Combined battery SOC/SOH estimation using a nonlinear adaptive observer

M. El Lakkis[1], O.Sename[1], M.Corno[2] and D. Bresch Pietri[1]

Abstract— This work presents a modeling and estimation techniques for State of Charge and State of Health estimation for Li-ion batteries. The analysis is done using an adaptive estimation approach for joint state and parameter estimation and by simplifying an existing nonlinear model previously obtained from experiments tests. A switching mechanism between two observers, one for the charging phase and one for the discharging phase, is done to avoid transients due to the discontinuity of model's parameters. Simulations on experimental data show that the approach is feasible and enhance the interest of the proposed estimation technique.

I. INTRODUCTION

Nowadays, Li-ion batteries are widely used in different applications that range from small sized portable devices to large sized equipment. This type of batteries has attracted the interest of several industries because of its advantages compared to other rechargeable batteries [17], [2]. Li-ion batteries have a higher energy density, require less maintenance, have a high nominal voltage, a low self discharge rate and exhibit no memory effect. These characteristics make the Li-ion batteries one of the most regarded technologies in energy solutions and automotive applications. On the other hand, the performance of Li-ion batteries degrades over time, they have high sensitivity to temperature and sometimes they are unsafe when overcharged. A battery management system (BMS) is needed to minimize and avoid these drawbacks. The BMS controls the charging and discharging of the battery while guaranteeing a reliable and safe operation and ensure longevity for the battery. It can also handle additional tasks like cell balancing and temperature control for the battery pack. Such tasks need to take into account two main parameters: the State of Charge (*SoC*) and the State of Health (*SoH*) of the battery.

The *SoC* is defined as the ratio between the saved energy in the battery and the total energy that can be saved in the battery [14]. The importance of this parameter is that it gives a picture of the current state of the battery, enables the BMS to safely charge/discharge the battery and helps the BMS to manage the usage of the battery in an optimal way. The *SoH* is defined as the ability of a battery to store energy,

[1] M. El Lakkis, O. Sename and D. Bresch-Pietri are with GIPSA-lab UMR CNRS 5216, Control Systems Department, Universit Grenoble Alpes, 11 rue des Mathmatiques, F-38402 St Martin d'Hères cedex, France, `olivier.sename@gipsa-lab.fr`
[2] M. Corno is with the Dipartimento di Elettronica, Informazione e Bioingegneria, Politecnico di Milano, Italy, `corno@elet.polimi.it`

source and sink high current and retain the charge over time compared to a new one [14]. Many factors accelerate the degradation of the battery like high C-rate, bad storage, high/low temperature and over charging/discharging. A good evaluation of the *SoH* should help to anticipate problems and to plan replacement. Also, it helps the BMS to adapt the *SoC* control based on the age of the battery.

Designing and building BMS algorithms for Li-ion batteries require a model that can describe the battery dynamics [2]. This model explains the relationship between the battery measurable output voltage and the current from one side, and the *SoC* and *SoH* form the other side which are internal states that are not directly measured. Different models exist and differ by their levels of complexity and accuracy. In general, battery models are divided into three categories: white box, black box and gray box models [30]. White box models such as electrochemical models [8], [10], [26], [4] use a set of equations consisting of spatial partial differential equations to model each electrochemical process. These models are very accurate, but their complexity can represent a computational burden and discards them from real time implementation. Further, they require a good understanding of the electrochemical processes in the cell. Black box models as fuzzy logic based ones or neural network structures [29], [30], [1] do not require any physical knowledge but require a learning phase using experimental data. In this paper, we consider an intermediate solution (gray box) which represents a reasonable compromise between accuracy and complexity. It consists of an electro-equivalent models where an analogy with electronic components is done to model the behavior of the cell. Different approaches exist like Thevenin circuit model, impedance based model, diffusive resistive capacitive model [5] and equivalent circuit based models [15], [14], [27]. These models are easy to understand and they are comprehensible even with little background on the electrochemical process in the battery.

A simple method to calculate the *SoC* is to use the Coulomb counting:

$$SoC(t) = SoC(0) - \frac{1}{3600 \times C} \times \int_0^t I dt \qquad (1)$$

where I is the current and C is the battery's capacity. However, the unknown value of the initial *SoC*, the noisy measurement of the current and the variable value of the capacity make this method inaccurate. An alternative method

to find the *SoC* is to measure the open circuit voltage V_{oc} of the battery since it only depends on the *SoC*. But the problem is that the V_{oc} can be measured directly only if the battery has been resting for a certain time, which makes this method inappropriate for vehicles applications. Besides direct measure, the *SoC* can be estimated using other techniques which exist in the literature. These techniques use a mathematical model (linear or nonlinear) that describes the dynamics of the battery in order to estimate its states and/or parameters. These methods like Kalman Filter (KF) [25], Extended Kalman Filter (EKF) [3], [1], [21], [7], H_∞, mixed kalman/H_∞ [27], adaptive Luenberger observer [12] and sliding mode observer [13] were used to estimate the internal states of the battery.

There is no common method to estimate the *SoH*. The simplest and more intuitive way is to use the number of charge/discharge cycles or time since manufacturing to determine the health of the battery. Other studies use the decrease in capacity [23], [6] or the increase of the internal resistance [9], [19], [28], [14] to find the *SoH* where a strong and direct relation between the health of the battery and these two parameter exists. In [11], the charging curves are the indicators used to predict the life of the battery. This method may not be suitable for EV and HEV since it requires a specific charging method. The weighted throughput method [20] uses the integration of the current (to find the throughput) multiplied by certain severity factors that depend on the temperature T and the Depth of Discharge (*DoD*). But this method requires a severity factor map which is not easy to construct.
Also, there exist some methods like extended Kalman filter [14], adaptive partial differential equation observer [18], fuzzy logic [22] and nonlinear least square regression technique [24] used for combined *SoC* and *SoH* estimations.

This paper aims at proposing a nonlinear adaptive observer in order to estimate the battery state of charge (state variable) and state of health from the internal resistance (time-varying parameter) simultaneously, which is an open problem. Besides the methodological application, the main contributions of the paper include the development of:

- an equivalent circuit battery model, where the parameters depend in a nonlinear manner on the state of charge and temperature, which is simplified for design purpose
- a 2-mode switching adaptive observer which accounts for the discontinuity between the charging/discharging phases.
- an evaluation of the observer efficiency using input experimental data from a realistic driving cycle on an electrical car.

The paper contents is as follows. In section II the battery model is presented together with its simplification. Section III is concerned with the development of the 2-modes nonlinear adaptive observer. In section IV the performances of the observer are analyzed using charging/discharging step inputs to show its efficiency to estimate the state of charge and state of health. Then tests with experimental input data are performed in section V to prove the robustness of the observer. Some conclusions are drawn in the last section.

II. MODELING FOR OBSERVER DESIGN

This work focuses on the electrical circuit models, motivated by the needs of EV applications. These models are simple enough for control and observation purpose, and have the capability to be connected to the rest of an electrical or electronic circuit system for simulations.

A. Equivalent circuit based model

The battery model used is this work is the equivalent circuit based model adopted in [15] and pictured in figure 1. C_{batt} is the usable capacity of the cell, it depends on the true capacity and the operating conditions. V_{oc} is the open circuit voltage of the cell when it is at electrochemical equilibrium and it is highly dependent on the *SoC*. R_0 is the ohmic resistance of the cell. R_s and C_s represent the short time transient behavior of the voltage where R_l and C_l represent the long time transient response. The self discharge resistance is neglected in this study.
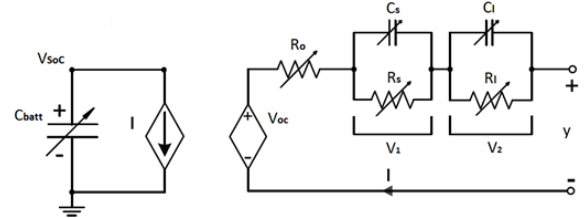

Fig. 1: Equivalent circuit based model

The state space model that describes the dynamics of the system is given by:

$$
\begin{cases}
\dot{x}_1 = \dfrac{1}{R_s(t)C_s(t)}x_1 + \dfrac{1}{C_s(t)}U & \text{(2a)} \\[2ex]
\dot{x}_2 = \dfrac{1}{R_l(t)C_l(t)}x_1 + \dfrac{1}{C_l(t)}U & \text{(2b)} \\[2ex]
\dot{x}_3 = \dfrac{\eta}{C_{batt}}U & \text{(2c)} \\[2ex]
y = V_{oc} - x_1 - x_2 - R_0(t)U & \text{(2d)}
\end{cases}
$$

where $x_1 = V_1$, $x_2 = V_2$, $x_3 = SoC$, $U = I$, η is the efficiency of the battery and y is the battery output voltage. $R_0(t)$, $R_s(t)$, $C_s(t)$, $R_l(t)$ and $C_l(t)$ are variable parameters and depend on the temperature (T) and on the state of charge (SoC). In addition, the values of these parameters differ between charging and discharging, thus the cell's voltage behavior will be described by two sets of parameters, one for charging and one for discharging as stated in [15].

B. Parameters modeling

As stated above, the battery's parameters are variable with respect to the temperature, the *SoC* and the current direction, making the overall model nonlinear. In [15], experimental data and curve fitting techniques are used to find empirical

equations relating the parameters with the operating conditions. For example, the equations below show the nonlinearity of V_{oc} and R_0 where the equations' parameters are constants values determined using curve fitting techniques.

$$V_{oc} = a_1 \times e^{(-a_2 \times SoC)} + a_3 + a_4 \times SoC + a_5 \times (SoC)^2 +$$
$$a_6 \times (SoC)^3 + a_7 \times e^{(-a_8/(1-SoC))} + a_9 \times SoC^4$$
$$+ a_{10} \times SoC^5 + a_{11} \times SoC^6 \qquad (3)$$

$$R_0 = \begin{cases} (b_1 \times SoC^4 + b_2 \times SoC^3 + b_3 \times SoC^2 & (4a) \\ + b_4 \times SoC + b_5) \times b_{t1} \times e^{(b_{t2}/(T-b_{t3}))} & \text{if } I < 0 \\ \\ (\beta_1 \times SoC^4 + \beta_2 \times SoC^3 + \beta_3 \times SoC^2 & (4b) \\ + \beta_4 \times SoC + \beta_5) \times \beta_{t1} \times e^{(\beta_{t2}/(T-\beta_{t3}))} & \text{if } I > 0 \end{cases}$$

Instead of using the nonlinear characteristics describing the parameter values provided by [15], the expressions of the battery model parameters are simplified to lower degree equations in order to reduce the computational complexity as well as to study the robustness of the observer to modeling uncertainties. This is done as follows:
- A simplification is done by studying separately each parameter and its variation with respect to SoC and T.
- The nonlinear equation for the specified parameter is used to generate data for a range of SoC and sometimes for different temperatures (depending on the case).
- These data will be introduced to a curve fitting procedure where an expression of lower order has to be found.
- The output of the high order equations model and the output of the lower order equations model are compared.
For instance, the following equations represent the simplification of (3) and (4) respectively. It is clear that the obtained equations are less complex than the original ones.

$$V_{oc} = V_{oc,nom} + \alpha SoC \qquad (5)$$

$$R_0 = \begin{cases} (a_1 \times SoC + a_2) \times a_{t1} \times e^{(a_{t2}/(T-a_{t3}))} & \text{if } I < 0 \\ & (6a) \\ (\alpha_1 \times SoC + \alpha_2) \times \alpha_{t1} \times e^{(\alpha_{t2}/(T-\alpha_{t3}))} & \text{if } I > 0 \\ & (6b) \end{cases}$$

Figure 2 shows a comparison between the parameters of the nonlinear model provided by [15] and the parameters of the simplified one. It can be seen that the simplified parameters are close to the real value to recast the existing difference as model uncertainties.

C. Ageing model

With time, the battery's parameters will change due to cycling, operation conditions and storage. Normally, the battery's capacity tends to decrease due to capacity fading while its impedance tends to increase. Different ageing models exist to describe the evolution of each parameter with cycling. In this study, only the changes in the capacity and the ohmic resistance will be considered since these two are the most important parameters that reflect the health of the battery. In addition, this change is chosen to be linear with
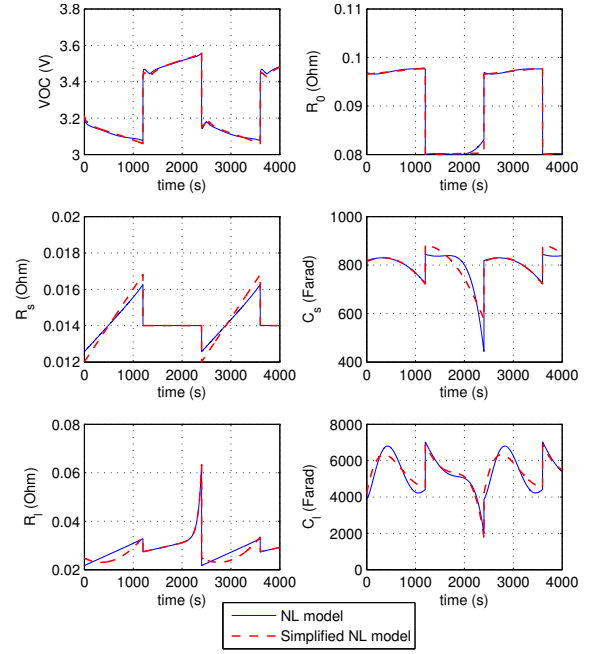

Fig. 2: Nonlinear original model in [15] VS nonlinear simplified model

usage. The table I below is used to define some thresholds regarding the Begin of Life (*BoL*) and the End of Life (*EoL*) of the battery. At the *BoL*, the parameters can be easily defined since for all new batteries the *SoH* is defined as 100% with 0 Ah delivered charge. The battery has its original capacity and resistance specified by the manufacturer. On the other hand, there is no common definition for the EoL and it depends on the battery type and application.

Parameter	Begin of Life	End of Life
State of Health (*SoH*)	100%	0%
Charge delivered	0 Ah	Max Ah
Battery capacity	C_{app}	$C_{app} - \Delta C_{EoL}$
Battery resistance	R_0	$R_0 + \Delta R_{EoL}$

TABLE I: Begin of Life Vs End of Life

Based on experimental information for an electrical vehicle available at Politecnico di Milano, the *EoL* is defined as follows:
- Under normal operation conditions, the battery used in the study cannot deliver more than 1200 Ah.
- The EoL is reached when the battery loses 20% of its capacity (i.e. $\Delta C_{EoL} = 0.2 \times C_{app}$).
- The increase of the internal resistance is equal to 0.0125 Ω for 1% of capacity loss.
Using these information, the evolution of the capacity and the resistance per Ah can be found and it is equal to 0.0166% /Ah and 2.08×10^{-4} ohm/Ah respectively.

III. ADAPTIVE OBSERVER

In this application, an observer is used for two main reasons. The first is to estimate the states of the system and eventually the *SoC* and the second is to estimate the internal resistance R_0, which is considered as unknown parameter in the output equation. The estimated value of R_0 will be used to predict the *SoH* of the battery (see Section IV).

A. Observer statement

The adaptive observer used in this study is the one stated in [16], where the unknown parameters are presented in both state and output equations. For a LTV system defined as:

$$\dot{x}(t) = A(t)x(t) + B(t)U(t) + \varphi(t)\theta(t) \tag{7a}$$
$$y(t) = C(t)x(t) + D(t)U(t) + \psi(t)\theta(t) \tag{7b}$$

the adaptive observer is given by:

$$\begin{aligned}
\dot{\Upsilon}(t) &= [A(t) - K(t)C(t)]\Upsilon(t) \\
&\quad + \varphi(t) - K(t)\psi(t)
\end{aligned} \tag{8a}$$

$$\begin{aligned}
\dot{\hat{x}}(t) &= A(t)\hat{x}(t) + B(t)U(t) + \varphi(t)\theta(t) \\
&\quad + [K(t) + \Gamma\Upsilon(t)(C(t)\Upsilon(t) + \psi(t))] \times \\
&\quad [y(t) - C(t)\hat{x}(t) - D(t)U(t) - \psi(t)\hat{\theta}(t)]
\end{aligned} \tag{8b}$$

$$\begin{aligned}
\dot{\hat{\theta}}(t) &= \Gamma[C(t)\Upsilon(t) + \psi(t)] \times \\
&\quad [y(t) - C(t)\hat{x}(t) - D(t)U(t) - \psi(t)\hat{\theta}(t)]
\end{aligned} \tag{8c}$$

where $\hat{x}(t)$ is the state estimation and $\hat{\theta}(t)$ is the parameter estimation. The gain matrix $K(t)$ is found by solving the riccati equation:

$$\begin{aligned}
\dot{P}(t) &= A(t)P(t) + P(t)A(t)^T \\
&\quad - P(t)C(t)^T V^{-1} C(t)P(t) + W
\end{aligned} \tag{9a}$$

$$K(t) = P(t)C(t)^T V^{-1} \tag{9b}$$

As shown in the equations, all the parameters are time dependent, except for Γ, V and W which are some tuning parameters.

B. Battery model

To implement the adaptive observer stated above, the matrices $A(t), B(t), C(t), D(t)$, $\varphi(t)$ and $\psi(t)$ should be specified. In the simplified model, the relation between V_{oc} and SoC is linear as in (5), thus (2d) can be reformulated as follows:

$$y = V_{oc} - x_1 - x_2 - R_0 U = V_{oc,nom} + \alpha SoC - x_1 - x_2 - R_0 U \tag{10}$$

Since $V_{oc,nom}$ is a constant value, it can be extracted from (10) and the model output equation will be:

$$y = -x_1 - x_2 + \alpha SoC - R_0 U \tag{11}$$

Based on (2) and (7), and under the consideration that the only unknown parameter is the ohmic resistance R_0 (i.e. $\theta = R_0$), the following matrices can be defined:

$$A(t) = \begin{bmatrix} \dfrac{1}{R_s(t)C_s(t)} & 0 & 0 \\ 0 & \dfrac{1}{R_l(t)C_l(t)} & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad B(t) = \begin{bmatrix} \dfrac{1}{C_s(t)} \\ \dfrac{1}{C_l(t)} \\ \dfrac{\eta}{C_{batt}} \end{bmatrix};$$

$$\varphi(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \quad C(t) = \begin{bmatrix} -1 & -1 & \alpha \end{bmatrix}; \quad D(t)=0; \quad \psi(t) = -U;$$

The matrices $A(t)$ and $B(t)$ are time-varying since R_s, C_s, R_l and C_l depend on the temperature T, the SoC and the current I. C is a constant matrix and $\psi(t)$ depends only on the input $U(t) = I(t)$.

C. Implementation: a two-mode switching observer

Due to the discontinuity in the value of R_0 between the charging and the discharging phase, both R_0 and SoC estimations could suffer a transient behavior when the direction of the current changes. To account for such a discontinuity, two observers are designed (one for the charging phase and one for the discharging one). Then a switching strategy between both observers is proposed keeping the continuity in the state estimation. When the current direction changes, the active observer will update its state from the last one estimated by the second observer while the parameter estimation will start from the last estimated value saved by the active observer. This method is illustrated in Figure 3. It worth noting that h, i, j, k and l denote the switching instants. The interest of this approach is not only to eliminate the peaks in the estimation when the current direction changes, but also to enhance the overall performance of the system and hence to better estimate the SoC and R_0. When fast switching between charging and discharging occurs, the system tends to converge faster since the required time to converge toward the new value of R_0 is heavily reduced thanks to the switching technique.
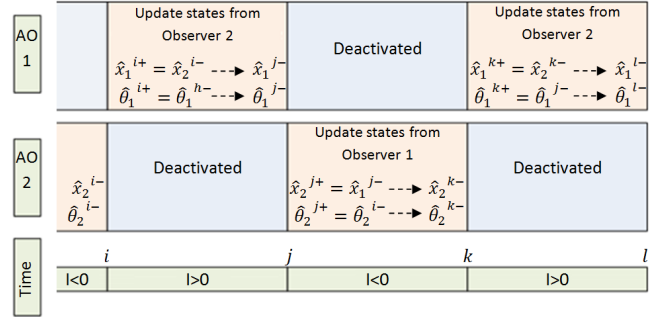

Fig. 3: Observer's implementation approach

D. Tuning parameters

In (8) and (9), the choice of V and W affects the value of the gain matrix $K(t)$ and Γ affects the convergence speed and the accuracy of the estimation. The values of the tuning parameters are found by trial and error. It was noticed that the solution of system (9) is almost constant thus there is no need to resolve the riccati equation in each iteration which will reduce the computational complexity of the algorithm. After several attempts, the chosen values for the tuning parameters are: $\Gamma = 500, V = 1$, and $W = diag[1, 1, 2500]$. These values will lead to a constant gain matrix $K = [-0.07, -0.07, 50]$.

IV. ANALYSIS AND SIMULATION

First the adaptive observer is tested under a constant temperature and constant charging/discharging current. The simulation results show an estimation error on SoC less than 5% while the estimation error on R_0 is very small as shown in Figure 4. The error on the SoC is considered as absolute error while the error on R_0 is considered as relative error:

$$\Delta SoC = SoC_{real}\% - SoC_{estimated}\% \tag{12}$$

$$\Delta R_0 = \frac{R_{0,real} - R_{0,estimated}}{R_{0,real}} \tag{13}$$
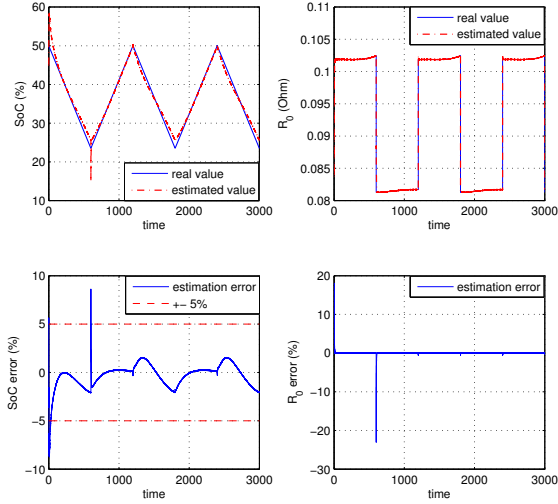
Fig. 4: *SoC* and R_0 estimation (up), ΔSoC and ΔR_0 (down)

The study of the *SoH* will be based on the estimated value of R_0. The method used is the one stated in [9], [14]:

$$SoH = \frac{R_{0,EoL} - R_0}{R_{0,EoL} - R_{0,BoL}} \times 100\% \quad (14)$$

$R_{0,EoL}$ is the ohmic resistance of a dead battery. It is a threshold chosen by the manufacturer and it depends on the battery's type and size. $R_{0,BoL}$ is the ohmic resistance of a new battery and R_0 is the current battery's resistance. Using Table I, a new model is built to test the performance of the observer and to check if it is able to track the change of R_0 with cycling. In the simulation, the ageing process is accelerated 100 times. It means that the values of R_0 and C_{batt} of the model will vary linearly with time and reach their threshold values after 12 Ah. In Figure 5, it is clear that the observer is able to track the change of R_0 with time which indicates that the *SoH* of the battery can be estimated using (14). The estimated *SoH* is shown in Figure 6 and indicates the evolution of the health of the battery with time, when a square wave signal (constant charging/discharging) is applied to the battery with a total charge equal to 6 Ah (i.e. *SoH* should be equal to 50%).
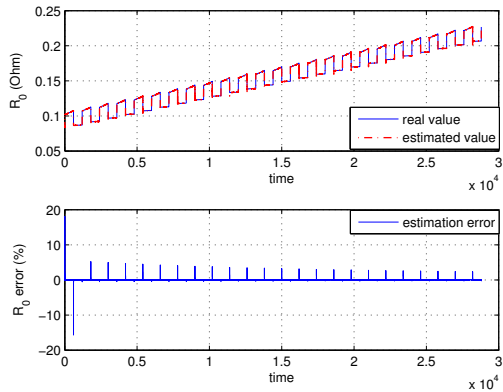

Fig. 5: R_0 estimation with ageing (accelerated 100 times)

Other tests were performed to check the performance of the observer. Some of them consider different cur-
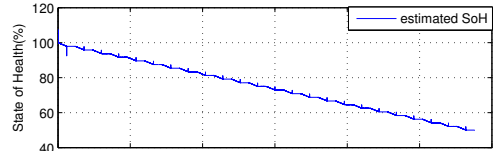

Fig. 6: State of Health estimation with ageing accelerated 100 times

rent/temperature shapes while others introduce some uncertainty on the measured temperature and the real capacity value. As results, the observer showed that a maximum absolute estimation error of 3.5% on the *SoC* estimation can be achieved even when certain uncertainties are introduced in the system. In addition, the adaptive observer is able to track the variation of R0 due to ageing, SoC and temperature variations. Even when uncertainties are introduced on the temperature and on the capacity, the observer seems to be robust and it is able to estimate the value of R_0 with a very small error which leads to good estimation of the *SoH*.

V. TEST WITH EXPERIMENTAL DATA

In this part, experimental current profiles are used to demonstrate the adaptive observer performance. These data represent a charging/discharging driving cycle for an electrical vehicle available at the Politecnico di Milano and it is shown in Figure 7. It is scaled by 0.1 from the original data to avoid over charging and discharging. Figure 8 presents the results obtained regarding the *SoC* and R_0 estimation.
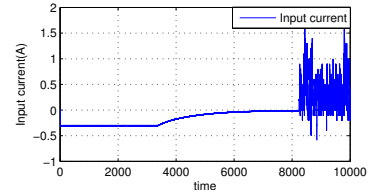

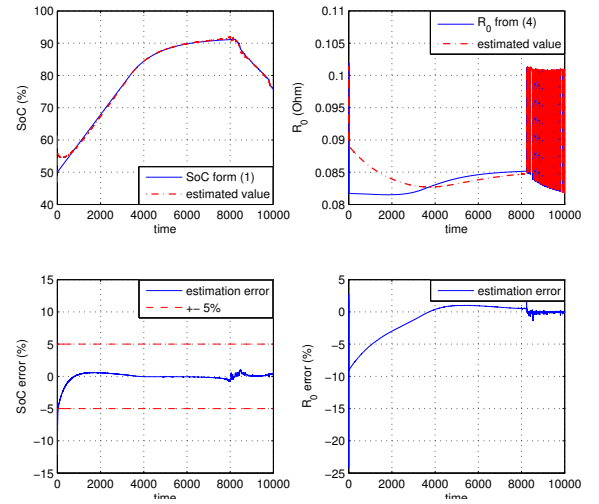Fig. 7: Charging/discharging driving cycle

Fig. 8: Validation of the proposed observer

As in Section IV, the adaptive observer showed is ability to estimate the *SoC* and this time for an absolute error less than 2%. Also, the observer was able to track the change of R_0 even when fast switching between charging and discharging

occurs. This is due to the implementation method proposed in this work where a switching between two different observer takes place to compensate the transients that can appear due to the discontinuity of R_0.

VI. CONCLUSION AND FUTURE WORKS

In this work, the states of an Li-ion battery model and the battery's internal variable resistance have been estimated using an adaptive observer. This method does not require any iterative calculation for the feedback gain and it provides an interesting alternative to the Kalman Filter. In addition, the advantage of this method is that it can be used for joint estimation of the states and the unknown parameters. The estimated values are used to evaluate the State of Charge and the State of Health of the battery.

Under different conditions, the simulations show that the error on the *SoC* estimation was less that 3.5%. On the other hand, the estimation of R_0 is very precise and the error is always less than 0.5% (except during the transitory phase between charge and discharge). Thus a *SoH* prediction can be made based on the internal resistance, if the thresholds of the End of Life are well defined. An implementation method is proposed to eliminate some undesired transitory behavior because of the discontinuity of the system. The method suggests a switching between two observers depending on the current direction. This method enhances the estimation of R_0 and minimizes the transitory peaks that could appear in the *SoC* estimation. The observer was tested under realistic current profiles and the results are very satisfactory.

As future work, comparative studies between this method and other ones stated in the literature can be done. In addition, this method can be tested for other driving cycles available from the MoVE team which represents different charging/discharging routines which can happen in an EV. Finally, real-time implementation can be considered.

REFERENCES

[1] M. Charkhgard and M. Farrokhi. State-of-charge estimation for lithium ion batteries using neural networks and ekf. *IEEE Transaction ON Industrial Electronics*, 57(12):41784187, December 2010.

[2] N. Chaturvedi, R. Klein, J. Christensen, J. Ahmed, and A. Kojic. Algorithms for advanced battery-management systems. *IEEE Control Systems Magazine*, 30(3):49 – 68, June 2010.

[3] M. Corno, N. Bhatt, S. M. Savaresi, and M. Verhaegen. Electrochemical model based state of charge estimation for li-ion cells. *IEEE Transaction on Control Systems Technology*, issue 99, 2014.

[4] M. Corno, N. Bhatt, and M. Verhaegen. Efficient control oriented modeling of lithium ion cells. *Proceedings of the 2012 American Control Conference*, pages 4733–4738, June.

[5] M. Corno and S.M. Savaresi. A diffusive electro-equivalent li-ion battery model. *Proceeding of the (2013) IEEE international conference on circuits and systems*, pages 2976 – 2979, May 2013.

[6] D. Doerffel and S. A. Sharkh. A critical review of using the peukert equation for determining the remaining capacity of lead-acid and lithium-ion batteries. *Journal of Power Sources*, 155(2):395 400, 2006.

[7] D.D. Domenico, A. Stefanopoulou, and G. Fiengo. Lithium-ion battery state of charge and critical surface charge estimation using an electrochemical model-based extended kalman filter. *Journal of Dynamic Systems, Measurement, and Control*, 132, 2010.

[8] F. Fuller, M. Doyle, and J. Newman. Simulation and optimization of the dual lithium ion insertion cell. *J. Electrochem. Soc.*, 141(1):1 – 10, 1994.

[9] M .Gholizadeh and F. R. Salmasi. Estimation of state of charge, unknown nonlinearities, and state of health of a lithium-ion battery based on a comprehensive unobservable model. *IEEE Transactions on Industrial Electronics*, 61(3):1335–1344, 2014.

[10] W. Gu and C. Wang. Thermal and electrochemical coupled modeling of a lithium-ion cell, in lithium batteries. *J. Electrochem. Soc.*, 147(8):29102922, 2000.

[11] Z. Guo, X. Qiu, G. Hou, B. Y. Liaw, and C. Zhang. State of health estimation for lithium ion batteries based on charging curves. *Journal of Power Sources*, 249(0):457 – 462, 2014.

[12] X. Hu, F. Sun, and Y. Zou. Estimation of state of charge of a lithium-ion battery pack for electric vehicles using an adaptive luenberger observer. *Energies*, 2010.

[13] I.-S. Kim. The novel state of charge estimation method for lithium battery using sliding mode observer. *Journal of Power Sources*, 163(1):584 – 590, 2006.

[14] J. Kim and B.-H. Cho. State-of-charge estimation and state-of-health prediction of a li-ion degraded battery based on an ekf combined with a per-unit system. *IEEE T. Vehicular Technology*, 60(9):4249–4260, 2011.

[15] L. Lam, P. Bauer, and E. Kelder. A practical circuit-based model for li-ion battery cells in electric vehicle applications. *Telecommunications Energy Conference (INTELEC), IEEE 33rd International*, pages 1 – 9, October 2011.

[16] X. Li, Q. Zhang, and H. Su. An adaptive observer for joint estimation of states and parameters in both state and output equations. *International Journal of Adaptive Control and Signal Processing*, 25(9):831–842, April 2011. 9 9.

[17] L. Lu, X. Han, J. Li, J. Hua, and M. Ouyang. A review on the key issues for lithium-ion battery management in electric vehicles. *Journal of Power Sources*, 226:272 – 288, March 2013.

[18] S. Moura, N. Chaturvedi, and M. Krstic. Adaptive partial differential equation observer for battery state-of-charge/state-of-health estimation via an electrochemical model. *Journal of Dynamic Systems, Measurement, and Control*, 136, January 2014.

[19] S. J. Moura, N. A. Chaturvedi, , and M. Krstic. Pde estimation techniques for advanced battery management systems - part ii: Soh identification. *Proceedings of the 2012 American Control Conference*, page 566571, June 2012.

[20] S. Onori, P. Spagnol, V. Marano, Y. Guezennec, and G. Rizzoni. A new life estimation method for lithiumion batteries in plugin hybrid electric vehicles applications. *International Journal of Power Electronics*, pages 302–319, May 2012.

[21] G. L. Plett. Extended kalman filtering for battery management systems of lipb-based hev battery packs: Part 3. state and parameter estimation. *Journal of Power Sources*, 134(2):277292, 2004.

[22] A. J. Salkind, C. Fennie, P. Singh, T. Atwater, and D. E. Reisner. Determination of state-of-charge and state-of-health of batteries by fuzzy logic methodology. *Journal of Power Sources*, 80:293 300, 1999.

[23] S. Santhanagopalan, Q. Guo, P. Ramadass, and R.E. White. Review of models for predicting the cycling performance of lithium ion batteries. *Journal of Power Sources*, 156(2):620–628, 2006.

[24] S. Santhanagopalan, Q. Zhang, K. Kumaresan, and R. E. White. Parameter estimation and life modeling of lithium-ion cells. *Journal of The Electrochemical Society*, 155(4).

[25] K. Smith, C. Rahn, , and C.-Y. Wang. Model-based electrochemical estimation and constraint management for pulse operation of lithium ion batteries. *IEEE Transactions on Control Systems Technology*, 18(3):654663, May 2010.

[26] K. A. Smith, C. D. Rahn, and C.-Y. Wang. Control oriented 1d electrochemical model of lithium ion battery. *Energy Conversion and Management*, 48(9):25652578, 2007.

[27] C. Unterrieder, R. Priewasser, S. Marsili, and M. Huemer. Battery state estimation using mixed kalman/h-infinity, adaptive luenberger and sliding mode observer. In *Proceedings of the IEEE Vehicle Power and Propulsion Conference (VPPC 2013)*, pages 71–76, 10 2013.

[28] W. Xuezhe and S. Zechang. A new soh prediction concept for the power lithium-ion battery used on hevs. *Vehicle Power and Propulsion Conference*, page 16491653, 2009.

[29] Z. Yang, K. Li, A. Foley, and C. Zhang. A new self-learning tlbo algorithm for rbf neural modelling of batteries in electric vehicles.

[30] C. Zhang, Z. Yang, and K. Li. Modeling of electric vehicle batteries using rbf neural networks. *International Conference on Computing, Management and Telecommunications*, pages 116 – 121, April 2014.