Prediction-based control of moisture in a convective flow

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***Abstract*— This work studies a convective flow system and presents experimental closed-loop results carried out on a test-bench representative of several industrial processes. This test bench consists of a horizontal column equipped with a mist actuator located at the inlet and fans generating an air flow circulating along the tube. Following our recent theoretical design, we implemented a prediction-based control strategy aiming at stabilizing the mist at the output of the tube actuating on the wind speed. Correspondingly, this set-up involves a transport input-dependent delay (between the inlet and the output of the tube). We propose a control-oriented model, in which the transport delay satisfies an integral equation, and compared our prediction-based design with a conventional Proportional-Integral controller. Experimental results underline the relevance of the proposed approach.**

I. Introduction

Ability to manipulate flow properties (concentration, temperature, density,etc.) in a transport of material is a question of major technological importance. Such a situation occurs for flow regulation in mining [23] or hydraulic networks [9], for airpath regulation in automotive engine [12] or control of after-treatment devices in exhaust lines [5][8], for blending in liquid or solid networks [7] and batch processes [20], to name a few. Remarkably, in all these examples, sensors and actuator are not collocated, which creates a lag depending on the speed of propagation of the fluid. The latter is the control variable in these problems which creates an inherently input-dependent delay.

Despite this record, this class of delay is still an under-developed topic. Surprisingly, it seems that stabilization of such processes with input-dependent delay acting on the input, that is, $D(u)$ or $D(u_t)$, where u_t denotes past values over a finite horizon, has seldom been theoretically studied. Indeed, in most of undergoing studies, delays are either represented by purely uncertain time-varying models, that is, $D(u_t) \approx D(t)$ or, in the worst case, by a constant mean value, that is, $D(u_t) \approx D$. Yet, in all the applications cited above, the lag variability is strong and the input-dependency raises concerns about stability, in both open and closed loop scenarii.

This paper focuses on an experimental test bench which is representative of this wide class of systems involving transport phenomena. It is schematically represented in Fig. 1. It consists of a horizontal tube equipped with two sets of actuators: fan located at the input and the output of the tube, creating a circulating air flow, and a mist injector located at the inlet of the tube. The mixture of dry inflow and mist injection generates a (distributed) change of moisture inside the tube. This is a mixing process similar to the shower problem [2] often used as a toy example in the delay literature [10][26]. Here, we are concerned with the regulation of the moisture at the output of the tube by acting on the dry air inflow, for a fixed mist injection. As this system involves transport of material between actuator and sensor and our control variable is the air flow, it is encompassed in the framework previously described, that it, the class of systems involving input-dependent transport delays.

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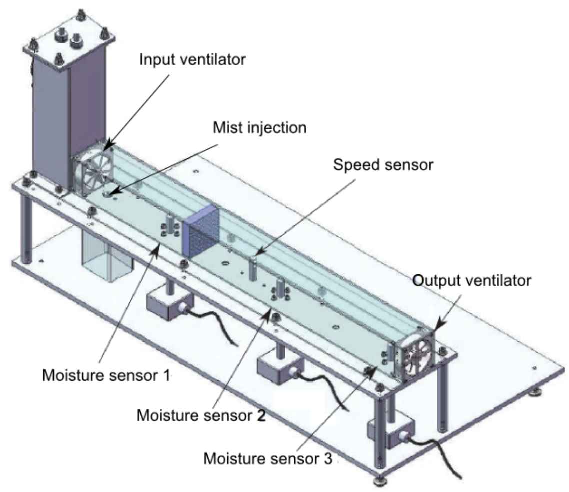


Fig. 1. Schematic view of the flow control test bench.

Recently, we have showed the practical relevancy of a particular class of integral equation to model delay arising from transport of material [6][24]. To improve transient performances, we investigated robust compensation of this particular class of input delay and obtained a sufficient condition for the stability of a linear system subject to a prediction-based controller [4]. Prediction-based control strategies [1][16][22] are already widely used for systems with constant input time-delays (see for instance [11] [14] [17] or [21] and the references therein) but are still not of general use for time-varying delays (see [18] or, more recently, [15]). In such cases, to compensate the varying input delay, the prediction has to be calculated over a time window of which length matches the value of the future delay. When the delay depends on the input, things are getting very involved: determining the required prediction horizon becomes an implicit question which is not solvable in all cases. This implicit nature is caused by the reciprocal interactions between the control (current and past) values and the delay, yielding a closed-loop dependency. For this reason, in lieu of seeking exact delay compensation, we proposed to use a prediction horizon

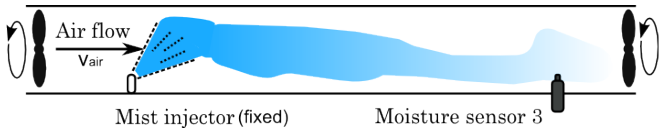


Fig. 2. Schematic view of the considered set-up: for a given fixed injection, we aim at controlling the output moisture by acting on the fan velocities.

equal to the current delay value. This does not aim at exactly compensating the input delay, but only at decreasing its impact in the closed-loop dynamics and, hence, at improving transient performances.

In this paper, we apply this tool for the control of the output moisture on the flow test-bench. From experimental data, we propose a simple first order model, subject to a transport input delay satisfying the integral time-varying equation previously discussed. In order to improve transient performances, we applied the prediction-based controller previously designed in [4]. The interest of this approach is emphasized by experimental results which favorably compare to the transient performances obtained with a conventional Proportional-Integral (PI) controller. Those practical developments are our main contributions.

The paper is organized as follows. In Section II, we present the experimental set-up along with the identified dynamical model. Then, in Section III, we present a solution to the moisture control problem before discussing experimental results obtained on test-bench in Section IV.

II. Experimental set-up, Modeling and Identification

The experimental test bench considered in this paper is depicted in Fig. 1. It consists of an horizontal tube equipped with two fans located at the inlet and the outlet of the tube and a mist injector located at the inlet.

A. Experimental set-up and control objective

We consider a configuration in which the mist injection is fixed and one is interested in regulating the moisture in the tube thanks to the air flow, that is, by controlling the (coupled) fans velocities (see Fig. 2). The test bench is also equipped with a speed sensor (sensitivity 0.01 m/s) located in the middle of the tube. The accuracy of the moisture sensor is around 0.2 points.

The overall system is monitored and controlled via an automaton (run in CX-programmer) with limited memory. Acquisition and control are performed with a sampling time equal to 0.1 s.

Variable	Quantity	Value	unity
L	Length of the tube	.8	m
m	Output moisture	-	%
P	Fan power	-	%
δ_0	Delay	2.8	s
τ	Time constant	9	s

TABLE I

Notations and values of the experimental variables

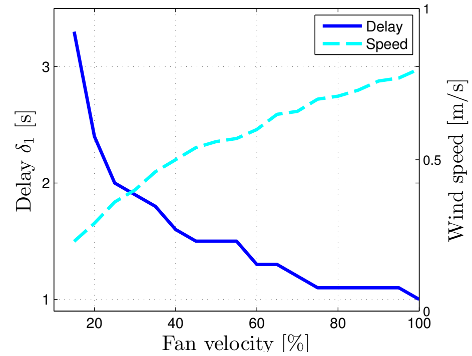


Fig. 3. Steady-state values of transport delay δ_1 and wind speed obtained experimentally for different fan powers.

B. Time-delay model with integral delay equation

As a first step in our analysis, a series of experiments were carried out to identify the dynamics relating the output moisture (Moisture 3) to changes in the fan velocities. To be representative of the whole operating range of the test bench, steps of fan velocity power of several magnitude between 100 % and 15 % (corresponding to an air flow rate circulating in the flowtube equal to 0.01 m/s, that is equal to the accuracy of the speed sensor, see Fig. 3) were performed. Those tests are pictured in Fig. 4.

In the sequel, we denote m the output moisture, that is, measured by the moisture sensor 3, P the fan power and Δm and ΔP changes around an operating point. Applying classical identification tools to the tests given in Fig. 4, the following linearized input-delay model was obtained

$$\tau \dot{\Delta m} = -\Delta m + G(t)\Delta P(t-\delta(t)) \quad (1)$$
$$\delta(t) = \delta_0 + \delta_1(t) \quad (2)$$

in which the static gain $G(t)$ depends on the operating point, the time constant τ and the delay δ_0 are constant and the transport delay δ_1 is defined implicitely by

$$\int_{t-\delta_1(t)}^{t} v_{air}(s)ds = L \quad (3)$$

in which $v_{air}(t)$ is the air velocity circulating in the tube at time t (assuming that the air flow is homogeneous) and L is the length of the flowtube. Values of all those parameters are gathered in Table I and comparison between experimental data and this model is provided in Fig. 4.

One can observe in Fig. 4 that this simple model encompasses the main features of the moisture dynamics. It is worth highlighting that, for a fan power between 50% and 100%, the magnitude of the moisture variation is small enough to let the sensor resolution appear in the dynamics and interfere with it. This explains the apparent poorer fit on those last two experiments.

The integral delay model (3) corresponds to a Plug-Flow assumption [19] in a transport phenomenon [20][24][25]. It can be understood as the time of propagation of the (non-compressible) fluid through the tube of length L with the

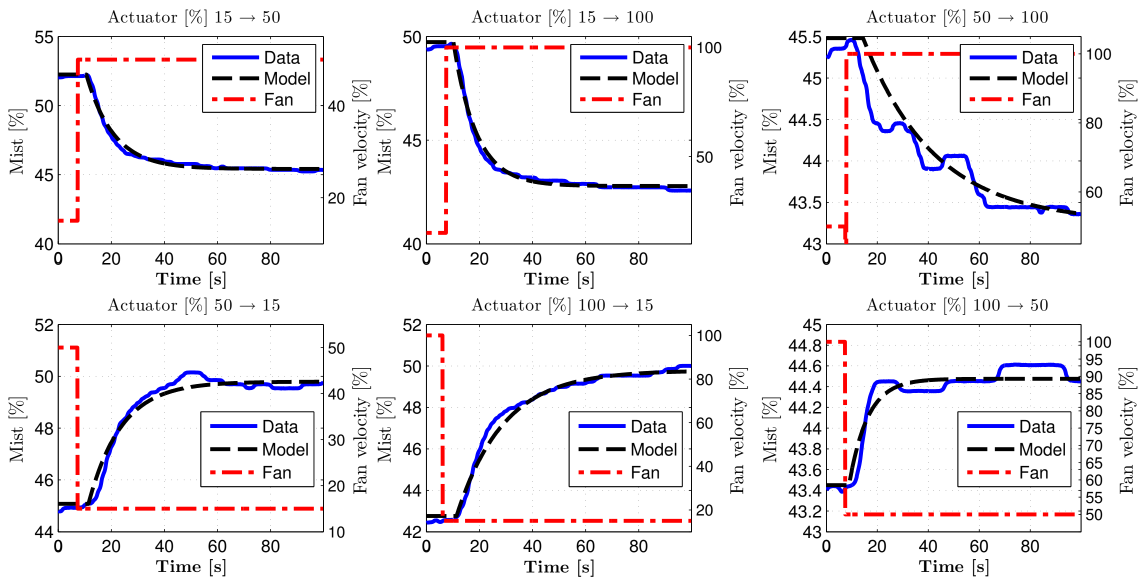


Fig. 4. Modeling of the moisture dynamics with the first-order delay model (1)–(3). The mist injection is fixed (maximum flow, 4 L/min)

time-varying speed v_{air}. This is the time-varying generalization of an intuitive propagation time model, for which the delay is defined as the ratio between the length and the gas speed. This situation actually corresponds to the steady-case, for which the gas speed is constant, which is indeed caught by (3). Conversely, the constant delay δ_0 arises from an actuator lag. Note that those two delays have similar scales (see Table I and Fig. 3). However, the variations of δ_1 are particularly challenging, from around 1 s up to 3 s on the whole operating range, as Fig. 3 illustrates it.

Also, it is worth highlighting the fact that, even if the transport delay defined through (3) cannot be analytically expressed, as underlined in [25], it can still be calculated numerically according to the history of the wind speed v_{air}. Indeed, the function $\delta_1 \geq 0 \mapsto \int_{t-\delta_1}^{t} v_{air}(s)ds$ is a strictly increasing function, equal to zero for a zero delay. Therefore, a simple procedure to evaluate the current value of the delay consists in evaluating the value of (a sampled version of) this function for increasing delay values, starting with $\delta_1 = 0$, until reaching the bound L. Such a procedure, which is real-time compliant but requires to keep in memory the history of the wind speed, is illustrated in Table II.

The first order dynamics obtained in (1) can be related to a dilution phenomenon of the water droplets into the inlet air. This is consistent with the type of model proposed in [13] for the same test-bench and a similar problem but with a different actuator. One can observe that scales of the time constant τ and the time delay δ are similar, which motivates the need of designing a control law specifically accounting for this input delay.

Finally, for a given operating point, the static gain G is slowly varying due to changes in the room thermodynamical conditions (as the temperature in the tube influences the moisture) and also to uncertainties on the fan position.

Algorithm 1 delta1 = CalculateDelay(v_{air}(1:N),L)

```
int = 0;
for i = 0:N−1
  %Calculate the integral
  %corresponding to a delay i*Ts
  int = int + Ts*v_air(N−i);
  % Check if the integral is equal to L
  if int >=L
    delta1 = i*Ts;
   break;
  end
end
delta1 = inf;
```

TABLE II

EXAMPLE OF DELAY CALCULATION PROCEDURE FOR THE INTEGRAL-TYPE RELATION (3). PURPOSELY, THE INTEGRAL SAMPLING DOES NOT INVOLVE THE CURRENT INPUT VALUE $v_{air}(t)$.

Nominal constant values of G have been identified and stored in a 1D look-up table. This is the gain model which is used in the sequel.

III. SOLUTION OF THE CONTROL PROBLEM

Our control objective is to compensate the input delay thanks to a prediction-based controller. Yet, the input-dependency of the delay makes control design significantly more complex. Indeed, as the control law depends on the delay, delay and control reciprocally interact in a malicious way. For this reason, we propose to use a prediction horizon equal to the current delay value, calculated following a procedure such as the one proposed in Table II. Recently, we showed the following robust prediction-based stabilization result in [3][4].

Theorem 1: Consider the closed-loop system

$$\begin{cases} \dot{X}(t) = AX(t) + B\varphi(t - D(t)) & (4)\\ \int_{t-D(t)}^{t} \varphi(s)ds = 1 \quad \text{with} \quad \varphi(t) = \text{Sat}_{[\underline{u},+\infty[}(u(t)) & (5)\\ u(t) = u^r + K\left[e^{AD(t)}X(t) + \int_{t-D(t)}^{t} e^{A(t-s)}B\varphi(s)ds - X^r\right] & (6)\end{cases}$$

where $X \in \mathbb{R}^n$, u is scalar, K is such that $A+BK$ is Hurwitz, X^r is a given state equilibrium and U^r is the corresponding (constant) reference control. Consider the functionals

$$\Theta(t) = |X(t) - X^r| + \max_{s\in[t-\overline{D},t]}\left[|u(s)-u^r| \quad |\dot{u}(s)| \ \ldots \ |u^{(n-1)}(s)|\right] \quad (7)$$

$$\Gamma(t) = |X(t)-X^r|^2 + \int_{t-D(t)}^{t}(u(s)-u^r)^2 ds + \int_{t-D(t)}^{t}\dot{u}(s)^2 ds \quad (8)$$

Then, there exists $\theta : \mathbb{R}^n \mapsto \mathbb{R}_+^\star$ such that, if $\Theta(0) < \theta(K)$, there exist $R, \rho > 0$ such that

$$\Gamma(t) \leq R\Gamma(0)e^{-\rho t}, \ t \geq 0$$

This result should be understood as follows. The prediction controller (6) aims at forecasting values of the state over a time window of varying length $D(t)$[1]. Of course, exact compensation of the delay is not achieved with this controller[2] and can be highly inaccurate when the delay is fast varying. In this context, it has been shown in [4] that a sufficient condition for robust delay compensation achievement is that the delay variations are sufficiently small. The spirit of this condition is that, if the delay varies sufficiently slowly, its current value $D(t)$ used for prediction remains close enough to its future values, and the corresponding prediction is accurate enough to guarantee the stabilization of the plant through the feedback loop.

Besides, the delay variations implicitly depend on the control input through the integral equation (5). Thus, their aggressiveness are scaled by the gain K. Then, to limit the delay variations, restricting the input variations seems like a natural requirement. This is achieved by choosing the initial conditions close enough to the desired equilibrium and in compliance with the feedback gain magnitude. This explains the sufficient condition bearing on the gain magnitude in Theorem 1. Under this condition, robust delay compensation is obtained, which should translate into improvement of transient performances.

This result can be applied to the moisture dynamics under consideration here, with $X = m$ and $u = P$. Note that our moisture model (1)–(3) does not explicitly fit into the framework considered in Theorem 1, as the transport delay (3) does not directly depend on our variable of actuation (fan power) but on the air velocity. Yet, those two variables are equivalent, in the sense that a bijection relates one to the other (see Fig. 3). Further, it is worth mentioning that appearance of a constant delay δ_0 in our process modeling in (2) does not modify the conclusion of this theorem, the proof of which given in [4] can be straightforwardly extended to handle this framework.

Finally, the plant considered in Theorem 1 involves a saturation operator, which simply accounts for a minimum flow rate constraint (otherwise, the delay would tend to infinity while the input tends to zero, which may reveal troublesome for unstable plants). This is directly translated in our context into the fact that we will not actuate the fans under a position of 15%, as the resulting air flow inside the tube would not then be large enough to be detected by the the wind speed sensor.

IV. Experimental results

To investigate the effectiveness of the proposed prediction-based control strategy, experimental results have been carried out on test-bench. To account for the static gain variations underlined before, we slightly changed the control law under consideration as follows

$$P(t) = P^r + K_I\int_0^t [m^r - m(s)]ds \quad (9)$$
$$+ K_P\left[e^{-(\delta_0+\delta_1(t))/\tau}\Delta m(t) + \int_{t-\delta_0-\delta_1(t)}^{t} e^{-(t-s)/\tau} G\frac{\Delta P(s)}{\tau}ds\right]$$

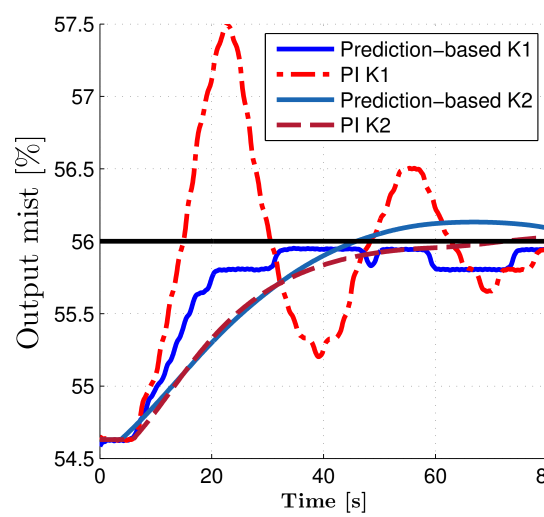


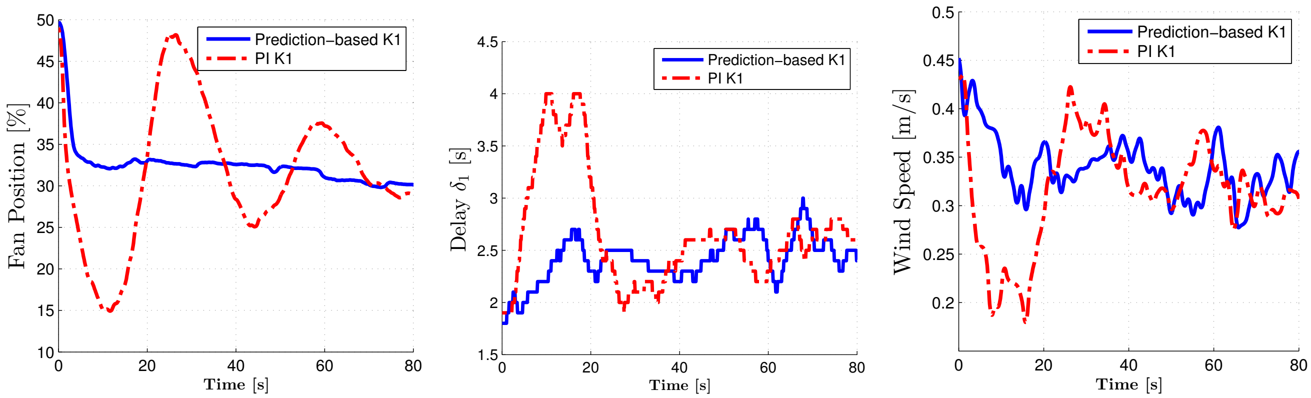
Fig. 5. Experimental moisture responses obtained, respectively, with the proposed prediction-based approach and with a PI controller, for two sets of gains $K1 = (K_P, K_I) = (8, 0.2)$ and $K2 = (K_P, K_I) = (5, 0.1)$.

[1]Note that this controller does not exactly match the predicted system state on a time-horizon $D(t)$. Indeed, using the variation of constant formula

$$\forall t \geq 0, \quad X(t+D(t)) = e^{AD(t)}X(t) + \int_{t-D(t)}^{t} e^{A(t-s)}Bu(s+D(t)-D(s))ds$$

Under the assumption that the variations of the delay are sufficiently small, this latter integral can be approximated by the one used in (6) as $D(t) - D(s) \approx 0$. As this assumption is already required to robustly compensate the delay, we rather use the prediction form (6) which is easier to implement instead of the true prediction given above.

[2]As discussed earlier, to do so, one would need to consider a time window of which length would exactly match the value of the future delay, as it is made in [18] and [15]. In details, defining $\eta(t) = t - D(t)$ and assuming that its inverse exists (which is the case if $\dot{D} < 1$), exact delay-compensation is obtained with the feedback law $U(t) = KX(\eta^{-1}(t))$. Yet, implementing this relation requires to predict the future variation of the delay via $\eta^{-1}(t)$, which may not practically achievable for an input-varying delay.



(a) Fan power. (b) Transport delay calculated according to (3). (c) Wind speed.

Fig. 6. Variations of the fans actuation, the resulting transport delay and wind speed corresponding to the closed-loop experiments pictured in Fig. 5, with feedback gains $K1 = (K_P, K_I) = (8, 0.2)$ (variations obtained with the second set of gains $K2 = (K_P, K_I) = (5, 0.1)$ have similar trends to those obtained with the prediction-based control law showed here and are not reported for the sake of clarity).

that is, we included an integral controller in our controller.

In the following, we compare this prediction-based controller with a conventional PI for two sets of gains. The first one $(K_P, K_I) = (8, 0.2)$ is referred to as high-gain when compared to the second one $(K_P, K_I) = (5, 0.1)$ referred to as low-gain. We start our closed-loop experiments with a moisture close to 54.5%, which corresponds to a fan position of 50%, and aim at stabilizing the output moisture at 56%. Corresponding results are pictured in Fig. 5 and intermediate variables of interest provided in Fig. 6.

On the one hand, for low gain, one can see that both types of controllers achieve convergence with similar transient performance. This can be reasonably interpreted as a consequence of the fact that the feedback gains values are reasonably low compared to the open-loop dynamics, which merely plays the main rule in transient here.

On the other hand, for high gain, one can observe that the prediction-based controller still achieves respectable performance while, with the PI controller, the system exhibits an oscillatory behavior. This illustrates the main advantage of our technique, which allows one to decrease the response time of the system. Those different behaviors can be interpreted in the light of the delay variations pictured in Fig. 6. With a PI controller, one can observe that the delay suddenly increases up to twice its initial value (as, correspondingly, the wind speed significantly decreases), which may no be compatible with the the chosen feedback gains. On the other hand, with the prediction-based control law, influence of control on the delay variation is limited and a smooth closed-loop response is obtained.

To understand more clearly the meaning of the sufficient condition on the feedback gain stated in Theorem 1, additional experiments should be carried out. This will reasonably imply to consider larger moisture steps.

V. Conclusion and perspectives

In this paper, we propose a simplified first-order model to represent the moisture dynamics and show on experimental data that a transport integral delay model encompasses the main features of the dynamics under consideration. We implemented a prediction-based control strategy and presented experimental results emphasizing its interest in the challenging context of an input-dependent input delay.

Future theoretical works will include the stability analysis in presence of an uncertain static gain and a corresponding integral term in the control law. Design of an adaptive controller to handle those uncertainties could also be an interesting direction of work. Finally, additional experimental tests should complete the preliminary obtained ones to compare the proposed prediction-based control law with e.g. an Internal Model Control carefully tuned to handle uncertainties [13]. Those will also allow to understand better the meaning of the sufficient condition stated in the stabilization result of this paper.

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