derivatives of $\hat{p}(x,t)$ can be expressed as follows:

$$\begin{aligned}\hat{p}_t(x,t) = {} & f\left(\hat{p}(0,t), u(0,t)\right) + \dot{\hat{D}}(t)\int_0^x f\left(\hat{p}(y,t), u(y,t)\right) dy \\ & + \hat{D}(t)\int_0^x \left[\frac{\partial f}{\partial \hat{p}}\left(\hat{p}(y,t), u(y,t)\right)\hat{p}_t(y,t) \right. \\ & \left. + \frac{\partial f}{\partial u}\left(\hat{p}(y,t), u(y,t)\right) u_t(y,t)\right] dy\end{aligned} \tag{25}$$

$$\begin{aligned}\hat{p}_x(x,t) = {} & \hat{D}(t) f\left(\hat{p}(x,t), u(x,t)\right) \\ = {} & \hat{D}(t) f\left(\hat{p}(0,t), u(0,t)\right) \\ & + \hat{D}(t)\int_0^x \left[\frac{\partial f}{\partial \hat{p}}\left(\hat{p}(y,t), u(y,t)\right)\hat{p}_x(y,t) \right. \\ & \left. + \frac{\partial f}{\partial u}\left(\hat{p}(y,t), u(y,t)\right) u_x(y,t)\right] dy.\end{aligned} \tag{26}$$

Therefore, using the governing equation of the distributed input u given in (6), one can obtain

$$\begin{aligned}& D\hat{p}_t(x,t) - \hat{p}_x(x,t) \\ & = \hat{D}(t)\int_0^x \frac{\partial f}{\partial \hat{p}}\left(\hat{p}(y,t), u(y,t)\right)\left[D\hat{p}_t(y,t) - \hat{p}_x(y,t)\right] dy \\ & + \dot{\hat{D}}(t) D\int_0^x f(\hat{p}(y,t), u(y,t))\, dy + \tilde{D}(t) f(\hat{p}(0,t), u(0,t)).\end{aligned} \tag{27}$$

Consider a given time t and define $r(x) = D\hat{p}_t(x,t) - \hat{p}_x(x,t)$. Taking a spatial derivative of the latter expression yields the following equation in x, parametrized in t

$$\begin{cases} \dfrac{dr}{dx}(x) = \hat{D}(t)\dfrac{\partial f}{\partial \hat{p}}\left(\hat{p}(x,t), u(x,t)\right) r(x) \\ \qquad\qquad + \dot{\hat{D}}(t) D f\left(\hat{p}(x,t), u(x,t)\right) \\ r(0) = \tilde{D}(t) f\left(\hat{p}(0,t), u(0,t)\right). \end{cases} \tag{28}$$

Introducing the transition matrix Φ associated with the corresponding homogeneous equation, one can solve this equation and finally get, using the backstepping transformation (16)

$$\begin{aligned} &D\hat{p}_t(x,t) \\ &= \hat{p}_x(x,t) + \tilde{D}(t)\Phi(x,0,t) f\left(X(t), \kappa\left(X(t)\right) + w(0,t)\right) \\ &+ \dot{\hat{D}}(t) D \int_0^x \Phi(x,y,t) f(\hat{p}(y,t), \kappa\left(\hat{p}(y,t)\right) + w(y,t))\, dy. \end{aligned} \tag{29}$$

Now, consider the backstepping transformation (16), whose time- and space-derivatives can be expressed as

$$w_t(x,t) = u_t(x,t) - \frac{d\kappa}{d\hat{p}}\left(\hat{p}(x,t)\right)\hat{p}_t(x,t) \tag{30}$$

$$w_x(x,t) = u_x(x,t) - \frac{d\kappa}{d\hat{p}}\left(\hat{p}(x,t)\right)\hat{p}_x(x,t). \tag{31}$$

From there, using both the previously obtained distributed predictor dynamics (29) and the governing equation of the distributed input u given in (6), one can obtain (22). Finally, the boundary condition (23) comes from the control law (12).

We are now ready to start the Lyapunov analysis. Define the following Lyapunov-Krasovskii functional candidate:

$$W(t) = D\log N(t) + \frac{b}{\gamma}\tilde{D}(t)^2 \tag{32}$$

$$N(t) = 1 + V(X) + b\int_0^1 (1+x) w(x,t)^2 dx \tag{33}$$

where the function V is the positive definite function introduced in Assumption 2. Taking a time-derivative of W, one can get with integration by parts and using Assumption 2 and 3

$$\begin{aligned} \dot{W}(t) \leq \frac{1}{N(t)} \Big(&-D\lambda\left|X(t)\right|^2 + Dc_2 M_L \left|X(t)\right|\left|w(0,t)\right| \\ &- bw(0,t)^2 - b\left\|w(t)\right\|_2^2 \\ &+ 2bD\dot{\hat{D}}(t)\int_0^1 (1+x) w(x,t) q_2(x,t) dx \Big) \\ &+ \frac{2b}{\gamma}\tilde{D}(t)\left(\gamma\tau_D(t) - \dot{\hat{D}}(t)\right) \end{aligned} \tag{34}$$

in which τ_D is defined through (15)–(17). We now introduce two lemmas whose proof is provided in Appendix.

Lemma 2: There exists a positive constant $M_5 > 0$ such that

$$\left|\hat{p}(x,t)\right| \leq M_5\left(|X| + \left\|w(t)\right\|_2\right), \quad x \in [0,1]. \tag{35}$$

Lemma 3: There exist positive constants M_6 and M_7 such that

$$2bD\left|\int_0^1 (1+x) q_2(x,t) w(x,t) dx\right| \leq M_6\left(|X|^2 + \left\|w(t)\right\|_2^2\right) \tag{36}$$

$$\left|\dot{\hat{D}}(t)\right| \leq \gamma M_7. \tag{37}$$

Using Lemmas 2 and 3 and Young's inequality, it follows that:

$$\begin{aligned} \dot{W}(t) \leq \frac{1}{N(t)} \Big(&-\eta\left(\left|X(t)\right|^2 + \left\|w(t)\right\|_2^2\right) - \left(b - \frac{c_2^2 D M_L^2}{2\lambda}\right) \\ &\times w(0,t)^2 + \gamma M_6 M_7\left(\left|X(t)\right|^2 + \left\|w(t)\right\|_2^2\right)\Big) \end{aligned} \tag{38}$$

in which $\eta = \min\{D\lambda/2, b\}$. Therefore, choosing $b > c_2^2 \bar{D} M_L^2/2\lambda$ and

$$\gamma < \gamma^* = \frac{\eta}{M_6 M_7} \tag{39}$$

one can finally obtain the existence of $\eta_0 > 0$ such that

$$\dot{W}(t) \leq -\frac{\eta_0}{N(t)}\left(\left|X(t)\right|^2 + \left\|w(t)\right\|_2^2\right) \tag{40}$$

which leads to the stability result

$$\forall t \geq 0, \quad W(t) \leq W(0) \tag{41}$$

To establish this stability result in terms of Γ, using Lemma 2 and Assumption 4, one can observe that there exist positive constant r_1, r_2, s_1, s_2 such that

$$\left\|u(t)\right\|_2^2 \leq r_1\left|X(t)\right|^2 + r_2\left\|w(t)\right\|_2^2 \tag{42}$$

$$\left\|w(t)\right\|_2^2 \leq s_1\left|X(t)\right|^2 + s_2\left\|u(t)\right\|_2^2. \tag{43}$$

With these inequalities, using Assumption 2 and the definition of Γ and W, respectively, in (18) and (32), one can obtain

$$\left|X(t)\right|^2 \leq V(t) \leq e^{\frac{W(t)}{D}} - 1 \tag{44}$$

$$\begin{aligned} \int_{t-D}^t U(s)^2 ds = D\left\|u(t)\right\|_2^2 &\leq D\left(r_1\left|X(t)\right|^2 + r_2\left\|w(t)\right\|_2^2\right) \\ &\leq D\left(r_1 + \frac{r_2}{b}\right)\left(e^{\frac{W(t)}{D}} - 1\right) \end{aligned} \tag{45}$$

$$\tilde{D}(t)^2 \leq \frac{\gamma}{b}W(t) \leq \frac{\gamma D}{b}\left(e^{\frac{W}{D}} - 1\right) \tag{46}$$

and therefore

$$\Gamma(t) \leq \left(D\left(r_1 + \frac{r_2}{b}\right) + 1 + \frac{\gamma D}{b}\right)\left(e^{\frac{W(t)}{D}} - 1\right). \tag{47}$$

Finally, using Assumption 2

$$W(t) \leq D\left(c_1 + 2b\left(s_1 + \frac{s_2}{D}\right) + \frac{b}{\gamma D}\right)\Gamma(t). \tag{48}$$

Matching these two last inequalities gives the stability result established in Theorem 1.

Finally, to prove convergence of both the system state and the control, we use Barbalat's lemma. From the Lyapunov analysis, integrating (40) from 0 to ∞, one can obtain that $|x(t)|$ and $U(t)$ are square integrable. Further

$$\frac{d}{dt}\left(|X(t)|^2\right) = 2X(t)^T f\left(X(t), u(0,t)\right). \tag{49}$$

From (41), it follows that $|x(t)|$ and $\|w(t)\|_2$ are uniformly bounded. Consequently, $\hat{p}(x,t)$ is uniformly bounded for any $x \in [0,1]$ and so is $u(x,t)$ and in particular $u(0,t)$. As f is continuous, we get the uniform boundedness of $|x(t)|^2$. Finally, we conclude with Barbalat's Lemma that $X(t) \to 0$ as $t \to \infty$.

Similarly

$$\frac{d}{dt}\left(U(t)^2\right) = 2U(t)\frac{d\kappa}{dx}\left(\hat{p}(1,t)\right)\hat{p}_t(1,t) \tag{50}$$

in which

$$\hat{p}_t(1,t) = \frac{1}{D}\left[\hat{D}(t)f\left(\hat{p}(1,t),u(1,t)\right) + \tilde{D}(t)f\left(\hat{p}(0,t),u(0,t)\right) + \dot{\hat{D}}(t)D\int_0^1 \Phi(1,y)f\left(\hat{p}(y,t),u(y,t)\right)dy\right]. \tag{51}$$

Applying Assumption 4 and using the continuity of f together with the previous considerations on Φ, one can show that this derivative is uniformly bounded. Therefore, one can conclude that $U(t) \to 0$ as $t \to \infty$.

C. Comments

Theorem 1 states a global asymptotic convergence result, provided that Assumption 3 and 4 are satisfied. In details, Assumption 3 is used in the Lyapunov analysis in (34) to bound the difference $(dV/dX)(X)[f(X,\kappa(X)+w(0,t)) - f(X,\kappa(X))]$, which arises from the comparison between the nominal delay-free dynamics $f(X,\kappa(X))$ and the actual one $f(X,\kappa(X)+w(0,t))$.[4] Therefore, this assumption is crucial to obtain a lower-bound for b which does not depend on the initial conditions.[5] Further, Assumption 4 plays a key role in the derivation of Lemma 2 and 3 and is therefore fundamental in obtaining a bound γ^* which does not depend on the initial conditions.

Yet, alternatively, one can obtain the following semi-global asymptotic result without requiring these (restrictive) assumptions.

Theorem 2: Consider the closed-loop system consisting of plant (2) satisfying Assumption 1 and 2, of the control law (12) and of the delay update law

$$\dot{\hat{D}}(t) = \mathrm{Proj}_{[\underline{D},\bar{D}]}\left\{\gamma\tau_D(t)\right\} \tag{52}$$

[4]In the same sense that usually, in adaptive control, one is led to consider the difference $f(t,\hat{\theta}(t)) - f(t,\theta)$ in which θ is the unknown parameter and to formulate similar Lipschitz assumption with regards to f [38].

[5]Alternatively, one can refer to the Lyapunov analysis developed in Section IV-B to see how this difference can be bounded thanks to the mean-value theorem, without requiring any Lipschitz assumption, but yields bounds depending on the initial conditions.

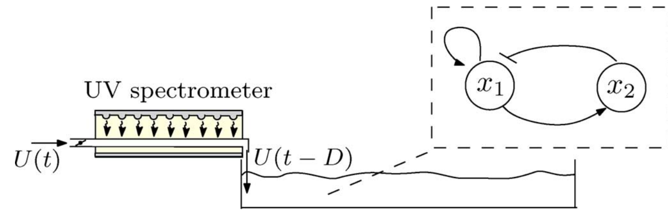


Fig. 1. Considered system set-up with an Activator-Repressor motif. The system is fed by a delayed activator protein addition $U(t-D)$. The activator protein (concentration x_1) promotes the expression of itself and of a repressor protein (concentration x_2) that represses the expression of the activator protein. This interaction is represented in the top-right corner, with directed edges (arrows stand for activation and vertices ended by a line stand for repression; an arrow linking a node x to a node y represents the effect of node x on node y).

$$\tau_D(t) = \frac{1}{1+V(x)+b\int_0^1(1+x)\left[|w(x,t)|+|w_x(x,t)|\right]dx} \times \Big[2\mathrm{sgn}\left(w_x(1,t)\right)q_1(1,t) + \int_0^1(1+x)\left[q_1(x,t)\mathrm{sgn}\left(w(x,t)\right) + q_3(x,t)\mathrm{sgn}\left(w_x(x,t)\right)\right]dx\Big] \tag{53}$$

in which the scalar function q_1 and q_3 are defined as follows:

$$q_1(x,t) = \frac{d\kappa}{d\hat{p}}\left(\hat{p}(x,t)\right)f\left(\hat{p}(0,t),u(0,t)\right) \tag{54}$$

$$q_3(x,t) = q_{1,x}(x,t). \tag{55}$$

Define the functional

$$\Gamma(t) = |X(t)| + \int_{t-D}^{t}|U(s)|\,ds + \int_{t-\bar{D}}^{t}\left|\dot{U}(s)\right|ds + \tilde{D}(t)^2. \tag{56}$$

There exists γ^*, b^* (depending on the initial conditions) and a class $\mathcal{K}_\infty$ function α^* such that, if $b > b^*$ and $\gamma < \gamma^*$

$$\forall t \geq 0, \quad \Gamma(t) \leq \alpha^*\left(\Gamma(0)\right) \tag{57}$$

$$X(t) \underset{t\to\infty}{\to} 0 \quad \text{and} \quad U(t) \underset{t\to\infty}{\to} 0. \tag{58}$$

For the sake of brevity, we do not detail here the proof of this theorem, which follows lines similar to those used in the following section, except from the derivation of the delay update law which follows from Lyapunov design. We simply highlight the fact that, in this result, the bounds γ^* and b^* depend on the initial conditions, while Theorem 1 provides bounds that do not depend on them. Therefore, in this sense, this result is weaker than Theorem 1 but does not require Assumptions 3 and 4 to be satisfied. Second, as in the proof below, without Assumption 4, the Lyapunov analysis has to be performed with a $\mathcal{L}_1$-norm functional instead of the $\mathcal{L}_2$ one employed above. This explains the unusual form of the delay update law (52), (53) in which the sign function appears (see [40]).

D. Simulation Example

In this section, we consider as illustration an activator/repressor system studied for example in [11], [37], composed by two proteins: an activator protein that promotes the expression of itself and of a repressor protein that represses the expression of the activator

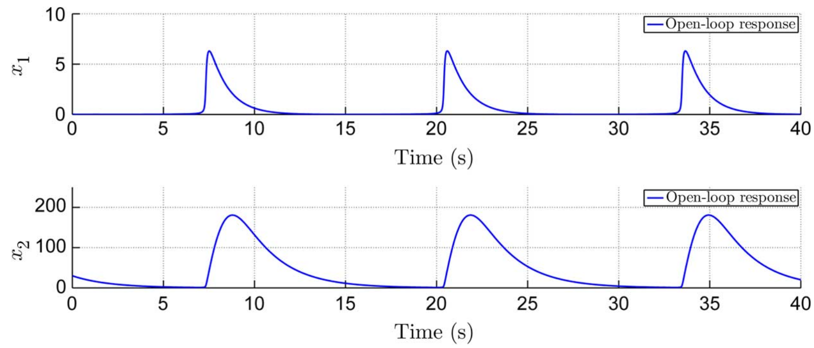


Fig. 2. Simulation of the open-loop response of plant (59) with $X(0) = [0.01\ 30]^T$. A stable periodic orbit appears.

protein. Such a circuit, represented in Fig. 1, is known to potentially exhibit a stable periodic orbit, depending on the values of the biochemical parameters. This behavior is of particular interest for synthetic biologists, as it can be used as a clock motif for other applications in the fields of Systems and Synthetic Biology. It has been interpreted in [11] as the result of the simultaneous existence of an unstable equilibrium point and of an attractive manifold for certain parameter values of the system.

Here, we consider that such a synthetic clock has been designed. We focus on the converse problem of stopping temporarily this clock (i.e., on stabilizing the system towards its unstable equilibrium), e.g., to synchronize it with another one for a given experimentation purposes. With this aim in view, we consider the experimental setup depicted in Fig. 1. The activator/repressor dynamics is controlled through the introduction of an additional amount of activator protein inside the system. As this setup involves transportation through an inlet pipe, an input delay arises which is constant (assuming that the inlet fluid has a constant flow rate and that only its activator concentration changes). This delay is uncertain, due to the involved flow rate scale. The laboratory experimental set-up is made of a UV spectrometer located along the inlet pipe and which enables to detect the (distributed) presence of activator protein along the pipe. Therefore, the distributed input $u(\cdot,t)$ can be assumed as known here.[6] Finally, we consider that the protein concentrations x_1 and x_2 are measured, with another spectrometer for example, assuming that concentrations in the solution are locally measured and that these signals are representative of the global distribution in the solution.

Following [37], the systems dynamics write:

$$\begin{cases} \dot{x}_1 = -x_1 + f_1(x_1,x_2) + U(t-D) \\ \dot{x}_2 = -\frac{x_2}{2} + f_2(x_1) \end{cases} \tag{59}$$

in which x_1 is the concentration of activator protein, x_2 the repressor one and in which f_1 and f_2 are Hill functions associated with proteins expressions given by

$$f_1(x_1,x_2) = \frac{K_1 x_1^2 + K_a}{1 + x_1^2 + x_2^2}$$
$$f_2(x_1) = \frac{K_2 x_1^2 + K_b}{1 + x_1^2}$$

[6]As underlined previously, however, it may be difficult to use this distributed input to directly estimate the delay value. Indeed, due to sampling and noise, the computation of its spatial- and time-derivative would likely result into a non-negligible estimation error. Yet, this information could be used to sharpen the adaptive technique, for example to improve the initial delay estimate.

where $K_1 = K_2 = 300$, $K_a = 0.04$ and $K_b = 0.004$. For this parameter, there exists an unstable equilibrium point $X^* = (x_1^\star, x_2^\star) \in \mathbb{R}_+^* \times \mathbb{R}_+^*$ and the system exhibits a periodic motif which is provided in Fig. 2.

A control law satisfying Assumption 2 is

$$\kappa(X(t)) = -f_1(x_1,x_2) + f_1(x_1^*, x_2^*) \tag{60}$$

for which an associated Lyapunov function is simply $V = (X - X^*)^T (X - X^*)$. Further, one can show that this control law and the vector field involved in the error dynamics

$$\tilde{f}(\tilde{X}) = \begin{pmatrix} f_1(x_1,x_2) - f_1(x_1^*, x_2^*) \\ f_2(x_1,x_2) - f_2(x_1^*, x_2^*) \end{pmatrix} \tag{61}$$

satisfy Assumption 4 in terms of the error variables $\tilde{X} = X - X^*$ and $\tilde{U} = U - U^*$.

Fig. 3 shows simulation results corresponding to the closed-loop response obtained with control law (12), (13) together with the delay update law defined through (14)–(17) and for initial conditions $X(0) = [0.01\ 30]^T$, $U_{|[-D,0]} = 0$. The delay update gain is chosen as $\gamma = 1000$, and the delay lower and upper bounds as $\underline{D} = 0.01$ and $\bar{D} = 4$. Two initial delay estimates are considered, $\hat{D}(0) = 0.01$ sec and $\hat{D}(0) = 2$ sec respectively. For the sake of comparison, a prediction-based control without delay adaptation, i.e. with $\hat{D}(t) = \hat{D}(0) = 2$ sec is also provided. For the sake of realism, the control law has been lower-saturated by zero, as it represents a non-negative variable; however, for the chosen set of initial conditions, this modification does not impact the significance of the obtained results. For implementation, the various integrals involved in the controller were discretized using a trapezoidal approximation and the transition matrix involved in the update law was numerically calculated by direct integration of the corresponding matrix differential equation.

First, one can notice that, as stated in Theorem 1, the proposed adaptive controller achieves asymptotic stabilization independently on the initial delay estimate the nominal prediction-based strategy fails (without delay adaptation, the system response continues to exhibit an oscillatory behavior). This is due to the state delay which appears here due to delay mismatch and induces a delay differential equation which is known to be responsible of oscillations [14]. Decreasing the delay estimation error would in all likelihood suppress these oscillations and achieve stabilization. This is the main interest of the proposed result, which enables stabilization despite significant initial delay estimation errors.

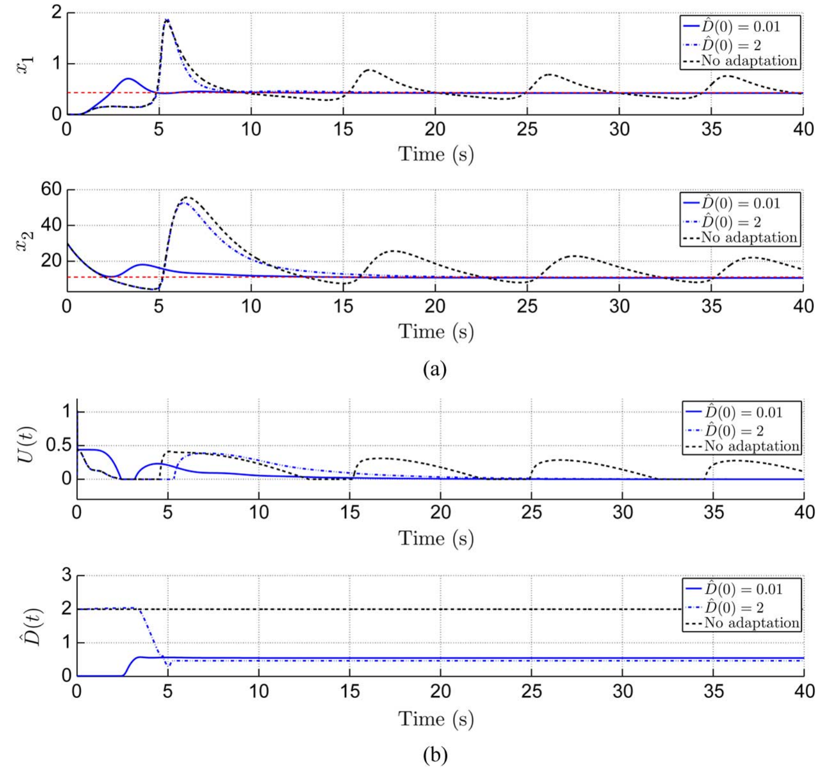


Fig. 3. Simulation results of the plant (59) with $D = 1$, $X(0) = [0.01\ 30]^T$, $U_{|[-D,0]} = 0$ and respectively $\hat{D}(0) = 0.01$ and $\hat{D}(0) = 2$. The update gain is chosen as $\gamma = 1000$. (a) Evolution of system state two different values of initial delay estimate. The red dotted curves represent the equilibrium set-point. (b) Control law and delay update law for two different values of initial delay estimate.

Second, it is worth noticing that the delay update law does not provide convergence of the delay estimate toward the unknown delay value. This phenomenon is well-known in adaptive control [15], [16], but does not prevent the plant to converge.

IV. Control Design in the Case of Unmeasured Distributed Input

In this section we consider the actuator state $u(\cdot, t)$ to be unmeasured, as is typically the case in applications.

A. Control Design

To deal with the fact that the actuator state is unmeasured, we introduce a distributed input estimate

$$\hat{u}(x,t) = U\left(t + \hat{D}(t)(x-1)\right), \quad x \in [0,1]. \tag{62}$$

Applying the certainty equivalence principle, the control law is chosen as

$$U(t) = \kappa\left(\hat{p}(1,t)\right) \tag{63}$$

in which the distributed predictor estimate is defined in terms of the actuator state estimate as

$$\hat{p}(x,t) = X\left(t + \hat{D}(t)x\right) = X(t) + \hat{D}(t)\int_0^x f\left(\hat{p}(y,t), \hat{u}(y,t)\right) dy \tag{64}$$

and the delay estimate satisfies one of the following assumptions.

Assumption 5: There exist a $\mathcal{C}^1$ function τ and positive parameters $\gamma > 0$ and $M > 0$ such that

$$\dot{\hat{D}}(t) = \gamma \mathrm{Proj}_{[\underline{D},\overline{D}]}\left\{\hat{D}(t), \tau(t)\right\} \tag{65}$$

$$\tilde{D}(t)\tau(t) \geq 0, \quad |\tau(t)| \leq M \quad \text{and} \quad |\dot{\tau}(t)| \leq M. \tag{66}$$

Example 1: Consider the following instantaneous cost function, proposed in [6] for a linear plant

$$\varphi : (t, \hat{D}) \in \left[t_0, \infty\right[\mapsto \left|X_P(t,\hat{D}) - X(t)\right| \tag{67}$$

where $X_P(t, \hat{D})$ is a $(t - t_0)$-units of time ahead prediction of the system state, starting from the initial condition $X(t_0)$ and assuming that the delay value is $\hat{D}$. Then, using a steepest descent argument, one can take

$$\tau_D(t) = -\left(X_P(t,\hat{D}) - X(t)\right)^T \frac{\partial X_P}{\partial \hat{D}}(t, \hat{D}) \tag{68}$$

with

$$X_P(t,\hat{D}) = X(t_0) + \int_{t_0}^t f\left(X_P(s), U(s-\hat{D})\right) ds \tag{69}$$

$$\frac{\partial X_P}{\partial \hat{D}}(t,\hat{D}) = -\int_{t_0}^t \frac{\partial f}{\partial U}\left(X_P(s), U(s-\hat{D})\right) \dot{U}(s-\hat{D}) ds. \tag{70}$$

This update law satisfies Assumption 5 [34], provided that the initial delay estimate is sufficiently close to the true delay value (no local minima interference). This condition is in accordance with the local condition stated in Theorem 3 below. This delay update law is further discussed in Section IV-D.

Assumption 6: There exist a $\mathcal{C}^1$ function τ, a positive parameter $\gamma > 0$ and class $\mathcal{K}$ functions $\alpha_{1,D}$ and $\alpha_{2,D}$ such that

$$\dot{\hat{D}}(t) = \gamma \text{Proj}_{[\underline{D},\overline{D}]} \left\{ \hat{D}(t), \tau(t) \right\} \quad (71)$$

$$|\tau(t)| \leq \alpha_{1,D}\left(\Gamma_0(t)\right) \quad \text{and} \quad |\dot{\tau}(t)| \leq \alpha_{2,D}\left(\Gamma_0(t)\right) \quad (72)$$

in which the functional Γ_0 is defined as

$$\Gamma_0(t) = |X(t)| + \int_{t-\max\left\{D,\hat{D}(t)\right\}}^{t} |U(s)|\, ds + \int_{t-\max\left\{D,\hat{D}(t)\right\}}^{t} \left|\dot{U}(s)\right| ds + \int_{t-\hat{D}(t)}^{t} \left|\ddot{U}(s)\right| ds. \quad (73)$$

Example 2: Applying the certainty equivalence principle to the delay update law stated in Theorem 2, one can choose

$$\tau_D(t) = 2\text{sgn}\left(\hat{w}_x(1,t)\right) q_1(1,t) + \int_0^1 (1+x)\left[q_1(x,t)\text{sgn}\left(\hat{w}(x,t)\right) + q_3(x,t)\text{sgn}\left(\hat{w}_x(x,t)\right)\right] dx \quad (74)$$

in which $\hat{w}$ is the following backstepping transformation of the distributed input estimate

$$\hat{w}(x,t) = \hat{u}(x,t) - \kappa\left(\hat{p}(x,t)\right) \quad (75)$$

and the scalar functions q_1 and q_3 are given by

$$q_1(x,t) = \frac{d\kappa}{d\hat{p}}\left(\hat{p}(x,t)\right) f\left(\hat{p}(0,t), \hat{u}(0,t)\right) \quad (76)$$

$$q_3(x,t) = q_{1,x}(x,t). \quad (77)$$

Applying elements from the Lyapunov analysis detailed below, one can show that this update law satisfies Assumption 6. We do not detail this point here, for sake of clarity and conciseness. This delay update law is further discussed in Section IV-D.

Theorem 3: Consider the closed-loop system consisting of plant (2) and control law (63) satisfying Assumption 1 and 2 with a delay estimate satisfying either Assumption 5 or 6. Define the functional

$$\Gamma(t) = |X(t)| + \int_{t-\max\left\{D,\hat{D}(t)\right\}}^{t} |U(s)|\, ds + \int_{t-\max\left\{D,\hat{D}(t)\right\}}^{t} \left|\dot{U}(s)\right| ds + \int_{t-\hat{D}(t)}^{t} |\ddot{U}(s)| ds + \tilde{D}(t)^2. \quad (78)$$

Then, there exist $\gamma^* > 0$ (potentially depending on the initial conditions), $\rho > 0$ and a class $\mathcal{K}_\infty$ function α^* such that, if $\Gamma(0) \leq \rho$ and if $\gamma < \gamma^*$, then

$$\forall t \geq 0, \quad \Gamma(t) \leq \alpha^*\left(\Gamma(0)\right) \quad (79)$$

$$X(t) \underset{t\to\infty}{\to} 0 \quad \text{and} \quad U(t) \underset{t\to\infty}{\to} 0. \quad (80)$$

Theorem 3 is a local result, requiring the initial delay estimate to be sufficiently close to the unknown delay value and the initial system state to small enough. Assumptions 5 and 6 allow one to consider many choices of delay update laws, consistent with the stability property (79) of the controller. In a nutshell, Assumption 5 requires the delay update law not to make the delay estimation worse which is in accordance with various identification techniques [31]. Then, employing such a delay on-line update law would hopefully provide delay identification, provided that persistence of excitation is guaranteed. This, ideally, would allow a large flexibility for control. On the other hand, Assumption 6 could be interpreted as a guarantee that computational errors while computing a delay update law satisfying Assumption 5 do not jeopardize closed-loop stability. Alternatively, other kinds of delay update laws such as the one proposed in Example 2 can be considered. It is worth noticing that the condition stated in Assumption 6 cannot be directly checked as it involves the unknown delay value. For strict implementation, a constructive but more restrictive choice is to employ $\underline{D}$ instead of D in definition (73). In both cases, Theorem 3 requires the delay update law to be slow enough, i.e., the update gain to be small enough, to be consistent with closed-loop stability.

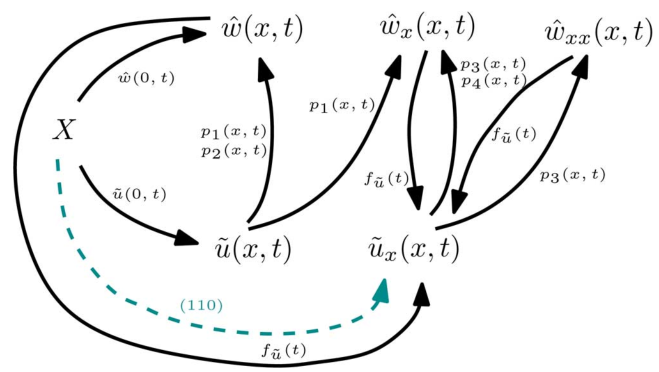


Fig. 4. Relations between the different variables involved in the Lyapunov analysis. The black solid edges represent relations inherent to the variable dynamics while the dashed cyan line represents a relation coming from Lyapunov derivation. Edges are oriented from the variable whose dynamics involves the designated one.

To prove this result, we introduce infinite-dimensional tools similar to the ones introduced in the previous section.

B. Proof of Theorem 3—Lyapunov Analysis

To ease the understanding of the Lyapunov analysis provided in this section and of its mechanism, the interested reader can refer to Fig. 4 as a reading guide, as it summarizes the main relations between the different involved variables, and to Section IV-C which contains several comments. One can also refer to [40] for examples of stability analysis in terms of $\mathcal{L}_1$-norms and to [6] in which a similar Lyapunov analysis is performed for linear systems.

As previously, to provide stability analysis, we first reformulate (6) using a suitable backstepping transformation.

Lemma 4: The backstepping transformation of the distributed input estimate (62)

$$\hat{w}(x,t) = \hat{u}(x,t) - \kappa\left(\hat{p}(x,t)\right) \quad (81)$$

in which the distributed predictor estimate is defined in (64), together with the control law (63), transforms plant (6) into

$$\dot{X}(t) = f\left(X(t), \kappa\left(X(t)\right) + \hat{w}(0,t) + \tilde{u}(0,t)\right) \quad (82)$$

$$\hat{D}(t)\hat{w}_t = \hat{w}_x + \dot{\hat{D}}(t)q_1(x,t) - q_2(x,t)f_{\tilde{u}}(t) \tag{83}$$
$$\hat{w}(1,t) = 0 \tag{84}$$
$$D\tilde{u}_t = \tilde{u}_x - \tilde{D}(t)p_1(x,t) - \dot{\hat{D}}(t)p_2(x,t) \tag{85}$$
$$\tilde{u}(1,t) = 0 \tag{86}$$

in which

$$\tilde{u}(x,t) = u(x,t) - \hat{u}(x,t) \tag{87}$$

is the distributed input estimation error and

$$p_1(x,t) = \frac{D}{\hat{D}(t)}\left[\hat{w}_x(x,t) + \hat{D}(t)\frac{d\kappa}{d\hat{p}}\left(\hat{p}(x,t)\right) \times f\left(\hat{p}(x,t), \hat{w}(x,t) + \kappa\left(\hat{p}(x,t)\right)\right)\right] \tag{88}$$

$$p_2(x,t) = \frac{D}{\hat{D}(t)}(x-1)\left[\hat{w}_x(x,t) + \hat{D}(t)\frac{d\kappa}{d\hat{p}}\left(\hat{p}(x,t)\right) \times f\left(\hat{p}(x,t), \hat{w}(x,t) + \kappa\left(\hat{p}(x,t)\right)\right)\right] \tag{89}$$

$$\begin{aligned} q_1(x,t) =& (x-1)\left[\hat{w}_x(x,t) + \hat{D}(t)\frac{d\kappa}{d\hat{p}}\left(\hat{p}(x,t)\right) \times f\left(\hat{p}(x,t), \hat{w}(x,t) + \kappa\left(\hat{p}(x,t)\right)\right)\right] \\ &- \hat{D}(t)\frac{d\kappa}{d\hat{p}}\left(\hat{p}(x,t)\right) \times \int_0^x \Phi(x,y,t) \\ &\times \left[f\left(\hat{p}(y,t), \hat{w}(y,t) + \kappa\left(\hat{p}(y,t)\right)\right) + \frac{\partial f}{\partial \hat{u}}\left(\hat{p}(y,t), \hat{w}(y,t) + \kappa\left(\hat{p}(y,t)\right)\right)(y-1) \right. \\ &\left. \times \left[\hat{w}_x(y,t) + \hat{D}(t)\frac{d\kappa}{d\hat{p}}\left(\hat{p}(y,t)\right) \times f\left(\hat{p}(y,t), \hat{w}(y,t) + \kappa\left(\hat{p}(y,t)\right)\right)\right]\right] dy \end{aligned} \tag{90}$$

$$q_2(x,t) = \hat{D}(t)\frac{d\kappa}{d\hat{p}}\left(\hat{p}(x,t)\right)\Phi(x,0,t) \tag{91}$$

$$f_{\tilde{u}}(t) = f\left(\hat{p}(0,t), u(0,t)\right) - f\left(\hat{p}(0,t), \hat{u}(0,t)\right) \tag{92}$$

where Φ is the transition matrix associated with the space-varying time-parametrized equation $(dr/dx)(x) = \hat{D}(t)(\partial f/\partial \hat{p})(\hat{p}(x,t), \hat{w}(x,t)+\kappa(\hat{p}(x,t)))r(x)$.

Proof: This proof follows lines similar to those used in the previous section. First, (82) can be directly obtained from definitions (7), (81) and the one of $\tilde{u}$. Second, one can easily obtain from (62) that the estimate distributed input satisfies

$$\hat{D}(t)\hat{u}_t(x,t) = \hat{u}_x(x,t) + \dot{\hat{D}}(t)(x-1)\hat{u}_x(x,t) \tag{93}$$
$$\hat{u}(1,t) = U(t). \tag{94}$$

As in the previous section, considering a given $t \geq 0$ and denoting $r(x) = \hat{D}(t)\hat{p}_t(x,t) - \hat{p}_x(x,t)$, one can show that $r(x)$ satisfies the following equation in x, parametrized in t

$$\begin{cases} \dfrac{dr}{dx}(x) = \hat{D}(t)\dfrac{\partial f}{\partial \hat{p}}\left(\hat{p}(x,t), \hat{u}(x,t)\right)r(x) + \dot{\hat{D}}(t)\hat{D}(t) \\ \qquad \times \left[f\left(\hat{p}(x,t), \hat{u}(x,t)\right) + \dfrac{\partial f}{\partial \hat{u}}\left(\hat{p}(x,t), \hat{u}(x,t)\right)(x-1)\hat{u}_x(x,t)\right] \\ r(0) = \hat{D}(t)\left[f\left(\hat{p}(0,t), u(0,t)\right) - f\left(\hat{p}(0,t), \hat{u}(0,t)\right)\right] \end{cases}$$

Defining the transition matrix Φ associated to the corresponding homogeneous equation, one can solve this equation and obtain

$$\begin{aligned} \hat{D}(t)\hat{p}_t =& \hat{p}_x + \dot{\hat{D}}(t)\hat{D}(t)\int_0^x \Phi(x,y,t) \\ &\times \left[f\left(\hat{p}(y,t), \hat{u}(y,t)\right) + \frac{\partial f}{\partial \hat{u}}\left(\hat{p}(y,t), \hat{u}(y,t)\right)(y-1)\hat{u}_x(y,t)\right] dy \\ &+ \Phi(x,0,t)\hat{D}(t)\left[f\left(\hat{p}(0,t), u(0,t)\right) - f\left(\hat{p}(0,t), \hat{u}(0,t)\right)\right]. \end{aligned} \tag{95}$$

Now, matching the time- and space-derivatives of the backstepping transformation (81) with the governing (93) and (95), one can obtain (83) and use the backstepping transformation (81) to express the functions q_1 and q_2 in terms of $\hat{w}$ and its spatial-derivative. ■

In the following, we also need the governing equation of its spatial derivative, which are given in the following lemma, the proof of which is provided in Appendix.

Lemma 5: The spatial derivatives of the distributed input estimation error (87) and of the backstepping transformation (81) satisfy

$$\begin{cases} D\tilde{u}_{xt} = \tilde{u}_{xx} - \tilde{D}(t)p_3(x,t) - \dot{\hat{D}}(t)p_4(x,t) \\ \tilde{u}_x(1,t) = \tilde{D}(t)p_1(1,t) \end{cases} \tag{96}$$

$$\begin{cases} \hat{D}(t)\hat{w}_{xt} = \hat{w}_{xx} + \dot{\hat{D}}(t)q_3(x,t) - q_4(x,t)f_{\tilde{u}}(t) \\ \hat{w}_x(1,t) = -\dot{\hat{D}}(t)q_1(1,t) + q_2(1,t)f_{\tilde{u}}(t) \end{cases} \tag{97}$$

$$\begin{cases} \hat{D}(t)\hat{w}_{xxt} = \hat{w}_{xxx} + \dot{\hat{D}}(t)q_5(x,t) - q_6(x,t)f_{\tilde{u}}(t) \\ \hat{w}_{xx}(1,t) = -\dot{\hat{D}}(t)q_3(1,t) + q_4(1,t)f_{\tilde{u}}(t) + \hat{D}(t)q_7(t) \end{cases} \tag{98}$$

in which $p_3 = p_{1,x}$, $p_4 = p_{2,x}$, $q_3 = q_{1,x}$, $q_4 = q_{2,x}$, $q_5 = q_{3,x}$, $q_6 = q_{4,x}$ and $q_7 = \dot{\hat{D}}(t)w_{xt}(1,t)$ whose expressions are not provided for the sake of brevity of the exposition but can be found in [7].

We are now ready to start the Lyapunov analysis, considering the following Lyapunov-Krasovskii functional candidate

$$\begin{aligned} W(t) =& V_0(X) + b_0 D\int_0^1 (1+x)\left|\tilde{u}(x,t)\right| dx \\ &+ b_1 D\int_0^1 (1+x)\left|\tilde{u}_x(x,t)\right| dx \\ &+ b_2\hat{D}(t)\int_0^1 (1+x)\left|\hat{w}(x,t)\right| dx \\ &+ b_3\hat{D}(t)\int_0^1 (1+x)\left|\hat{w}_x(x,t)\right| dx \\ &+ b_4\hat{D}(t)\int_0^1 (1+x)\left|\hat{w}_{xx}(x,t)\right| dx + \tilde{D}(t)^2 \end{aligned} \tag{99}$$

in which $V_0 = \sqrt{V}$ and V is defined in Assumption 2. Taking a time-derivative of V_0 and using the properties stated in Assumption 2, one can obtain the following inequality:

$$\dot{V}_0 \leq -\frac{\lambda}{2}|X| + \frac{c_2}{2}\left|\frac{\partial f}{\partial u}(X, d_1(X,t))\right| |\tilde{u}(0,t) + \hat{w}(0,t)| \tag{100}$$

in which we have applied the mean-value theorem to introduce $d_1(X,t)$ which is a variable between $\kappa(X)$ and $\kappa(X) + \hat{u}(0,t)$. Therefore, taking a time-derivative of W and using integration by parts, one can get

$$\begin{aligned}
&\dot{W}(t)\\
&\leq -\frac{\lambda}{2}|X| + \frac{c_2}{2}\left|\frac{\partial f}{\partial u}(X, d_1(X,t))\right| |\tilde{u}(0,t) + \hat{w}(0,t)|\\
&\quad - b_0\|\tilde{u}(t)\|_1 - b_0|\tilde{u}(0,t)| + b_0\left|\tilde{D}(t)\right|\int_0^1(1+x)|p_1(x,t)|dx\\
&\quad + b_0\left|\dot{\hat{D}}(t)\right|\int_0^1(1+x)|p_2(x,t)|dx - b_1\|\tilde{u}_x(t)\|_2^2\\
&\quad + b_1|\tilde{u}_x(1,t)| - b_1|\tilde{u}_x(0,t)| + b_1\left|\tilde{D}(t)\right|\int_0^1(1+x)|p_3(x,t)|dx\\
&\quad + b_1\left|\dot{\hat{D}}(t)\right|\int_0^1(1+x)|p_4(x,t)|dx - b_2\|\hat{w}(t)\|_1 - b_2|\hat{w}(0,t)|\\
&\quad + b_2\left|\dot{\hat{D}}(t)\right|\int_0^1(1+x)|q_1(x,t)|dx\\
&\quad + b_2\int_0^1(1+x)|q_2(x,t)f_{\tilde{u}}(t)|dx\\
&\quad - b_3\|\hat{w}_x(t)\|_2^2 - b_3\hat{w}_x(0,t)^2 + b_3|\hat{w}_x(1,t)|\\
&\quad + b_3|\dot{\hat{D}}(t)|\int_0^1(1+x)|q_3(x,t)|dx\\
&\quad + b_3\int_0^1(1+x)|q_4(x,t)f_{\tilde{u}}(t)|dx\\
&\quad + b_4|\hat{w}_{xx}(1,t)| - b_4|\hat{w}_{xx}(0,t)| - b_4\|\hat{w}_{xx}(t)\|_1\\
&\quad + b_4\left|\dot{\hat{D}}(t)\right|\int_0^1(1+x)|q_5(x,t)|dx\\
&\quad + b_4\int_0^1(1+x)|q_6(x,t)f_{\tilde{u}}(t)|dx\\
&\quad + b_2\dot{\hat{D}}(t)\int_0^1(1+x)|\hat{w}(x,t)|dx\\
&\quad + b_3\dot{\hat{D}}(t)\int_0^1(1+x)|\hat{w}_x(x,t)|dx\\
&\quad + b_4\dot{\hat{D}}(t)\int_0^1(1+x)|\hat{w}_{xx}(x,t)|dx - 2\dot{\hat{D}}(t)\tilde{D}(t).
\end{aligned} \tag{101}$$

To bound the positive terms remaining in this last expression, we use technical lemmas which are provided in Appendix for sake of clarity of the main exposition and define the following alternative functional:

$$\begin{aligned}
W_0(t) = {}& |X(t)| + \|\tilde{u}(t)\|_1 + \|\hat{w}(t)\|_1 + \|\tilde{u}_x(t)\|_1\\
&+ \|\hat{w}_x(t)\|_1 + \|\hat{w}_{xx}(t)\|_1.
\end{aligned} \tag{102}$$

As f is continuously differentiable and the variable $d_1(X,t)$ is between $\kappa(X)$ and $\kappa(X) + \hat{u}(0,t)$, applying Lemma 7 given in Appendix, one can obtain the existence of a $\mathcal{K}_\infty$ function α_{21} such that

$$\frac{c_2}{2}\left|\frac{\partial f}{\partial u}(X, d_1(X,t))\right| \leq \alpha_{21}(W_0(t)). \tag{103}$$

Then, introducing $\eta = \min\{\lambda/2, b_0, b_1, b_2, b_3, b_4\}$ and using Lemma 8 provided in Appendix, one can obtain

$$\begin{aligned}
&\dot{W}(t)\\
&\leq -\Big(\eta W_0(t) - \left|\dot{\hat{D}}(t)\right|\\
&\qquad \times [b_0\alpha_4(W_0(t)) + b_1\alpha_7(W_0(t))\\
&\qquad\quad + b_2\alpha_8(W_0(t)) + b_3\alpha_{12}(W_0(t)) + b_4\alpha_{14}(W_0(t))\\
&\qquad\quad + 2(b_3 + b_3 + b_4)W_0(t) + b_3\alpha_{10}(W_0(t))]\\
&\qquad - b_4\left(\left|\dot{\hat{D}}(t)\right| + \dot{\hat{D}}(t)^2 + \left|\dot{\hat{D}}(t)\right|^3\right)\alpha_{16}(W_0(t))\\
&\qquad - b_4|\ddot{\hat{D}}(t)|\alpha_{19}(W_0(t)) - \left|\tilde{D}(t)\right|\\
&\qquad \times [b_0\alpha_3(W_0(t)) + b_1\alpha_6(W_0(t)) + b_1\alpha_{11}(W_0(t))\\
&\qquad\quad + b_1\alpha_5(W_0(t)) + b_1|\dot{\hat{D}}(t)|\alpha_{10}(W_0(t))\\
&\qquad\quad + b_4\alpha_{20}(W_0(t))]\Big)\\
&\quad - (b_1 - b_4\alpha_{17}(W_0(t)))|\tilde{u}_x(0,t)|\\
&\qquad - (b_0 - b_2\alpha_9(W_0(t)) - b_3\alpha_{11}(W_0(t)) - b_3\alpha_{13}(W_0(t))\\
&\qquad\quad - b_4\alpha_{15}(W_0(t)) - b_4\alpha_{18}(W_0(t)) - \alpha_{21}(W_0(t)))\\
&\quad \times |\tilde{u}(0,t)| - (b_2 - \alpha_{21}(W_0(t)))|\hat{w}(0,t)|\\
&\quad - (b_3 - b_4\alpha_{17}(W_0(t)))|\hat{w}_x(0,t)| - b_4\left(1 - \left|\dot{\hat{D}}(t)\right|\right)|\hat{w}_{xx}(0,t)|\\
&\quad - 2\dot{\hat{D}}(t)\tilde{D}(t).
\end{aligned} \tag{104}$$

We now distinguish two cases depending on the type of delay update law that is considered.

1) Lyapunov Analysis the First Class of Delay Update Law (Assumption 5): We use Assumption 5 to rewrite (104). Denote $R = W_0(0)$. To ensure that the last terms in (104) are initially negative, we choose the parameters such that

$$b_1 > b_4\alpha_{17}(R),\ b_2 > \alpha_{21}(R),\ b_3 > b_4\alpha_{17}(R) \tag{105}$$

$$\begin{aligned}
b_0 > {}& b_2\alpha_9(R) + b_3\alpha_{11}(R) + b_3\alpha_{13}(R)\\
&+ b_4\alpha_{15}(R) + b_4\alpha_{18}(R) + \alpha_{21}(R).
\end{aligned} \tag{106}$$

Introduce the functions

$$\begin{aligned}
\alpha_1^*(W_0) = {}& M[b_0\alpha_4(W_0(t)) + b_1\alpha_7(W_0(t)) + b_2\alpha_8(W_0(t))\\
&\quad + b_3\alpha_{12}(W_0(t)) + b_4\alpha_{14}(W_0(t))\\
&\quad + 2(b_3 + b_3 + b_4)W_0(t)]\\
&+ b_3M^2\alpha_{10}(W_0(t)) + b_4(M^2 + M^4 + M^6)\\
&\times \alpha_{16}(W_0(t)) + b_4M\alpha_{19}(W_0(t))
\end{aligned} \tag{107}$$

$$\alpha_2^*(W_0) = b_0\alpha_3(W_0(t)) + b_1\alpha_6(W_0(t)) + b_1\alpha_{11}(W_0(t)) + b_1\alpha_5(W_0(t)) + b_1M^2\alpha_{10}(W_0(t)) + b_4\alpha_{20}(W_0(t)) \quad (108)$$

and choose $\gamma < 1$. Then, for $|W_0(t)| \leq R$

$$\dot{W}(t) \leq -\left(\eta W_0(t) - \gamma\alpha_1^*(W_0(t)) - |\tilde{D}(t)|\alpha_2^*(W_0(t))\right) - b_4(1-\gamma M)\left|\hat{w}_{xx}(0,t)\right|. \quad (109)$$

Now, using the following inequality, which results from Young's inequality

$$\left|\tilde{D}(t)\right| \leq \frac{\varepsilon}{2} + \frac{1}{2\varepsilon}\tilde{D}(t)^2 \leq \frac{\varepsilon}{2} + \frac{1}{2\varepsilon}W(t) \quad (110)$$

one can obtain

$$\dot{W}(t) \leq -\left(\eta W_0(t) - \gamma\alpha_1^*(W_0(t)) - \left(\frac{\varepsilon}{2} + \frac{1}{2\varepsilon}W(t)\right)\alpha_2^*(W_0(t))\right) - b_4(1-\gamma M)|\hat{w}_{xx}(0,t)|. \quad (111)$$

Therefore, by choosing for a given $\nu \in]0,1[$

$$\gamma < \gamma^* = \max\left\{1, \frac{1}{M}, \frac{\nu\eta}{\max_{x\in[0,R]} \alpha_1^{*\prime}(x)}\right\} \quad (112)$$

$$\epsilon < 2\frac{\nu\eta - \gamma\max_{x\in[0,R]}\alpha_1^{*\prime}(x)}{\max_{x\in[0,R}\alpha_2^{*\prime}(x)} \quad (113)$$

$$W(0) \leq 2\epsilon\frac{\nu\eta - \gamma\max_{x\in[0,R]}\alpha_1^{*\prime}(x) - \frac{\varepsilon}{2}\max_{x\in[0,R]}\alpha_2^{*\prime}(x)}{\max_{x\in[0,R]}\alpha_2^{*\prime}(x)} \quad (114)$$

one can ensure that

$$\dot{W}(t) \leq -(1-\nu)\eta W_0(t) \quad (115)$$

and consequently the following inequality which concludes the Lyapunov analysis:

$$W(t) \leq W(0). \quad (116)$$

2) Lyapunov Analysis for the Second Class of Delay Update Law (Assumption 6): Observing that there exists two class $\mathcal{K}_\infty$ functions such that W_0 and Γ_0 are equivalent, Assumption 6 can easily be reformulated in terms of $W_0(t)$. Under this assumption, one can rewrite (104) Denote $R = W_0(0)$. To ensure that the last terms in (104) are negative, we choose the parameters as stated in (105), (106). Now, introduce the functions

$$\begin{aligned}\alpha_1^*(W_0) = {} & \alpha_{1,D}(W_0(t)) \\ & \times [b_0\alpha_4(W_0(t)) + b_1\alpha_7(W_0(t)) + b_2\alpha_8(W_0(t)) \\ & \quad + b_3\alpha_{12}(W_0(t)) + b_4\alpha_{14}(W_0(t)) \\ & \quad + 2(b_3 + b_3 + b_4)W_0(t)] \\ & + b_3\alpha_{1,D}(W_0(t))^2\alpha_{10}(W_0(t)) \\ & + b_4\left(\alpha_{1,D}(W_0(t))^2 + \alpha_{1,D}(W_0(t))^4 \right.\\ & \quad \left. + \alpha_{1,D}(W_0(t))^6\right) \\ & \times \alpha_{16}(W_0(t)) + b_4\alpha_{2,D}(W_0(t))\,\alpha_{19}(W_0(t))\end{aligned} \quad (117)$$

$$\alpha_2^*(W_0) = b_0\alpha_3(W_0(t)) + 2\alpha_{1,D}(W_0(t)) + b_1\alpha_6(W_0(t)) + b_1\alpha_{11}(W_0(t)) + b_1\alpha_5(W_0(t)) + b_1\alpha_{1,D}(W_0(t))^2\alpha_{10}(W_0(t)) + b_4\alpha_{20}(W_0(t)) \quad (118)$$

and choose $\gamma < 1$. Then, for $|W_0(t)| < R$

$$\dot{W}(t) \leq -\left(\eta W_0(t) - \gamma\alpha_1^*(W_0(t)) - |\tilde{D}(t)|\alpha_2^*(W_0(t)))\right) - b_4(1-\gamma\alpha_{1,D}(W_0(t)))\,|\hat{w}_{xx}(0,t)|. \quad (119)$$

Therefore, by choosing for a given $\nu \in]0,1[$

$$\gamma < \gamma^* = \max\left\{1, \frac{1}{\alpha_{1,D}(R)}, \frac{\nu\eta}{\max_{x\in[0,R]}\alpha_1^{*\prime}(x)}\right\} \quad (120)$$

the same choices and conclusions than in the previous section can be made and yield (115) and (116). This concludes the Lyapunov analysis.

To provide a stability result in terms of Γ, using Assumption 2, one can show that there exist two class $\mathcal{K}_\infty$ functions α_3^* and α_4^* such that $\alpha_3^*(t)(\Gamma(t)) \leq W(t) \leq \alpha_4^*(\Gamma(t))$. Then

$$\Gamma(t) \leq \alpha_3^{*-1}(W(0)) \leq \alpha_3^{*-1}(\alpha_4^*(\Gamma(0))). \quad (121)$$

The stability result stated in Theorem 3 follows with $\alpha^* = \alpha_3^{*-1} \circ \alpha_4^*$.

We now use Barbalat's lemma to prove convergence. From the Lyapunov analysis, integrating (115) from 0 to ∞, one can obtain that $|x(t)|$ and $|U(t)|$ are integrable. Further

$$\frac{d}{dt}\left(|X(t)|^2\right) = 2X(t)^T f(X(t), u(0,t)). \quad (122)$$

From (116), it follows that $|x(t)|$, $\|\tilde{u}_x(t)\|_2$, $\|\hat{w}(t)\|_1$ and $\|\hat{w}_x(t)\|_2$ are uniformly bounded. Consequently, applying Lemma 7, $u(0,t)$ is uniformly bounded. As f is continuous, we get the uniform boundedness of $|x(t)|^2$ and consequently of $|x(t)|$. Finally, we conclude with Barbalat's Lemma that $X(t) \to 0$ as $t \to \infty$.

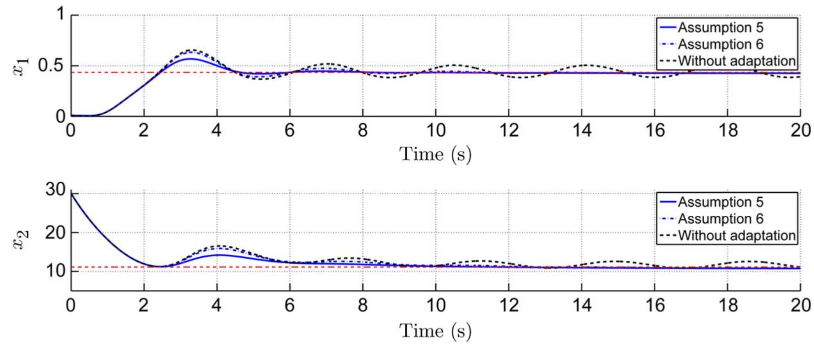
Similarly

$$\frac{d}{dt}(U(t)) = \frac{d\kappa}{dx}(\hat{p}(1,t))\,\hat{p}_t(1,t) \quad (123)$$

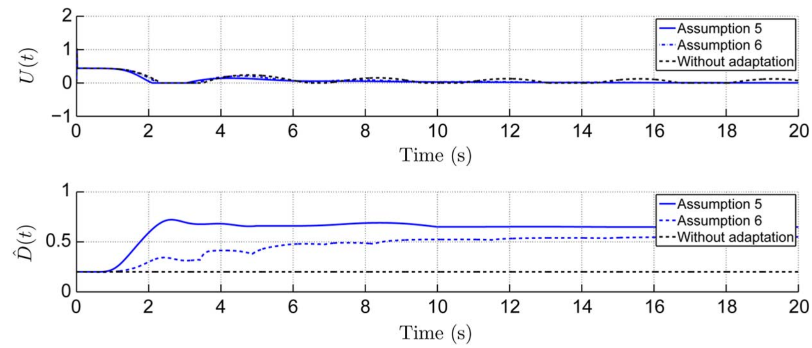
in which

$$\begin{aligned}\hat{p}_t(1,t) = {} & \frac{1}{\hat{D}(t)}\Big[\hat{D}(t)f(\hat{p}(1,t),\hat{u}(1,t)) + \Phi(1,0)\hat{D}(t)f_{\tilde{u}}(t) \\ & + \dot{\hat{D}}(t)\hat{D}(t)\int_0^1 \Phi(1,y) \\ & \times\left[f(\hat{p}(y,t),\hat{u}(y,t)) + \frac{\partial f}{\partial \hat{u}}(\hat{p}(y,t),\hat{u}(y,t))\right. \\ & \times\left.(y-1)\hat{u}_x(y,t)\right]dy\Big].\end{aligned} \quad (124)$$

Applying Lemma 7 and 6, one can bound this quantity in terms of W_0 which is uniformly bounded. Therefore, one can conclude that $U(t) \to 0$ as $t \to \infty$.



(a)



(b)

Fig. 5. Simulation results of the plant (59) with $D = 1$, $X(0) = [0.01\ 30]^T$, $U_{[-D,0]} = 0$ and $\hat{D}(0) = 0.2$ and with three different delay update laws: (68) satisfying Assumption 5, (74)–(77) satisfying Assumption 6 and $\dot{\hat{D}} = 0$. The update gains are chosen as $\gamma = 50000$ and $\gamma = 0.5$ for the two first delay update laws. (a) Evolution of system state three different delay update laws. The red dotted curves represent the equilibrium set-point. (b) Control law and delay update law for three different delay update laws.

C. Comments

Consider Fig. 4, summarizing the main relations between the dynamics (82)–(86) and (96)–(98). From there, one can see that to study the system state stability, it is necessary to take into account the dynamics of $\hat{w}(\cdot, t)$ and $\tilde{u}(\cdot, t)$ and, sequentially, the ones of $\hat{w}_x(\cdot, t)$ and $\tilde{u}_x(\cdot, t)$ and therefore the one of $\hat{w}_{xx}(\cdot, t)$. This explains the choice of the Lyapunov functional (99).

Finally, a technical point that is worth noticing is the reason why the analysis is based on $\mathcal{L}_1$-norms and not $\mathcal{L}_2$-norms like in the previous section. One can observe that most of the bounding operations realized in the Lyapunov analysis are based on Lemma 6 and 7 given in Appendix. These two lemmas involve the system state norm $|x|$ and not its square. Without additional assumptions, like the growth condition stated in Assumption 4 in Section III, these bounds cannot be sharpened and it is therefore necessary to include a term in $|x|$ in the Lyapunov analysis and not its square. This yields the choice of the first term $V_0 = V$ in the Lyapunov functional (99). Observing (100), it is therefore necessary to study $\mathcal{L}_1$-norms of the spatial variables in order to bound the remaining positive term $|\tilde{u}(0, t) + \hat{w}(0, t)|$. This also explains why Theorem 2, obtained by relieving Assumption 4, is stated with $\mathcal{L}_1$-norms.

D. Simulation Example

To illustrate the merits of our approach, consider again the illustrative example considered in Section III-D. Now, we consider a slightly different experimental setup in which no UV spectrometer is used. The distributed input $u(\cdot, t)$ is therefore unmeasured.

Fig. 5 reports the simulation results obtained for the same initial conditions, $X(0) = [0.01\ 30]^T$, $U_{[-D,0]} = 0$ and $\hat{D}(0) = 0.1$ while $D = 1$, and for the two delay update laws presented in Examples 1 and 2. For the sake of comparison, the closed-loop response without adaptation, i.e., with $\hat{D}(t) = \hat{D}(0) = 0.2$ is also provided.

First, one can observe that the first two adaptive prediction-based controller achieve stabilization while the third one fails. Like previously, this is due to the fact that the delay mismatch obtained in the third case results is too important to enable stabilization. On the other hand, the proposed adaptive strategy allows to reduce this mismatch and to stabilize the plant. This is the main interest of the proposed result, which increases the stability domain, meaning that, for similar (relatively small) initial estimation errors, the adaptive prediction-based controller provides stabilization where a constant-delay one can fail.

Second, it is interesting to notice that the first delay update law provides notable delay estimation improvements at the beginning of the simulation, i.e., before 2.5 s. This is consistent with the employed update law as the cost function exhibits its most pronounced gradient on this time interval. Yet, this update law does not monotonically increase, as should be the case from its definition. This can be reasonably attributed to discretization. Neither this update law nor the second converges to the true delay, but the first one could be reasonably expected to do so, would persistence of excitation exist.

Finally, we wish to stress that the selection of the update gain γ is a key step in the controller tuning, influencing the closed-loop performance. According to the presented results (both Theorems 1 and Theorem 3), this gain has to be chosen sufficiently small to guarantee stabilization. However, choosing an extremely small update gain results in slow delay adaptation and therefore in very poor closed-loop performance. Consequently, a tradeoff has to be reached while selecting the adaptation parameter. In all likelihood, the expressions of $\gamma^\star$ provided in the Lyapunov analysis, respectively, in (39) and (112) are conservative and cannot be used in practice to choose the update gain value. Analysis of the closed-loop performance induced by the proposed controller is a direction of future work.

V. Conclusion

In this paper, we have presented the first systematic delay-adaptive prediction-based control design for nonlinear systems with unknown but constant long actuator delay. We proposed two different asymptotic results, according to availability of the actuator state measurement. For the full-state feedback case, the designed adaptive controller has been proven to be globally stabilizing while, when the actuator state is replaced by its adaptive estimate, local stability and regulation are achieved. The proposed results may be of particular interest in the case of an integral-type transport delay systems (corresponding to a Plug-Flow assumption [5]), which, as shown in [41], can be reformulated as nonlinear plant with constant input-delay. Extension to the problems with unknown plant parameters, trajectory tracking or output feedback are directions for future work.

Appendix

A. Proof of Technical Lemmas Used in Section III

1) Proof of Lemma 2: The distributed predictor satisfies the spatial ODE

$$\hat{p}_x = \hat{D}(t) f\left(\hat{p}(x,t), w(x,t) + \kappa\left(\hat{p}(x,t)\right)\right) \quad (125)$$

$$\hat{p}(0,t) = X(t). \quad (126)$$

Therefore, as $\hat{D}(t) \in [\underline{D}, \overline{D}]$, as f is continuous and satisfies Assumption 1 and 4 and as κ satisfies the growth condition introduced in Assumption 1, there exists M_5 such as defined in the Lemma.

2) Proof of Lemma 3: Consider the expression of q_2 given in (24) which involves the transition matrix Φ. By definition, for given $y \in [0,1]$ and $t \geq 0$, Φ satisfies the differential equation

$$\frac{d\Phi}{dx}(x,y,t) = \hat{D}(t) \frac{\partial f}{\partial \hat{p}}(\hat{p}(x,t), u(x,t)) \Phi(x,y,t), \quad x \in [y,1] \quad (127)$$

$$\Phi(y,y,t) = I \quad (128)$$

and therefore its norm satisfies, for given $y \in [0,1]$ and $t \geq 0$, and for any $x \in [y,1]$

$$\frac{d|\Phi|}{dx}(x,y,t) \leq \left|\frac{d\Phi}{dx}(x,y,t)\right| \leq \bar{D}\left|\frac{\partial f}{\partial \hat{p}}(\hat{p}(x,t), u(x,t))\right| \times |\Phi(x,y,t)| \quad (129)$$

with $|\Phi(y,y,t)| = 1$. Consequently, using Assumption 4, one can obtain the following upper bound:

$$|\Phi(x,y,t)| \leq \overline{D} \max_{x \in [0,1]} \left|\frac{\partial f}{\partial \hat{p}}\left(\hat{p}(x,t), u(x,t)\right)\right| \leq \bar{D} M_2. \quad (130)$$

From there, using Assumption 4, Lemma 2 and Cauchy-Schwartz's inequality, one can obtain the existence of M_6 such as stated in the Lemma. With the same arguments, using (4) projector operator properties and straightforward bounding techniques, one obtains the existence of $M_7 > 0$ such that

$$|\dot{\hat{D}}(t)| \leq \gamma \left| \frac{\int_0^1 (1+x) q_1(x,t) w(x,t) dx}{1 + V(X) + b\int_0^1 (1+x) w(x,t)^2 dx} \right| \leq \gamma M_7.$$

3) Proof of Lemma 5: Taking a spatial derivative of (85), one can obtain the governing equation in (96) and, from the boundary condition (86), that $\tilde{u}_t(1,t) = 0$ which gives, replacing in (85), the boundary condition in (96).

The exact same arguments applied to (83), (84) governing the backstepping transformation give system (97).

Taking a spatial derivative of the first equation in (97) give the one in (98). Finally, using the first equation in (97) for $x = 1$, one can obtain

$$\hat{w}_{xx}(1,t) = -\dot{\hat{D}}(t) q_3(x,t) + q_4(1,t) f_{\tilde{u}}(t) + \hat{D}(t) \hat{w}_{xt}(1,t) \quad (131)$$

in which $\hat{w}_{x,t}(1,t) = q_7(t)$ can be reformulated by taking a time derivative of the boundary condition in (97).

B. Technical Lemmas Used in the Lyapunov Analysis in Section IV

Lemma 6: There exists class $\mathcal{K}_\infty$ functions α_1 and α_2 such that

$$|\hat{p}(x,t)| \leq \alpha_1\left(|X| + \|\hat{u}(t)\|_1\right), \quad x \in [0,1] \quad (132)$$

$$|\hat{p}(x,t)| \leq \alpha_2\left(|X| + \|\hat{w}(t)\|_1\right), \quad x \in [0,1]. \quad (133)$$

Proof: The distributed predictor estimate satisfies the spatial ODE

$$\hat{p}_x = \hat{D}(t) f\left(\hat{p}(x,t), \hat{u}(x,t)\right) \quad (134)$$

$$\hat{p}(0,t) = X(t). \quad (135)$$

Therefore, as $\hat{D}(t) \in [\underline{D}, \overline{D}]$, as f is continuous and as the plant is strictly forward complete, there exists a class $\mathcal{K}_\infty$ function α_1 such as introduced in the Lemma. Alternatively, using the definition (81), one can obtain

$$\hat{p}_x = \hat{D}(t) f\left(\hat{p}(x,t), \hat{w}(x,t) + \kappa\left(\hat{p}(x,t)\right)\right) \quad (136)$$

$$\hat{p}(0,t) = X(t) \quad (137)$$

and, from the same reason that previously, obtain the existence of α_2 such as defined in the lemma. ■

Lemma 7: There exist $\mathcal{K}_\infty$ functions α_{16}, α_{17} such that, for all $x \in [0,1]$

$$|\hat{u}(x,t)| \leq \alpha_{21}\left(|X(t)| + \|\hat{w}(t)\|_1 + \|\hat{w}_x(t)\|_1\right) \quad (138)$$

$$|\tilde{u}(x,t)| \leq \alpha_{22}\left(|X(t)| + \|\hat{w}(t)\|_1 + \|\tilde{u}_x(t)\|_1 + \|\hat{w}_x(t)\|_1\right) \quad (139)$$

Proof: By integration, one gets the following inequality:

$$|\hat{u}(x,t)| \leq |\hat{u}(1,t)| + \int_0^1 |\hat{u}_x(x,t)|\, dx. \quad (140)$$

Rewriting $\hat{u}_x$ thanks to (81) and using Lemma 6 yield the existence of the desired function α_{21}. Similar arguments yield the existence of α_{22}. ■

Lemma 8: There exist $\mathcal{K}_\infty$ functions $\alpha_1, \ldots, \alpha_{13}$ and a constant $M_0 > 0$ such that

$$\int_0^1 (1+x)\,|p_1(x,t)|\,dx \leq \alpha_3\left(|X(t)| + \|\hat{w}(t)\|_1 + \|\hat{w}_x(t)\|_1\right) \tag{141}$$

$$\int_0^1 (1+x)\,|p_2(x,t)|\,dx \leq \alpha_4\left(|X(t)| + \|\hat{w}(t)\|_1 + \|\hat{w}_x(t)\|_1\right) \tag{142}$$

$$|\tilde{u}_x(1,t)| \leq \left|\tilde{D}(t)\right| \alpha_5\left(|X(t)| + \|\hat{w}(t)\|_1\right) + \left|\tilde{D}(t)\right| M_0\, |\hat{w}_x(1,t)| \tag{143}$$

$$\int_0^1 (1+x)\,|p_3(x,t)|\,dx \leq \alpha_6\left(|X(t)| + \|\hat{w}(t)\|_1 + \|\hat{w}_{xx}(t)\|_1\right) \tag{144}$$

$$2\int_0^1 (1+x)\,|p_4(x,t)|\,dx \leq \alpha_7\left(|X(t)| + \|\hat{w}(t)\|_1 + \|\hat{w}_x(t)\|_1 + \|\hat{w}_{xx}(t)\|_1\right) \tag{145}$$

$$\int_0^1 (1+x)\,|q_1(x,t)|\,dx \leq \alpha_8\left(|X(t)| + \|\hat{w}(t)\|_1 + \|\hat{w}_x(t)\|_1\right) \tag{146}$$

$$\int_0^1 (1+x)\,|q_2(x,t) f_{\tilde{u}}(t)|\,dx \leq \alpha_9\left(|X(t)| + \|\hat{w}(t)\|_1 + \|\tilde{u}_x(t)\|_1 + \|\hat{w}_x(t)\|_1\right) |\tilde{u}(0,t)| \tag{147}$$

$$\begin{aligned}|\hat{w}_x(1,t)| \leq{}& \left|\dot{\hat{D}}(t)\right| \alpha_{10}\left(|X(t)| + \|\hat{w}(t)\|_1 + \|\hat{w}_x(t)\|_1\right)\\ &+ \alpha_{11}\left(|X(t)| + \|\hat{w}(t)\|_1 + \|\tilde{u}_x(t)\|_1 + \|\hat{w}_x(t)\|_1\right) |\tilde{u}(0,t)|\end{aligned} \tag{148}$$

$$\int_0^1 (1+x)\,|q_3(x,t)|\,dx \leq \alpha_{12}\left(|X(t)| + \|\hat{w}(t)\|_1 + \|\hat{w}_x(t)|_1\right) \tag{149}$$

$$\begin{aligned}2\int_0^1 (1+x)\,|q_4(x,t) f_{\tilde{u}}(t)|\,dx \leq{}& \alpha_{13}\left(|X(t)| + \|\hat{w}(t)\|_1 + \|\tilde{u}_x(t)\|_1 + \|\hat{w}_x(t)\|_1\right)\\ &\times |\tilde{u}(0,t)|\end{aligned} \tag{150}$$

$$\begin{aligned}\int_0^1 (1+x)\,|q_5(x,t)|\,dx \leq{}& |\hat{w}_{xx}(0,t)|\\ &+ \alpha_{14}\left(|X(t)| + \|\hat{w}(t)\|_1 + \|\hat{w}_x(t)\|_1 + \|\hat{w}_{xx}(t)|_1\right)\end{aligned} \tag{151}$$

$$\begin{aligned}\int_0^1 (1+x)\,|q_6(x,t) f_{\tilde{u}}(t)|\,dx \leq{}& \alpha_{15}\left(|X(t)| + \|\hat{w}(t)\|_1 + \|\tilde{u}_x(t)\|_1 + \|\hat{w}_x(t)\|_1 + \|\hat{w}_{xx}(t)|_1\right)\\ &\times |\tilde{u}(0,t)|\end{aligned} \tag{152}$$

$$\begin{aligned}|\hat{w}_{xx}(1,t)| \leq{}& \left(\left|\dot{\hat{D}}(t)\right| + \left|\dot{\hat{D}}(t)\right|^2 + \left|\dot{\hat{D}}(t)\right|^3\right)\\ &\times \alpha_{16}\left(|X(t)| + \|\hat{w}(t)\|_1 + \|\tilde{u}_x(t)\|_1 + \|\hat{w}_x(t)\|_1 + \|\hat{w}_{xx}(t)\|_1\right)\\ &+ \alpha_{17}\left(|X(t)| + \|\hat{w}(t)\|_1 + \|\tilde{u}_x(t)\|_1 + \|\hat{w}_x(t)\|_1\right)\\ &\times \left(|\tilde{u}_x(0,t)| + |\hat{w}_x(0,t)|\right)\\ &+ \alpha_{18}\left(|X(t)| + \|\hat{w}(t)\|_1 + \|\tilde{u}_x(t)\|_1 + \|\hat{w}_x(t)\|_1\right) |\tilde{u}(0,t)|\\ &+ \left|\ddot{\hat{D}}(t)\right| \alpha_{19}\left(|X(t)| + \|\hat{w}(t)\|_1 + \|\tilde{u}_x(t)\|_1 + \|\hat{w}_x(t)\|_1\right)\\ &+ \left|\tilde{D}(t)\right| \alpha_{20}\left(|X(t)| + \|\hat{w}(t)\|_1 + \|w_x(t)\|_1\right).\end{aligned} \tag{153}$$

Proof: From the expression of p_1 and the backstepping transformation (81), one can obtain

$$p_1(x,t) = \frac{D}{\hat{D}(t)}\left[\hat{w}_x(x,t) + \frac{d\kappa}{d\hat{p}}\left(\hat{p}(x,t)\right)\hat{D}(t) f\left(\hat{p}(x,t), \hat{w}(x,t) + \kappa\left(\hat{p}(x,t)\right)\right)\right]. \tag{154}$$

From there, Lemma 6 together with the continuity of f, κ and of its derivative give the existence of α_3 such as introduced in (141). The exact same arguments yield to the existence of α_4 introduced in (142). From the definition of $\tilde{u}_x(1,t)$, one can get

$$\tilde{u}_x(1,t) = \tilde{D}(t)\frac{D}{\hat{D}(t)} \times \left[\hat{w}_x(1,t) + \hat{D}(t)\frac{d\kappa}{d\hat{p}}\left(\hat{p}(x,t)\right) f\left(\hat{p}(x,t), \hat{u}(x,t)\right)\right]. \tag{155}$$

Applying Lemma 6 to this expression give (143).

Now, consider the expression of q_1. The same arguments that in the proof of Lemma 3 yield

$$|\Phi(x,y,t)| \leq \overline{D} \max_{x\in[0,1]} \left|\frac{\partial f}{\partial \hat{p}}\left(\hat{p}(x,t), \hat{u}(x,t)\right)\right|. \tag{156}$$

From there, using the backstepping transformation (81) to express $\hat{u}$ and $\hat{u}_x(x,t)$ as functions of respectively $\hat{w}(x,t)$ and $\hat{p}(x,t)$ and $\hat{w}_x(x,t)$, $\hat{w}(x,t)$ and $\hat{p}(x,t)$, Lemma 6 and the continuity of f, κ and of their derivatives, give the existence of α_7 such as introduced in (146).

From the expression of $f_{\tilde{u}}$, applying the mean-value theorem, one can obtain the existence of d_2 between $u(0,t)$ and $\hat{u}(0,t)$ such that

$$f_{\tilde{u}}(t) = \frac{\partial f}{\partial \hat{u}}\left(X(t), d_2(t)\right)\tilde{u}(0,t) \tag{157}$$

Using Lemma 7 for d_2, Lemma 6, the previous considerations on $|\Phi|$ and the continuity of the derivative of κ, one can then bound $|q_2(x,t)(\partial f/\partial\hat{u})(X(t), d_2(t))|$ as desired and obtain the existence of α_9 as stated in (147). Similar considerations applied to $|\hat{w}_x(1,t)|$ yield the existence of α_{10} and α_{11} as defined in (148).

From the definitions of p_3, p_4, q_3 and q_4 (one can refer to [7] for complete calculation of these functions), one can express these functions in terms of $\hat{p}(x,t)$, $\hat{w}(x,t)$ and its derivative and, using Lemma 6 together with the fact that f and κ are class C^2 functions, obtain the existence of α_6 such as introduced in (144), of α_7 in (145) and similarly derive (149) and (150). Then, observing that the higher order spatial derivative term appearing in the product q_5 can be bounded, using integration by part

$$\begin{aligned}\int_0^1 &(x^2-1)\,|\hat{w}_{xxx}(x,t)|\,dx\\ &= \left[(x^2-1)\text{sign}\left(w_{xxx}(x,t)\right) w_{xx}(x,t)\right]_0^1\\ &\quad - \int_0^1 2x\text{sign}\left(w_{xxx}(x,t)\right) w_{xx}(x,t)dx\\ &\leq |\hat{w}_{xx}(0,t)| + \int_0^1 2x\,|\hat{w}_{xx}(x,t)|\,dx. \end{aligned} \tag{158}$$

Equations (151) and (152) can be obtained from the previously presented arguments.

Finally, the expression (153) can be obtained by lengthy calculations applying triangle inequality to the definition of $\hat{w}_{xx}(1,t)$ (provided in details in [7]). Indeed, $q_3(1,t)$ and $q_4(1,t)$ can easily be bounded as desired and the same considerations as previously apply to $f_{\tilde{u}}$. The main difficulties arise while bounding $q_7(t)$. Applying the triangle inequality to $q_7(t)$ and previous arguments, one obtain a term in $|\dot{\hat{D}}(t)|$ as stated in (153) and one in $|\tilde{D}(t)|$. Two terms in $|\tilde{u}_x(0,t)| + |\hat{w}_x(0,t)|$ and $|\tilde{u}(0,t)|$ can also be obtained, as the following terms appear in the expression of q_7:

$$\begin{aligned} f_{dp}(t) &= \frac{\partial f}{\partial \hat{p}}\left(\hat{p}(0,t), u(0,t)\right) - \frac{\partial f}{\partial \hat{p}}\left(\hat{p}(0,t), \hat{u}(0,t)\right)\\ &= \frac{\partial^2 f}{\partial \hat{p}\partial \hat{u}}\left(\hat{p}(0,t), d_3(t)\right)\tilde{u}(0,t) \end{aligned} \tag{159}$$

$$\begin{aligned} f_{du}(t) &= \frac{\partial f}{\partial \hat{u}}\left(\hat{p}(0,t), u(0,t)\right) - \frac{\partial f}{\partial \hat{u}}\left(\hat{p}(0,t), \hat{u}(0,t)\right)\\ &= \frac{\partial^2 f}{\partial \hat{u}^2}\left(\hat{p}(0,t), d_4(t)\right)\tilde{u}(0,t) \end{aligned} \tag{160}$$

with d_3 and d_4 between $\tilde{u}(0,t)+\hat{u}(0,t)$ and $\hat{u}(0,t)$ and applying the same considerations as previously. Finally, the first term in (153) can be obtained by a careful analysis of the expression of $p_{1,t}$ involved in q_7. ■

REFERENCES

[1] D. Angeli and E. D. Sontag, "Forward completeness, unboundedness observability, and their Lyapunov characterizations," *Syst. Control Lett.*, vol. 38, no. 4, pp. 209–217, 1999.

[2] Z. Artstein, "Linear systems with delayed controls: A reduction," *IEEE Trans. Autom. Control*, vol. 27, no. 4, pp. 869–879, 1982.

[3] N. Bekiaris-Liberis and M. Krstic, "Compensation of time-varying input and state delays for nonlinear systems," *J. Dyn. Syst., Meas. Control*, vol. 134, 2012.

[4] N. Bekiaris-Liberis and M. Krstic, *Nonlinear Control Under Nonconstant Delays*. Philadelphia, PA: Society for Industrial and Applied Mathematics (SIAM), 2013, vol. 25.

[5] D. Bresch-Pietri, J. Chauvin, and Petit, "Sufficient condition for prediction-based stabilization of linear system subject to input-dependent input-delay," in *Proc. Amer. Control Conf.*, 2013, pp. 144–151.

[6] D. Bresch-Pietri, J. Chauvin, and N. Petit, "Adaptive control scheme for uncertain time-delay systems," *Automatica*, vol. 48, pp. 1536–1552, 2012.

[7] D. Bresch-Pietri and M. Krstic, Backstepping Transformation of Nonlinear Systems Subject to Unknown Input Delay [Online]. Available: http://arxiv.org/abs/1305.5305

[8] D. Bresch-Pietri and M. Krstic, "Adaptive trajectory tracking despite unknown input delay and plant parameters," *Automatica*, vol. 45, no. 9, pp. 2074–2081, 2009.

[9] D. Bresch-Pietri and M. Krstic, "Delay-adaptive predictor feedback for systems with unknown long actuator delay," *IEEE Trans. Autom. Control*, vol. 55, no. 9, pp. 2106–2112, Sep. 2010.

[10] M. Chèbre, Y. Creff, and N. Petit, "Feedback control and optimization for the production of commercial fuels by blending," *J. Process Control*, vol. 20, no. 4, pp. 441–451, 2010.

[11] D. Del Vecchio, "Design and analysis of an activator-repressor clock in *E. coli*," in *Proc. Amer. Control Conf.*, 2007, pp. 1589–1594.

[12] J.-Y. Dieulot and J.-P. Richard, "Tracking control of a nonlinear system with input-dependent delay," in *Proc. 40th Conf. Decision Control*, 2001, vol. 4, pp. 4027–4031.

[13] S. Diop, I. Kolmanovsky, P. E. Moraal, and M. Van Nieuwstadt, "Preserving stability/performance when facing an unknown time-delay," *Control Eng. Practice*, vol. 9, no. 12, pp. 1319–1325, 2001.

[14] T. Erneux, *Applied Delay Differential Equations*. New York: Springer Verlag, 2009, vol. 3.

[15] P. A. Ioannou and B. Fidan, *Adaptive Control Tutorial*. Philadelphia, PA: Society for Industrial Mathematics, 2006.

[16] P. A. Ioannou and J. Sun, *Robust Adaptive Control*. Englewood Cliffs, NJ: Prentice Hall, 1996.

[17] M. Jankovic and I. Kolmanovsky, "Developments in control of time-delay systems for automotive powertrain applications," in *Delay Differential Equations, Recent Advances and New Directions, Balachandran, B. and Kalmar-Nagy, T. and Gilsinn D. E.*. New York: Springer Science, 2009, pp. 55–92.

[18] I. Karafyllis, "Stabilization by means of approximate predictors for systems with delayed input," *SIAM J. Control Optim.*, vol. 49, no. 3, pp. 1100–1123, 2011.

[19] H. Khalil, *Nonlinear Systems*, 3rd ed. Englewood Cliffs, NJ: Prentice Hall, 2002.

[20] M. Krstic, *Boundary Control of PDEs: A Course on Backstepping Designs*. Philadelphia, PA, USA: Society for Industrial and Applied Mathematics, 2008.

[21] M. Krstic, *Delay Compensation for Nonlinear, Adaptive, and PDE Systems*. Boston, MA: Birkhauser, 2009.

[22] M. Krstic and A. Banaszuk, "Multivariable adaptive control of instabilities arising in jet engines," *Control Eng. Practice*, vol. 14, no. 7, pp. 833–842, 2006.

[23] M. Krstic and D. Bresch-Pietri, "Delay-adaptive full-state predictor feedback for systems with unknown long actuator delay," in *Proc. Amer. Control Conf.*, 2009, pp. 4500–4505.

[24] W. Kwon and A. Pearson, "Feedback stabilization of linear systems with delayed control," *IEEE Trans. Autom. Control*, vol. 25, no. 2, pp. 266–269, 1980.

[25] A. Manitius and A. Olbrot, "Finite spectrum assignment problem for systems with delays," *IEEE Trans. Autom. Control*, vol. 24, no. 4, pp. 541–552, 1979.

[26] F. Mazenc and S. I. Niculescu, "Generating positive and stable solutions through delayed state feedback," *Automatica*, vol. 47, pp. 525–533, 2011.

[27] F. Mazenc, S.-I. Niculescu, and M. Krstic, "Some remarks on Lyapunov-Krasovskii functionals and reduction approach for input delay compensation," in *Proc. Amer. Control Conf.*, 2012, pp. 6629–6634.

[28] S. Mondié and W. Michiels, "A safe implementation for finite spectrum assignment: Robustness analysis," in *Proc. 42nd IEEE Conf. Decision Control*, 2003, vol. 5, pp. 4569–4576.

[29] S. I. Niculescu, *Delay Effects on Stability: A Robust Control Approach*. New York: Springer Verlag, 2001, vol. 269, Lecture notes in control and information sciences.

[30] M. T. Nihtila, "Finite pole assignment for systems with time-varying input delays," in *Proc. 30th IEEE Conf. Decision Control*, 1991, pp. 927–928.

[31] A. O'Dwyer, A Survey of Techniques for the Estimation and Compensation of Processes With Time Delay 2000.

[32] Y. Orlov, I. Kolmanovsky, and O. Gomez, "Adaptive identification of linear time-delay systems: From theory toward application to engine transient fuel identification," *Int. J. Adaptive Control Signal Processing*, vol. 23, no. 2, pp. 150–165, 2009.

[33] Z. Palmor, "Time-delay compensation-smith predictor and its modifications," *The Control Handbook*, vol. 1, pp. 224–229, 1996.

[34] E. Polak, *Optimization: Algorithms and Consistent Approximations*. New York: Springer, 1997, vol. 7.

[35] L. Praly, Fonctions de Lyapunov, Stabilité et Stabilisation Classe notes, 2008.

[36] J.-P. Richard, "Time-delay systems: An overview of some recent advances and open problems," *Automatica*, vol. 39, no. 10, pp. 1667–1694, 2003.
[37] A. Rosenberg, S. Jayanthi, and D. Del Vecchio, "Tuning an activator-repressor clock employing retroactivity," in *Proc. Amer. Control Conf.*, 2011, pp. 2308–2313.
[38] S. S. Sastry and A. Isidori, "Adaptive control of linearizable systems," *IEEE Trans. Autom. Control*, vol. 34, no. 11, pp. 1123–1131, 1989.
[39] M. Sbarciog, R. De Keyser, S. Cristea, and C. De Prada, "Nonlinear predictive control of processes with variable time delay. A temperature control case study," in *Proc. IEEE Int. Conf. Control Appl.*, 2008, pp. 1001–1006.
[40] G. Tao and P. V. Kokotovic, *Adaptive Control of Systems With Actuator and Sensor Nonlinearities*. New York: Wiley, 1996.
[41] K. Zenger and R. Ylinen, "Simulation of variable delays in material transport models," *Math. Comp. Simul.*, vol. 37, pp. 52–72, 1994.

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